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# TESTS OF BOND BETWEEN CONCRETE AND STEEL

BY

DUFF A. ABRAMS

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 71 DECEMBER, 1913

# TESTS OF BOND BETWEEN CONCRETE AND STEEL

BY DUFF A. ABRAMS, ASSOCIATE IN THEORETICAL AND APPLIED MECHANICS

## CONTENTS

### I. INTRODUCTION

Page
1. Preliminary ........ 7
2. Scope of Bulletin ........ 7
3. Acknowledgment ........ 9

### II. MATERIALS, TEST PIECES AND METHODS OF TESTING

4. Concrete Materials ........ 9
5. Concrete ........ 11
6. Reinforcing Steel ........ 13
7. Pull-out Specimens ........ 13
8. Reinforced Concrete Beams ........ 15
9. Auxiliary Test Specimens ........ 17
10. Storage of Test Specimens ........ 17
11. Method of Making Pull-out Tests ........ 17
12. Method of Making Beam Tests ........ 19
13. Auxiliary Tests ........ 20

### III. EXPERIMENTAL DATA AND DISCUSSION

14. Preliminary Discussion of the Nature of Bond Resistance.... 20
15. Strength of Concrete ........ 22

### A. PULL-OUT TESTS

16. Classification of Pull-out Tests ........ 25
17. Stresses and Deformations in a Pull-out Specimen ........ 26
18. Phenomena of Pull-out Tests ........ 28

*a. Effect of Variations in the Dimensions of Pull-out Specimens*

Page
19. Effect of Size of Plain Bars; Embedment Variable.......... 31
20. Effect of Size of Corrugated Bars; Embedment Variable...... 35
21. Effect of Size of Plain Bars; Embedment 8 in.............. 37
22. Effect of Length of Embedment; 1¼-in. Plain Rounds....... 37
23. Effect of Length of Embedment; 1⅛-in. Corrugated Rounds.. 42
24. Effect of Variation in Diameter of the Concrete Block........ 45

*b. Effect of Shape of Section and Condition of Surface of Bar*

25. Bond Resistance with Rusted Bars........................ 47
26. Bond Resistance with Flat Bars.......................... 49
27. Bond Resistance with T-Bars............................ 49
28. Bond Resistance with Polished Round Bars................ 50
29. Tests with Tapered Bars................................ 51

*c. Effect of Condition of Storage*

30. Preliminary ............................................ 53
31. Batches 3 and 4......................................... 53
32. Batch 35 ............................................... 57
33. Batch 41 ............................................... 58
34. Batch 39 ............................................... 59
35. Discussion of Effect of Storage......................... 59

*d. Bond Tests with Deformed Bars*

36. Preliminary ............................................ 60
37. Phenomena of Pull-out Tests with Deformed Bars.......... 60
38. Basis of Comparison of the Bond Resistance of Deformed Bars. 65
39. Discussion of Deformed Bar Tests........................ 66
40. Threaded Bars .......................................... 72
41. Twisted Square Bars .................................... 73
42. Elements of the Bond Resistance of Deformed Bars......... 76

*e. Effect of Age and Mix*

43. Preliminary ............................................ 81
44. Effect of Age on Bond Resistance ....................... 84
45. Effect of Mix on Bond Resistance........................ 90
46. Mixes of the Form 1-2-*m*................................. 90
47. Mixes of the Form 1-*n*-2*n*................................ 90
48. Relation of Bond Resistance to the Compressive Strength of Concrete ........................................... 94
49. Effect of Age on the Compressive Strength of Concrete....... 95

*f. Effect of Anchoring Ends of Bars*

Page
50. Preliminary .......... 97
51. Bars Anchored with Nuts and Washer.......... 97
52. Bars Anchored by Means of Hooks and Bends.......... 99
53. Discussion of Tests with Anchored Bars..........100

*g. Miscellaneous Tests*

54. Preliminary ..........101
55. Repeated Loads on Pull-out Specimens..........101
56. Effect of Distribution of Load on the Face of the Concrete Block ..........104
57. Effect of Position of Bar During Molding..........104
58. Effect of Method of Applying Load in Pull-out Tests..........105
59. Autogenous Healing in Concrete..........107
60. Effect of Loads Reapplied after Failure of Bond..........108
61. Bond Resistance of Concrete which Set under Pressure..........115
62. Compression Tests of Concrete which Set under Pressure..........117
63. Effect of Loads Reapplied after Failure in Compression..........121
64. Effect of Reinforcing the Pull-out Specimen against Bursting.122

B. REINFORCED CONCRETE BEAM TESTS

65. Preliminary ..........124
66. Measurements of Slip of Bar..........124
67. Computation of Stresses in a Reinforced Concrete Beam..........125
68. Phenomena of Beam Tests..........126

*a. 1909 Beam Tests*

69. Outline of Series ..........130
70. Bond with Plain Round Bars..........131
71. Bond with Deformed Bars ..........131

*b. 1911 Beam Tests*

72. Outline of Series ..........133
73. Basis of Comparison of Bond Resistance in Beams and in Pull-out Tests ..........133
74. Bond with Plain Round Bars..........140
75. Effect of Vertical Stirrups on Bond Resistance..........142
76. Effect of Depth of Concrete Below the Reinforcement..........143
77. Effect of Span Length..........144
78. Effect of Length of Overhang of Ends of Beams..........145
79. Bond with Corrugated Bars..........147
80. Bond Resistance in 1911 Beam and Pull-out Tests..........149

### *c. 1912 Beam Tests*

Page
81. Outline of Series .......................................150
82. Bond with Plain Round Bars...............................150
83. Bond with Plain Square Bars..............................151
84. Bond with Twisted Square Bars............................159
85. Bond with Corrugated Bars................................159
86. Effect of Span Length....................................163
87. Effect of Position of Loads on Beams.....................164
88. Effect of Auxiliary Reinforcement in Ends of Beam........165
89. Effect of Variation in Number and Size of Reinforcing Bars...166
90. Effect of Repeated and Continued Loads on Beams..........168
91. Pull-out Tests with 1912 Beam Series.....................173
92. Bond Resistance in 1912 Beam and Pull-out Tests..........174
93. Variation in Results of Cube and Pull-out Tests..........175

### *d. Discussion of Reinforced Concrete Beam Tests*

94. Slip of Reinforcing Bar at Internal Points in Beams......177
95. Distribution of Bond and Tensile Stress along Bar........191
96. Relation of Slip of Bar to Diagonal Tension Cracks.......202
97. Influence of Slip of Bar upon Beam Deflection............204
98. Critical Bond Stress in Reinforced Concrete Beams........205
99. Working Stresses for Bond................................205

## IV. SUMMARY

100. Summary ...............................................207
101. Concluding Remarks ....................................220

## LIST OF TABLES

Page
1. Briquette Tests of Cements.......................................... 10
2. Mechanical Analysis of Sand.......................................... 12
3. Mechanical Analysis of Stone.......................................... 12
4. Record of Batches of Concrete Used in 1909 Pull-out Tests.......................... 23
5. Compressive and Flexural Strength of Concrete.......................................... 24
6. Classification of Pull-out Tests.......................................... 26
7. Effect of Size of Bar; Embedment Variable.......................................... 31
8. Effect of Size of Plain Round Bars; Embedment 8 in.......................................... 36
9. Effect of Length of Embedment.......................................... 39
10. Effect of Variation in Diameter of Concrete Block.......................................... 46
11. Effect of Shape of Section and Condition of Surface of Bar.......................................... 48
12. Effect of Condition of Storage.......................................... 54
13. Notes on Deformed Bars Used in 1909 Pull-out Tests.......................................... 62
14. Pull-out Tests with Deformed Bars (1909).......................................... 68
15. Bearing Stresses Developed by Deformed Bars.......................................... 79
16. Effect of Age and Mix.......................................... 82
17. Comparative Values for Bond on Plain Round Bars.......................................... 89
18. Effect of Varying Quantity of Cement and Mortar.......................................... 92
19. Effect of Anchoring Ends of Bars.......................................... 96
20. Miscellaneous Pull-out Tests .......................................... 103
21. Effect of Loads Reapplied after Failure of Bond.......................................... 110
22. Bond Resistance of Concrete which Set under Pressure.......................................... 116
23. Compression Tests of Concrete which Set under Pressure.......................................... 119
24. Effect of Reinforcing the Concrete Block against Splitting.......................................... 123
25. Data of Reinforced Concrete Beams—1909 Series.......................................... 130
26. Tests of Reinforced Concrete Beams—1909 Series.......................................... 131
27. Data of Reinforced Concrete Beams—1911 Series..........................................134, 136
28. Tests of Reinforced Concrete Beams—1911 Series..........................................135, 137
29. Comparison of Bond Resistance in Beams and Pull-out Tests—1911 Series........ 138
30. Summary of Bond Stresses in Beam and Pull-out Tests—1911 Series............ 141
31. Data of Reinforced Concrete Beams—1912 Series..........................................152, 154
32. Tests of Reinforced Concrete Beams—1912 Series..........................................153, 155
33. Comparison of Bond Resistance in Beams and Pull-out Tests—1912 Series....... 156
34. Summary of Bond Stresses in Beam and Pull-out Tests—1912 Series........... 160
35. Bond Resistance of 6-ft. Beams Reinforced with Plain Round Bars............ 162
36. Bond Resistance with Twisted Square, Square, and Round Bars................ 163
37. Variation in the Compressive Strength of 6-in. Concrete Cubes................ 176
38. Variation of Values of Bond Resistance in Pull-out Tests.......................... 176
39. Applied Load at First Slip of Bars at Intermediate Points in Reinforced Concrete Beams .......................................... 180
40. Distribution of Bond Stress in Reinforced Concrete Beams.......................... 193

## LIST OF FIGURES

1. Types of Pull-out Speciments Used in the Tests.......................................... 14
2. Typical Forms of Reinforced Concrete Beams Used in the Tests............... 16
3. Pull-out Specimen in Machine Ready for Test.......................................... 18
4. Diagram Showing Principal Stresses in a Pull-out Specimen.................. 27
5. Load-Slip Curves after Eliminating Size of Bar and Length of Embedment...... 29
6. Load-Slip Curves from Pull-out Tests with Plain Round Bars.................. 30
7. Load-Slip Curves for Plain Round Bars of Different Size.................. 32
8. Bond Resistance of Plain Round Bars; Embedment Variable.................. 33
9. Bond Resistance of Corrugated Bars; Embedment Variable.................. 36
10. Bond Resistance of Plain Round Bars of Different Size; Embedment 8 in...... 38
11. Load-Slip Curves for 1¼-in. Plain Round Bars; Embedment Variable........... 40
12. Bond Resistance of 1¼-in. Plain Round Bars; Embedment Variable........... 41

Page
13. Load-Slip Curves for 1⅛-in. Corrugated Round Bars; Embedment Variable..... 43
14. Bond Resistance of 1⅛ in. Corrugated Round Bars; Embedment Variable........ 44
15. Load-Slip Curves for ¾-in. Plain Round Bars in Cylinders of Different Diameters 46
16. Bond Resistance of ¾-in. Plain Round Bars in Cylinders of Different Diameters 47
17. Load-Slip Curves for Plain Round, Flat and T-Bars.......................... 49
18. Bond Resistance with Polished Bars of Wedging Taper....................... 52
19. Load-Slip Curves for Different Conditions of Storage...................... 56
20. Maximum Bond Resistance for Specimens Stored in Air and in Water.......... 58
21. Deformed Bars Used in Pull-out Tests....................................... 61
22. Composite Load-Slip Curve from Pull-out Tests with Deformed Bars........... 64
23. Load-Slip Curves from Pull-out Tests with Deformed Bars.................... 70
24. Load-Slip Curves from Pull-out Tests with Deformed Bars (Enlargement of a Portion of Fig. 23)............................................ 71
25. Relative Values of Bond Resistance of Deformed Bars........................ 72
26. Load-Slip Curves for Twisted Square Bars................................... 75
27. Effect of Age and Mix on Bond Resistance of Plain Round Bars............... 84
28. Load-Slip Curves for Plain Round Bars Embedded in 1-2-4 Concrete........... 85
29. Effect of Age and Mix on Bond Resistance of Corrugated Bars................ 87
30. Load-Slip Curves for Corrugated Bars Embedded in 1-2-4 Concrete............ 88
31. Load-Slip Curves for Plain Round Bars...................................... 91
32. Bond Resistance for Different Mixes........................................ 93
33. Relation of Bond Stress at End Slip of 0.0005 in. to the Compressive Strength of 6-in. Concrete Cubes............................................. 94
34. Relation of Maximum Bond Resistance to the Compressive Strength of 6-in. Concrete Cubes................................................... 95
35. Effect of Age on the Compressive Strength of Concrete...................... 96
36. Forms of Pull-out Specimens with Anchored Bars............................ 97
37. Load-Slip Curves for Bars Anchored with Nuts and Washers................... 98
38. Load-Slip Curves for Repeated Loads on Pull-out Specimens.................. 102
39. Load-Slip Curves for Repeated Loads on Pull-out Specimens.................. 102
40. Load-Slip Curves for Miscellaneous Pull-out Specimens...................... 104
41. Load-Slip Curves for Specimens of the Form Shown........................... 106
42. Load-Slip Curves for Specimens of the Form Shown........................... 107
43. Load-Slip Curves for Loads Reapplied after Failure of Bond................. 112
44. Load-Slip Curves for Loads Reapplied after Failure of Bond................. 113
45. Load-Slip Curves for Miscellaneous Pull-out Specimens...................... 117
46. Stress-Deformation Curves for 8 by 16-in. Concrete Cylinders............... 120
47. Effect of Pressure During Setting on the Properties of Concrete............ 122
48. Effect of Length of Overhang of Ends of Beams.............................. 146
49. Comparison of Bond Stresses in Beams and Pull-out Specimens—1911 Series..... 148
50. Effect of Position of Loads on Reinforced Concrete Beams................... 165
51. Load-Slip and Load-Deflection Curves for a Beam under Repeated Loads....... 169
52. Slip of Bar in Beams under Constant Loads.................................. 170
53. Slip of Bar and Center Deflection of Beam No. 1055.8....................... 171
54. Load-Slip Curves for Beam No. 1055.8....................................... 172
55. Load-Slip Curves for Pull-out Tests from 1912 Beam Series.................. 174
56. Reinforced Concrete Beams in Machine Ready for Test........................ 183
57-63. Reinforced Concrete Beams after Test...............................184 to 190
64. Diagrams from Test of Beam No. 1055.6. One 1-in. Plain Round Bar; Span 10 ft. 194
65. Distribution of Tensile Stress in Bar in Beam No. 1055.3. One 1-in. Plain Round Bar; Span 10 ft............................................. 197
66. Diagrams from Test of Beam No. 1049.3. One 1⅛-in. Corrugated Round Bar; Span 10 ft..................................................... 198
67. Diagrams from Test of Beam No. 1049-3. One 1⅛-in. Corrugated Round Bar; Span 10 ft..................................................... 199
68. Idealized Diagram Showing the Distribution of Tensile and Bond Stresses in a Simple Reinforced Concrete Beam.................................... 203
69-76. Load-Slip Curves for Bars in Reinforced Concrete Beams..............221 to 228
77-86. Load-Deflection and End-Slip Curves for Reinforced Concrete Beams......229 to 238

# TESTS OF BOND BETWEEN CONCRETE AND STEEL

## I. Introduction.

1. *Preliminary.*—The usefulness of reinforced concrete as a structural material depends on the strength and permanency of the bond between the concrete and the reinforcing metal, and for this reason bond resistance has received much attention from engineers and experimenters. It is said that Thaddeus Hyatt made tests to determine the bond between concrete and iron bars as early as 1876. During the past decade numerous bond tests have been reported. These tests have been characterized by a lack of uniformity in the form of the test specimen and in the methods of conducting the tests, as well as by the wide variations in the values reported for bond resistance. In nearly all the tests thus far published values of maximum bond resistance only have been given. These test results and the discussions called forth by them have furnished the basis for a great variety of opinions as to the value of bond resistance. Many explanations of the source and nature of bond resistance have been given. Various methods have been advocated for increasing bond resistance and numerous devices have been employed for this purpose.

Present American practice is fairly standardized as to the bond stresses to be used in designing, but a rational basis for the stresses used is lacking and there is a great diversity of practice in the methods of calculating these stresses. There are many phases of bond action which are not now understood. It is evident that the distribution of bond stress in reinforced concrete members under load and the nature and value of bond resistance under given conditions may well be the subject of experimental investigation.

2. *Scope of Bulletin.*—The tests reported in this bulletin were undertaken with a view to securing additional information on the nature of the bond resistance of reinforcing bars in concrete, to determining values of bond resistance for a wide range of conditions, and to studying bond

action in specimens of different forms. Tests were made on pull-out specimens and on reinforced concrete beams. In both forms of specimen attention was given to obtaining accurate measurement of the slip of bar through the concrete as the loading progressed. In many of the beam tests the slip of bar at various points along its length was measured for different loads. In the discussion of bond resistance the load-slip-of-bar relation has been utilized to a considerable extent. These measurements are useful in indicating the distribution of bond stress. They are particularly significant in the beam tests. In a few of the beam tests the distribution of bond stress was studied by measuring the changes in the stress in the longitudinal reinforcement throughout the length. The values found for bond resistance and the relative bond resistance found in beam tests and pull-out specimens are also interesting features of the investigation.

The pull-out tests consisted in applying load to a short reinforcing bar embedded in a block of concrete. The concrete block was generally 8 in. in diameter and 8 in. long, with the bar embedded axially. In certain groups of tests these dimensions were varied. The size of bar used varied between ¼ in. and 1¼ in. The pull-out tests covered a wide range and included effect of dimensions of specimen, effect of form of bar, effect of conditions of storage, effect of age and mix, using both plain and deformed bars, effect of different methods of loading, bond resistance of concrete setting under pressure, effect of reapplied loads, comparison with the bond resistance of reinforced concrete beams, etc. The deformed bars used included most of the forms in use at the time the work was begun, but it should be noted that the tests with deformed bars were intended to bring out the action of the deformed bar as contrasted with the plain bar and not to determine the value of particular forms of bars.

A special effort was made to determine the behavior of beams subjected to high bond stresses. The beams tested were 8 by 12 in. in section with an effective depth of 10 in. The span length was generally 6 ft.; a few beams were tested with span lengths of 5 to 10 ft. All beams were tested with two symmetrical loads, generally at the one-third points of the span. With the exception of six tests, the longitudinal reinforcement consisted of a single bar of large diameter placed horizontally throughout the length of the beam. Both plain and deformed bars were used.

3. *Acknowledgment.*—The tests reported herein were made in the Laboratory of Applied Mechanics of the University of Illinois and formed a part of the investigations of reinforced concrete and other structural materials which are being conducted by the Illinois Engineering Experiment Station. These tests cover the experiments which were designed with special reference to a study of bond between concrete and steel during the period 1909-1912. The work was done under the direction of A. N. Talbot, Professor in Charge of the Department of Theoretical and Applied Mechanics. The writer is indebted to Professor Talbot for many helpful suggestions in planning the tests and in interpreting the data.

In the 1912 beam series the work of testing was done under the writer's supervision by Messrs. W. W. Manspeaker and A. W. Wand, senior civil engineering students of the class of 1912. These tests furnished the subject-matter of their baccalaureate thesis, where a very creditable report on the tests was presented. These men exercised great care in carrying out the routine of the work and they are to be commended for the way in which they met the demands of an unusually arduous program of tests. The other tests were made by the writer with the assistance of various members of the Laboratory staff.

## II. MATERIALS, TEST PIECES AND METHODS OF TESTING.

4. *Concrete Materials.*—The materials were the same as used for other concrete and reinforced concrete specimens made and tested by the Engineering Experiment Station during the period 1909 to 1912. The quality of the materials may be considered as representative of that used in first-class concrete work in the central states.

Cement. Most of the test specimens were made with Universal portland cement, which was furnished by the manufacturers. Chicago AA portland cement was used in the 1909 series of beam tests and in Batch No. 4 of the 1909 pull-out tests. Lehigh cement was used in part of the 1911 beam series. The Chicago AA and Lehigh cements were purchased from a local dealer. The results of briquette tests of these cements are given in Table 1. Samples were taken at intervals throughout the season. Each value given is the average of five briquette tests. Vicat needle tests on the three samples of the 1909 lot of Universal cement showed initial set to occur at 1 hr. 45 m., and final set at 3 hr. 45 m. after mixing. Sieve analysis showed 96.5% passing a No. 100 sieve and 81.9% passing a No. 200 sieve. The 1912 lot of Universal cement (7 samples) gave the following average values: initial set, 3 hr. 5 m., final set, 6 hr. 32 m.; 97.2% passing a No. 100 sieve and 81.8% passing a No. 200 sieve. All cement tests recorded in this bulletin were made by Mr. B. L. Bowling, Assistant in Charge of Cement Laboratory, University of Illinois.

## TABLE 1.

### Briquette Tests of Cements.

Each value is the average of five tests.
Unless otherwise noted, standard Ottawa sand was used in the 1–3 briquettes.
The results are expressed in pounds per square inch.

| Sample No. | Neat | | 1–3 | | Neat | | 1–3 | |
|---|---|---|---|---|---|---|---|---|
| | 7 days | 28 days | 7 days | 28 days | 7 days | 28 days | 7 days | 28 days |
| | Universal Cement | | (1909) | | Chicago AA Cement. | | | |
| 1 | 607 | 732 | 197 | 281 | 742 | 783 | 205 | 270 |
| 2 | 595 | 772 | 179 | 280 | 716 | 807 | 232 | 306 |
| | | | | | | | 288* | 331* |
| 3 | 617 | 853 | 160 | 278 | 725 | 768 | 176 | 254 |
| Average | 606 | 786 | 179 | 280 | 728 | 786 | 204 | 277 |
| | Universal Cement | | (1911) | | Lehigh Cement. | | | |
| 1 | 589 | 674 | 198 | 278 | 719 | 805 | 248 | 329 |
| | | | 265* | 323* | | | | |
| 2 | 684 | 709 | 227 | 283 | | | | |
| 3 | 653 | 731 | 240 | 319 | | | | |
| 4 | 662 | 696 | 214 | 282 | | | | |
| Average | 647 | 702 | 220 | 290 | | | | |
| | Universal Cement | | (1912) | | | | | |
| 1 | 585 | 685 | 239 | 315 | | | | |
| 2 | 577 | 694 | 225 | 297 | | | | |
| 3 | 691 | 715 | 242 | 306 | | | | |
| 4 | 617 | 792 | 231 | 326 | | | | |
| 5 | 588 | 672 | 246 | 333 | | | | |
| 6 | 612 | 758 | 253 | 323 | | | | |
| 7 | 698 | 884 | 287 | 372 | | | | |
| Average | 624 | 743 | 246 | 325 | | | | |

* Made with sand used in making concrete; not included in average.

Sand. The sand came from a deposit of glacial drift near the Wabash River at Attica, Indiana. Nearly all the clay or loam had been removed by washing. Fineness tests of samples from the three lots of sand are given in Table 2. A single set of briquette tests made from a sample of the 1909 lot of this sand, using Chicago AA cement, gave values about 10% higher than briquettes made from the same sample of cement using standard Ottawa sand. A set of briquettes made from the 1911 sand using Universal cement gave values about 25% higher than the standard sand. The values from these tests are included in Table 1. The sand was well graded, but it will be noted that the 1912 sand was somewhat coarser than the other lots.

Stone. The crushed limestone came from Kankakee, Illinois. It had been screened through a 1-in. screen and over a ¼-in. screen. Mechanical analyses of a number of samples of this stone are given in Table 3. It contained about 48% voids.

5. *Concrete.*—The concrete was proportioned by loose volume. The material was weighed also in order to obtain an independent check on the proportions of each batch. The ratios of the weights of the materials used in most of the specimens are given in Tables 4, 25, 27 and 31. In making the 1909 specimens the cement was measured in buckets, as was done for the other materials. This resulted in considerable variation in the batches. The practice of considering 95 lb. of cement equal to 1 cu. ft. was adopted for the 1911 and 1912 tests. This is in reality a weight proportioning for the cement and is more rational than the older method.

The work of mixing and placing the concrete was done by men of considerable experience in concrete work. The foreman has been employed in the laboratory since 1905, but has spent five or six months each year on contract work in concrete. All of the concrete except that used in the last two-thirds of the 1912 beam series was mixed by hand. The hand-mixing was done directly on the floor of the concrete laboratory. The cement and sand were first mixed dry; the stone, which had previously been thoroughly moistened, was added and the batch then turned until it presented a uniform appearance. The first operation usually required five or six turnings, and the last two or three. Water was added and the material then turned until thoroughly mixed.

In December, 1911, a 9-cu. ft., motor-driven, batch-mixer, made by the Marsh-Capron Mfg. Co., Chicago, Illinois, was installed. The second and third beams in each set of three in the 1912 series and the corresponding auxiliary specimens, as well as some later miscellaneous pull-out specimens, were made of machine-mixed concrete. With the machine running continuously, the stone and sand were placed in the mixer and about one-half the required amount of water admitted. The cement was then added and the remainder of the water admitted at the same time. The amount of water used in each batch was measured and recorded. The drum of the mixer operated at about 22 revolutions per minute. Each batch was mixed for about 5 minutes after adding the cement. When the mixing was complete, the batch was dis-

### TABLE 2.

### MECHANICAL ANALYSIS OF SAND.

| Sieve No. | Separation Size inches | Per cent Passing Each Sieve | | |
|---|---|---|---|---|
| | | 1909 | 1911 | 1912 |
| 3 | 0.28 | 99.7 | 99.8 | 100.0 |
| 5 | .174 | 95.9 | 96.7 | 88.0 |
| 10 | .091 | 77.5 | 74.4 | 54.3 |
| 12 | .067 | 70.3 | 67.6 | 47.5 |
| 16 | .... | 62.8 | 61.0 | 41.7 |
| 18 | .043 | 51.1 | 50.2 | 32.9 |
| 30 | .027 | 28.2 | 30.3 | 21.2 |
| 40 | .019 | 16.9 | 17.0 | 13.3 |
| 50 | .013 | 5.8 | 7.0 | 5.1 |
| 74 | .009 | 3.1 | 3.3 | 2.7 |
| 150 | .... | 0.8 | 0.8 | 1.0 |

### TABLE 3.

### MECHANICAL ANALYSIS OF STONE.

| Size of Square Opening | Separation Size inches | Per cent Passing Each Sieve | | |
|---|---|---|---|---|
| | | 1909 22 Samples | 1911 8 Samples | 1912 5 Samples |
| 1 in. | ...... | 100 | 100 | 100 |
| ¾ in. | ...... | 87 | 95 | 95 |
| ½ in. | ...... | 46 | 57 | 67 |
| ⅜ in. | ...... | 29 | 32 | 46 |
| No. 3 | 0.280 | 14 | 18 | 26 |
| No. 5 | .174 | 2 | 3 | 8 |
| No. 10 | .091 | 1 | 2 | 3 |

charged onto the concrete floor, and was later removed to the forms directly with shovels or by means of a wheelbarrow. The concrete was mixed rather wet so that very little ramming was necessary after placing it in the forms.

6. *Reinforcing Steel.*—The plain round bars of ¾-in. diameter and larger sizes used in the 1909 and 1911 tests were of high-carbon steel. The smaller sizes of plain round bars used in a few of the pull-out specimens and all the plain bars used in the 1912 series were of mild steel. The corrugated bars and most of the other types of deformed bars used were of high-carbon steel. The Thacher bars were of mild steel. See Table 13 and Fig. 21 for details of deformed bars. Additional notes concerning the character and preparation of the steel are given with the discussion of the various groups of tests.

Tensile tests on the steel are not given in this bulletin. In only a few of the tests was the steel stressed to the yield point, and these are noted in the tables.

7. *Pull-out Specimens.*—The specimens for pull-out tests consisted of a cylindrical block of concrete with a steel bar embedded axially. The blocks were generally 8 in. in diameter with embedment of 8 in. In some of the tests both the diameter of the block and length of embedment varied from these figures. The pull-out specimens were usually cast in a vertical position with the bar projecting about 16 in. below and ¼ in. above the finished block. The forms, consisting of galvanized sheet-steel, were set up on a platform made of two 8-in. or 12-in. I-beams placed with their flanges about 2 in. apart. A planed cast-iron base plate made the bottom of the form for the concrete cylinder. A central hole in the base plate allowed the rod to pass through. These plates were removed when the specimens were taken from the forms. The specimens were tested in the same position, the load being applied to the longer end of the bar. The usual form of pull-out specimen is shown in Fig. 1 (a). Other pull-out specimens of unusual form are shown in Figs. 36, 41 and 42.

Nearly all of the pull-out specimens with deformed bars in the 1909 series and certain other groups of specimens were reinforced against bursting by means of six or seven turns of ¼-in. wire in the form of a spiral. This spiral was set inside the form before placing the concrete and was near the outer surface of the concrete block; see Fig. 1 (b). In general, the spiral reinforcement was not used in the pull-out specimens made with the reinforced concrete beams.

Generally, the test pieces were made in sets of five; but for a few of the tests the specimens were made in sets of two to ten. The pull-out specimens made as companion pieces to the 1911 and 1912 series of beams were made in sets of three. When practicable, the individual specimens of a set in the 1909 pull-out series were made from different batches, thus minimizing accidental effects in a given set. Frequent duplicate sets were made in order that comparison might be made between concretes from different batches. The numbers of the batches from which the pull-out specimens of the 1909 series were made are given with the tables of test data.

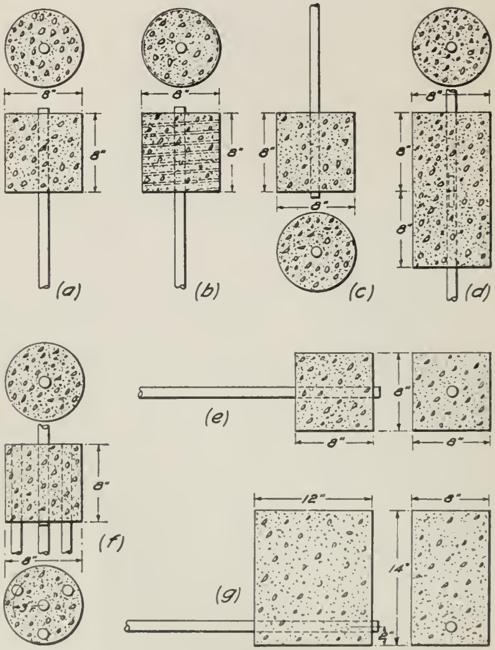


Fig. 1. Types of Pull-out Specimens Used in the Tests.

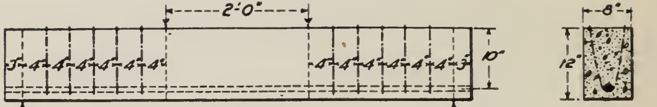
In the group of tests on deformed bars (Table 14) one specimen of each set was made from each of five batches which were mixed at intervals of about two weeks. In the series on "Effect of Age and Mix," (Table 16) three specimens with plain round bars and two with corrugated bars for each age were made from one batch, and the remaining specimens from a second batch.

8. *Reinforced Concrete Beams.*—The 1909 series of tests included 11 reinforced concrete beams; the 1911 series, 36 beams, and the 1912 series, 63 beams. All of the beams (with the exceptions mentioned in Table 27) were 8 by 12 in. in section. The length was generally 6½ ft. A few of the beams in the 1911 and 1912 series were made in lengths of 5½, 7½, 8½ and 10½ ft. The arrangement of the reinforcement for typical forms of beams is indicated in Fig. 2. The ends of the bars were squared, and extended flush with the ends of the beam. In all but two sets of beams the longitudinal reinforcement consisted of a single straight bar placed in the middle of the width of the beam, with its center 10 in. below the top. One set of three beams in the 1912 series was reinforced with 3 ¾-in. rounds and one set with 4 ⅝-in. rounds.

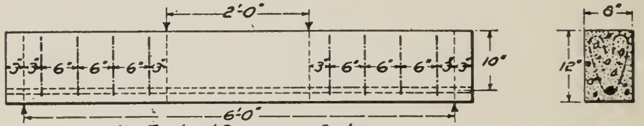
All of the beams, except a part of the 1911 series, were reinforced with vertical stirrups of plain round bars in sizes varying from ¼ to ⅝ in. The stirrups engaged the longitudinal reinforcement and extended to the top surface of the beams; they were placed 4 in. apart throughout the outer thirds of the beam in the 1909 series and 6 in. apart in other beams in which stirrups were used. In the beams in which the longitudinal reinforcement consisted of a single bar, the stirrups were V-shaped; in the beams in which three or four smaller rods were used for longitudinal reinforcement, the stirrups were U-shaped; in the 1911 and 1912 beams the ends of the stirrups were curved inward.

The beams were made in wooden forms directly on the concrete floor of the laboratory, a sheet of building paper having been placed under the form. Generally enough concrete was mixed at one time to make two beams and the corresponding auxiliary specimens. Enough concrete was placed in the form at first to fill it a little above the point where the center of the reinforcing bar should be. After placing the reinforcing steel in position, the form was filled in layers of about 3 in. depth. From the first few batches of machine-mixed concrete only one beam was made; later it became the practice to discharge two batches together on the cement floor of the laboratory before commencing the making of the beams. This supplied enough concrete for two or three beams and the corresponding auxiliary specimens.

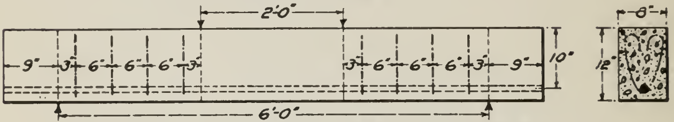
The 1909 beams were considered a preliminary group. The beams of the 1911 and 1912 series were generally made and tested in sets of three.



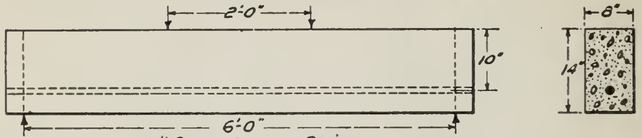
(a) Typical Beam; 1909 Series.



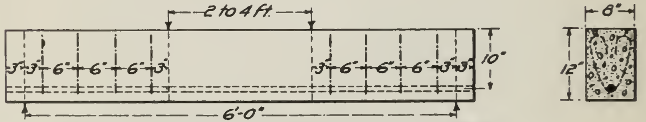
(b) Typical Beam, 1911 Series.



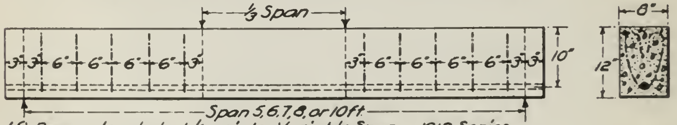
(c) Beam with Ends Overhanging Supports 9 in. 1911 Series.



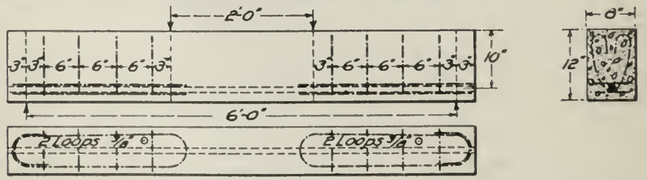
(d) Group 6; 1911 Series.



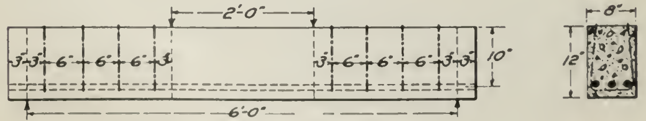
(e) 6 ft Beams, Variable Spacing of Loads; 1912 Series.



(f) Beams loaded at ⅓ points; Variable Span, 1912 Series.



(g) Group 16, 1912 Series.



(h) Group 28; 1912 Series

Fig. 2. Typical Forms of Reinforced Concrete Beams Used in the Tests.

9. *Auxiliary Test Specimens.*—From each batch of concrete three or more 6-in. cubes were made for compression tests. With several groups of the 1909 pull-out specimens in which the age at test or condition of storage were varied, sets of cubes were made for each variation. The 6-in. cubes were made in metal forms. Plain concrete beams 6 by 8 in. in section and 42 in. long, for flexure tests, were made from each batch of the 1909 pull-out tests. Pull-out specimens were made from the same material as was used in many of the reinforced concrete beams of the 1911 and 1912 series.

10. *Storage of Concrete Specimens.*—The 1909 pull-out specimens and the corresponding 6-in. cubes were stored in damp sand unless otherwise noted. The plain concrete flexure beams were stored in open air. These forms were removed after four days except for the specimens tested at age of two days. The specimens were stored in a damp room where the temperature range was about 65° to 75° F.

All of the reinforced concrete beams were stored in the open air of a room in which the temperature varied from 50° to 75° F. They were wet daily with water from a hose until about two months old. The pull-out specimens made with the 1911 and 1912 series of beams were stored under the same conditions as the beams. The 6-in. cubes made with the beams were stored in damp sand. The forms of the beams and their auxiliary specimens remained in place seven days.

The variations in the conditions of storage of the pull-out specimens made at different times should be borne in mind when comparing the bond stresses developed in the different series of tests.

The 1909 pull-out specimens were made between Jan. 1 and May 5; the 1909 beams between Feb. 1 and April 15; the 1911 series of beams between Jan. 1 and April 15; and the 1912 series of beams between Nov. 1, 1911, and Jan. 22, 1912. Other smaller groups included under "Miscellaneous Tests" were made during the season 1911-1912. The 1909 pull-out tests were generally made at age of about 60 days. In a few of the groups of pull-out tests the age ranged from 2 days to about 3½ years. The 1909 beams were tested at about 100 days; the 1911 beams and pull-out tests at about 8 months; the 1912 beams and companion specimens at about 60 days.

11. *Method of Making Pull-out Tests.*—In testing, the pull-out specimen was placed above the weighing head of the testing machine as shown in Fig. 3. The lower end of the embedded bar was engaged by the grips of the pulling head of the testing machine, the load being

transmitted from the concrete cylinder which rested on a planed cast-iron base plate, through a rubber cushion to a spherical bearing block through which it passed to the weighing head of the machine. The rubber cushion served to reduce the rate of application of the load during the earlier stages of the tests and to minimize the effect of shocks arising from the slipping of the grips or vibration of the testing machine. The spherical bearing block allowed the bar to take a vertical position and tended to prevent bending action due to the bar being non-central in the machine or not parallel to the axis of the cylinder.

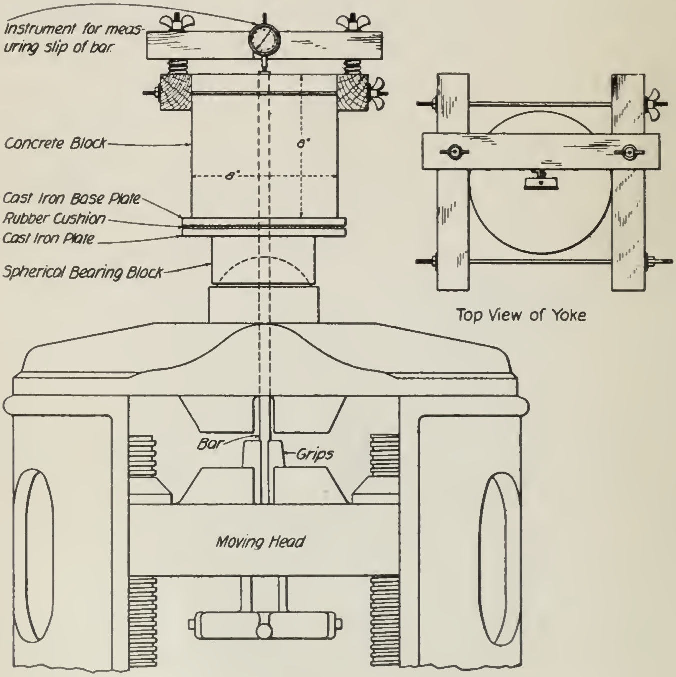


FIG. 3. PULL-OUT SPECIMEN IN MACHINE READY FOR TEST.

A 100,000-lb. Riehle testing machine was used for all the pull-out tests. In these tests the moving head of the testing machine moved at the rate of 0.05 in. per minute.

In the pull-out tests the amount of movement of the free end of the embedded bar was measured by means of an Ames gage in contact with the upper or free end of the bar. The instrument was mounted on a yoke which was attached near the top of the concrete cylinder, as shown in Fig. 3. This instrument is self-indicating and requires no manipulation during the test. It is graduated directly to 0.001 in., and fractional parts of a division may readily be estimated. In order to test the stability of the yoke and to determine whether the top face of the cylinder remains a plane section during the test, four additional Ames gages were attached to the yoke at different points along the diameter of the cylinder in a number of tests. The yoke remained perfectly stable. The concrete ¼ in. from the bar showed a measurable depression, about 0.0002 in. at loads near the maximum, but no depression could be measured at points further than 1 in. from the edge of the bar.

The load was applied continuously, except in a few of the tests in which the load was released and reapplied after the beginning of slip at the free end of the bar. As the test progressed the loads on the testing machine corresponding to an end slip of 0.0005 in., 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, and 0.10 in., were noted. A slip of 0.0005 in. is about the smallest amount that should be used in making comparison, although smaller amounts can readily be measured.

Two men were required to conduct a test; one man operated the testing machine and observed the loads, while the other observed the amount of movement of the bar and kept the test notes.

12. *Method of Making Beam Tests.*—The reinforced concrete beams were loaded in a 200 000-lb. Olsen testing machine. The beams were tested under two equal loads applied generally at the one-third points of the span length; exceptions to this method of loading are noted in the tables. In general the ends of the beams overhung the supports 3 in.; in some of the 1911 tests the overhang was 9 and 15 in., respectively. The supports consisted of roller or rocker bearings. Rollers were used also to transmit the load to the top of the beams. Fig. 56 shows a beam set up in the testing machine ready for loading.

Center deflections and the movement of the ends of the reinforcing bars were measured by means of Ames gages. For measuring deflections the Ames gage was attached at the middle of a wooden bar which was carried by metal points at the mid-depth of one face of the beam at points directly over the supports. A small metal bracket attached to the beam at mid-span, transmitted the movement of the beam to the gage. For measuring the movement of the ends of the reinforcing bar, the gages were carried by yokes in such a way that the plunger had a direct bearing against the end of the bar.

In many beams of the 1911 and 1912 series observations were made on the amount of movement of the reinforcing bar with respect to the adjacent concrete at several points along the length of the beam as described in Art. 66. The method of attaching the instruments to the beam is shown in Fig. 56.

The load was applied in increments of 1000 or 2000 lb. In all but five of the beam tests the load was increased progressively to failure.

13. *Auxiliary Tests.*—The 6-in. cubes were tested for compressive strength only. About one day before testing, the faces to be loaded were bedded in a thin layer of plaster of paris. At least one set of three cubes made with each batch of the 1909 pull-out specimens was tested at age of 60 days.

The 6 by 8 in. plain concrete beams made with the 1909 pull-out specimens were loaded at the one-third points of a 3-ft. span with the 8-in. dimension vertical. The age at test was generally 60 days.

## III. Experimental Data and Discussion.

14. *Preliminary Discussion of the Nature of Bond Resistance.*—In this preliminary discussion it will be necessary to anticipate certain conclusions which appear in the following pages. The tests therein reported indicate that if a bar embedded in concrete is subjected to a tensile stress sufficient to overcome the bond resistance and withdraw the bar, certain definite relations exist between the amount of movement of the bar and the bond stresses developed. In the case of plain bars of ordinary mill surface there is no appreciable movement of the bar until a bond stress about 60% of the maximum bond resistance has been developed. If the bar is further stressed until the slip amounts to, say, 0.1 in. it will be seen that the bond-slip relations have undergone numerous changes. After slipping begins, the bond stress increases with further movement of the bar, very rapidly at first, then more slowly until the maximum bond resistance is reached. After the maximum bond resistance is reached, the bond stress drops off with further slip and at a slip of, say, 0.1 in. amounts to about 50 to 60% of the maximum.

For purposes of this discussion bond resistance may be divided into two elements which we shall designate as (1) adhesive resistance and (2) sliding resistance. The term adhesive resistance is used to designate the bond resistance developed before movement of bar with respect to the adjacent concrete begins. Sliding resistance is applied to the resistance developed as a result of the movement of the bar. Adhesive resistance may be considered as due to tangential adhesion between the concrete and steel and to static friction. The origin of tangential adhesion is not well understood, but its presence is a matter of universal experience with materials of the nature of mortar and concrete. It is recognized that tangential adhesion does not account for all the resistance developed before slipping begins, but the difference may be attributed to static friction developed by the pressure or grip of the concrete on the bar. As soon as the adhesive resistance is overcome, that is, as soon as the bond stress exceeds the sum of the tangential adhesion plus the static friction, there is a movement of the bar with respect to the adjacent concrete. Although we cannot accurately divide this adhesive resistance into its component elements, the sum of these elements or the total adhesive resistance can be determined, and it is found that under certain conditions it bears a definite ratio to the ultimate bond resistance.

The bond resistance which is developed after slip of bar begins may be said to be due entirely to sliding resistance. Friction between bodies in contact arises primarily from roughnesses of their surfaces; its value is expressed as the product of the coefficient of friction into the normal pressure which exists at the surfaces of contact. The static friction mentioned above is due to the same cause. The roughness of surface in the case of a bar embedded in concrete is due to inequalities in the surfaces of contact which arise from irregularities of section and alignment of the bar and the corresponding conformation of the concrete. The coefficient of friction between concrete and steel for the conditions present in these tests has not been determined. The normal pressure at the surface of the bar may be due to the following causes: (a) initial stresses generated by shrinkage of the concrete during setting and hardening; (b) wedging of the bar in the concrete following a movement from its original position. This wedging is caused by the inequalities in the bar mentioned above and in itself will give an added resistance. Apparently it is augmented under certain conditions by concrete adhering to the bar.

In the above discussion the various components of bond resistance have been mentioned in the order in which they become effective in resisting bond stress. It is evident from these considerations that what has been termed adhesive resistance, that is, the amount of bond resistance which may be utilized before slipping begins, is by far the more significant element. While frictional resistance is of importance, reliance should not be placed on this element of bond resistance.

It was stated above that we cannot accurately divide adhesive resistance into its component elements; however, certain tests do give some indication of the values of these components. A large number of specimens with polished round bars embedded 8 in. in 1-2-4 concrete tested at age of 60 days gave a maximum bond resistance of about 160 lb. per sq. in. In these tests frictional resistance (both static and sliding) was reduced to a minimum and bond resistance after slipping began was almost nil. We may conclude then that 160 lb. per sq. in. represents about what may be expected for the tangential adhesion between steel and concrete of this quality. It seems probable that the same value may be used for the tangential adhesion between any clean steel surface and concrete of the quality used. Round bars with ordinary mill surface tested under the same conditions as the polished bars show slip to begin at, say, 260 lb. per sq. in. with a maximum bond resistance of about 440 lb. per sq. in., developed after a slip of about 0.01 in. had occurred. The difference of about 100 lb. per sq. in. between the bond resistance developed by the polished bars and by the ordinary bars when slipping begins (corresponding also to the maximum for the polished bars) may be said to represent the value of static friction for the bars of ordinary mill surface above that of polished bars embedded in the same concrete. The additional bond resistance developed by the ordinary bars after slipping begins is due to frictional resistance.

The above observations refer primarily to the general case of bond between concrete and plain bars; for deformed bars, certain obvious modifications in these statements would be necessary. It will be seen later that the conditions under which the specimen is molded and tested have an important bearing on the bond resistance developed in any case.

In reinforced concrete beams, where the reinforcing bars are considered to take the main tensile stresses, the phenomena of bond action are complicated by the stiffness of the adjoining concrete in resisting stretching concurrently with the steel. This results in anti-stretch slip, a term which is discussed at length in Art. 68. The presence of this stress makes it desirable to distinguish between the phenomenon of anti-stretch slip and the slip produced by ordinary beam action.

15. *Strength of Concrete.*—The compressive and flexural strength of the various batches of concrete used in the 1909 series of pull-out specimens are given in Table 4. For convenience of reference a sum-

## TABLE 4.

### RECORD OF BATCHES OF CONCRETE USED IN THE 1909 PULL-OUT TESTS.

The average compressive strength of 69 6-in. cubes from 19 batches of 1-2-4 concrete is 2150 lb. per sq. in.; the average modulus of rupture of 18 6 by 8-in. plain concrete beams loaded at the ⅓ points of a 36-in. span, is 296 lb. per sq. in. The average values with the mean variations for the cube and plain beam tests may be expressed as 2150±58 and 296±16 lb. per sq. in. respectively.

The tests given in this table were made at age of about 60 days.

Stresses are given in pounds per square inch.

| Batch No. | Date Made (1909) | Number of Specimens from Batch | | | Mixture by Loose Volume | Mixture by Weight | Compressive Strength of 6-in. Cubes | Modulus of Rupture of 6 x 8 x 36-in. Plain Beams |
|---|---|---|---|---|---|---|---|---|
| | | Pull-out | 6-in. Cubes | Flexure Beams | | | | |
| 1 | Jan. 1 | 47 | 27 | 1 | 1-2-4 | 1-2.4-4.3 | 1815 | 376 |
| 2 | Jan. 7 | 48 | 18 | 1 | 1-1½-3 | 1-1.8-3.3 | 3060 | 359 |
| 3 | Jan. 12 | 25 | 15 | 1 | 1-2-4 | 1-2.5-4.0 | 2898 | 550 |
| 4° | Jan. 13 | 40 | 15 | 1 | 1-2-4 | 1-2.4-5.5 | 1913° | 338° |
| 5 | Jan. 18 | 39 | 39 | 1 | 1-3-6 | 1-3.6-6.3 | 1908 | 282 |
| 6 | Jan. 25 | 5 | 6 | 1 | 1-1-2 | 1-1.5-2.0 | 4061* | 374 |
| 7 | Jan. 22 | 5 | 6 | 1 | 1-4-8 | 1-5.0-8.8 | 1155* | 151 |
| 8 | Jan. 22 | 5 | 6 | 1 | 1-2-2 | 1-2.1-1.9 | 2696* | 332 |
| 9 | Jan. 22 | 5 | 6 | 1 | 1-5-10 | 1-6.3-10.2 | 533* | 62 |
| 10 | Jan. 27 | 43 | 6 | 1 | 1-2-4 | 1-2.4-4.2 | 2180* | 255 |
| 11 | Feb. 1 | 24 | 3 | 1 | 1-2-4 | 1-2.4-4.3 | 1840 | 375 |
| 12 | Feb. 9 | 47 | 27 | 1 | 1-2-4 | 1-2.2-3.8 | 2637 | 303 |
| 13 | Feb. 15 | 25 | 18 | 1 | 1-1½-3 | 1-1.9-3.3 | 2688 | 385 |
| 14 | Feb. 15 | 5 | 3 | 1 | 1-2½-5 | 1-3.0-5.1 | 1555 | 223 |
| 15 | Feb. 15 | 5 | 3 | 1 | 1-2-3 | 1-1.8-2.4 | 1733 | 302 |
| 16 | Feb. 25 | 5 | 3 | 1 | 1-2-5 | 1-2.1-5.0 | 1685 | 122 |
| 17 | Feb. 19 | 50 | 6 | 1 | 1-2-4 | 1-2.3-4.3 | 1683* | 240 |
| 18 | Feb. 25 | 5 | 3 | 1 | 1-2-1 | 1-2.1-1.1 | 2870 | 336 |
| 19 | Feb. 25 | 30 | 21 | 1 | 1-3-6 | 1-3.4-6.2 | 1242 | 183 |
| 20 | March 1 | 42 | 18 | 1 | 1-1-2 | 1-1.2-2.1 | 3000 | 367 |
| 21 | March 6 | 45 | 21 | 1 | 1-4-8 | 1-4.6-8.3 | 1298 | 167 |
| 22 | March 10 | 23 | 3 | 1 | 1-2-4 | 1-2.3-4.2 | 2043 | 213 |
| 23 | March 11 | 33 | 18 | 1 | 1-1-2 | 1-1.2-2.0 | 3488 | 335 |
| 24 | March 15 | 51 | 6 | 1 | 1-2-4 | 1-2.0-3.5 | 1468* | 151 |
| 25 | March 19 | 25 | 3 | 1 | 1-2-4 | 1-2.4-4.3 | 1950* | 165 |
| 26 | March 20 | 30 | 18 | 1 | 1-4-8 | 1-4.8-8.5 | 800 | ... |
| 27 | March 23 | 57 | 6 | 1 | 1-2-4 | 1-2.4-4.3 | 1675 | 247 |
| 28 | March 30 | 5 | 3 | 1 | 1-2-6 | 1-2.4-6.6 | 1108 | 113 |
| 29 | March 30 | 10 | 3 | 1 | 1-2-0 | 1-4.2-0. | .... | 357 |
| 30 | March 27 | 53 | 6 | 1 | 1-2-4 | 1-2.4-4.2 | 1715 | 172 |
| 31 | March 31 | 38 | 3 | 1 | 1-2-4 | 1-2.3-4.1 | 2300 | 332 |
| 32 | March 31 | 5 | 3 | 1 | 1-2-8 | 1-2.4-8.0 | 1240 | 117 |
| 33 | April 8 | 40 | 24 | 1 | 1-1½-3 | 1-1.6-3.1 | .... | 371 |
| 34 | April 19 | 42 | 3 | 1 | 1-2-4 | 1-2.5-4.5 | 1950 | 202 |
| 35 | April 23 | 45 | 9 | 1 | 1-2-4 | 1-3.1-5.9 | 3040 | 427 |
| 36 | April 16 | 26 | 3 | 1 | 1-2-4 | 1-2.4-4.5 | 1770 | 238 |
| 37 | April 24 | 48 | 3 | 1 | 1-2-4 | 1-2.4-4.2 | 2460 | 357 |
| 38 | May 5 | 7 | 3 | 1 | 1-3-6 | 1-3.7-6.5 | 967 | 257 |
| 39 | April 29 | 60 | 21 | 1 | 1-2-4 | 1-2.2-4.1 | 2190 | 379 |
| 40 | May 3 | 55 | 36 | .. | 1-2-4 | 1-2.4-4.3 | 2560 | ... |
| Total | | 1198 | 444 | 39 | | | | |

° Batch No. 4 was made from Chicago AA cement; all others from Universal cement.

* Average of 6 tests; all other values for cube strength are the averages of 3 tests.

mary of the strength of concrete from the most important groups of tests is given in Table 5. This table includes all the tests on 6-in. cubes which were stored in damp sand and tested at age of about 60 days. The average compressive strength of 69 6-in. cubes from 19 batches of 1-2-4 concrete in Table 4 is 2150 lb. per sq. in.; the average flexural strength of the same concrete, 296 lb. per sq. in. Values for the compressive strength of the concrete used in each group of specimens are given in the tables of test data. Other data of compressive strength for different mixes and ages are shown in Tables 16 and 18 and in Fig. 35. Values for different conditions of storage are given in Table 12.

## TABLE 5.

### COMPRESSIVE AND FLEXURAL STRENGTH OF CONCRETE.

All cubes given in this table were stored in damp sand. The plain beams were stored in the open air.

Cube tests for other ages and conditions of storage are given in Tables 12 and 16. Stresses are given in pounds per square inch.

| Mix | Cement | Method of Mixing | Number of Batches | Age at Test days | 6 x 8 x 36-in. Plain Beams | | 6-in. Cubes | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Number of Tests | Modulus of Rupture | Number of Tests | Compressive Strength |
| | | | | 1909 | | | | |
| 1-1-2 | Universal | Hand | 3 | 61 | 3 | 362 | 12 | 3653 |
| 1-1½-3 | Universal | Hand | 3 | 62 | 3 | 338 | 6 | 2874 |
| 1-2-4 | Universal | Hand | 19* | 64 | 18 | 296 | 69 | 2150 |
| 1-2½-5 | Universal | Hand | 1 | 61 | 1 | 223 | 3 | 1555 |
| 1-3-6 | Universal | Hand | 3 | 62 | 3 | 241 | 9 | 1372 |
| 1-4-8 | Universal | Hand | 3 | 60 | 2 | 159 | 12 | 1102 |
| 1-5-10 | Universal | Hand | 1 | 60 | 1 | 62 | 6 | 533 |
| 1-2-0 | Universal | Hand | 1 | 72 | 1 | 357 | .. | .. |
| 1-2-1 | Universal | Hand | 1 | 60 | 1 | 336 | 3 | 2870 |
| 1-2-2 | Universal | Hand | 1 | 60 | 1 | 332 | 6 | 2696 |
| 1-2-3 | Universal | Hand | 1 | 61 | 1 | 302 | 3 | 1733 |
| 1-2-5 | Universal | Hand | 1 | 61 | 1 | 122 | 3 | 1685 |
| 1-2-6 | Universal | Hand | 1 | 65 | 1 | 113 | 3 | 1108 |
| 1-2-8 | Universal | Hand | 1 | 73 | 1 | 117 | 3 | 1240 |
| 1-2-4 | Chicago AA | Hand | 6† | 100 | .. | .. | 18 | 1700 |
| | | | | 1911 | | | | |
| 1-2-4 | Universal | Hand | 11 | 240 | .. | ... | 33 | 3100 |
| 1-2-4 | Lehigh | Hand | 6 | 240 | .. | .. | 18 | 2937 |
| | | | | 1912 | | | | |
| 1-2-4 | Universal | Hand | 12 | 63 | .. | ... | 36 | 2200 |
| 1-2-4 | Universal | Machine | 31 | 63 | .. | .. | 93 | 2800 |
| 1-2-4 | Universal | Machine | 4 | 240 | .. | ... | 12 | 3774 |

* Includes one batch (No. 4) made of Chicago AA cement.

† From the concrete used in the 1909 beams.

The compressive strength of 6-in. cubes from the 1912 1-2-4 concrete tested at average age of 63 days was as follows: 36 cubes of hand-mixed concrete, 2200 lb. per sq. in.; 93 cubes of machine-mixed concrete, 2800 lb. per sq. in. The machine-mixed cubes gave about 30% higher strength than the hand-mixed. However, it will be seen later that all of the tests do not bear out this conclusion; in some of the beam tests the hand-mixed concrete gave the higher values for bond and vertical shearing stress.

In making some of the larger batches of concrete for the 1909 pull-out tests a period of from three to four hours elapsed between the making of the first and last specimens. From five batches of this kind a set of three 6-in. cubes was made as soon as the mixing was completed and a second set after about 50 pull-out specimens had been finished. The portion of the batch remaining on the mixing floor was turned two or three times during the interval between making the two sets of cubes. The average compressive strength of five sets of cubes made as soon as the mixing was complete, was 1756 lb. per sq. in. The average for cubes made 3½ hours after mixing, was 1733 lb. per sq. in., a difference of about 1%. It will be remembered that the Vicat needle test showed this cement to reach final set at 3¾ hr.

## A. Pull-out Tests.

16. *Classification of Pull-out Tests.*—A total of 1500 pull-out tests are reported in this bulletin. The 1909 series included about 1000 pull-out specimens. Forty-two pull-out specimens were made as companion pieces to the 1911 series of reinforced concrete beams, and 180 with the 1912 series of beams. The remainder of the pull-out tests are grouped under "Miscellaneous Tests," Art. 54 to 64.

The pull-out tests will be discussed under eight different headings as shown in Table 6, which indicates the number of tests included under each sub-division and the tables and figures in which details of the tests may be found. An inspection of the table will show that these divisions are not exclusive, as such elements as effect of age, proportions of concrete, condition of storage, etc., are found to be important variables in more than one of the groups. A few tests are included under two sub-divisions, but this is indicated in the tables of test data. However, each sub-division may be considered as a more or less complete series of tests, and they were so considered in designing and making the specimens. In making comparisons between different groups, the conditions of storage and the quality of the concrete as shown by the cube tests, should be taken into consideration.

TABLE 6.

CLASSIFICATION OF PULL-OUT TESTS.

| Item | Number of Tests | Test Data in | |
|---|---|---|---|
| | | Tables No. | Figures No. |
| a. Effect of Variations in the Dimensions of Pull-out Specimens. | 163 | 7, 8, 9, 10 | 5 to 16 |
| b. Effect of Shape of Section and Condition of Surface of Bar.... | 90 | 11 | 17, 18 |
| c. Effect of Condition of Storage.......................... | 189 | 1 | 19, 20 |
| d. Bond Tests with Deformed Bars........................ | 116 | 13, 14, 15 | 21 to 26 |
| e. Effect of Age and Mix.................................. | 265 | 16, 17, 18 | 27 to 35 |
| f. Effect of Anchoring End of Bar......................... | 97 | 19 | 36, 37 |
| g. Miscellaneous Tests..................................... | 357 | 20, 21, 22, 24 | 38 to 45 |
| Companion Tests to Reinforced Concrete Beams........... | 222 | 28, 29, 30, 32, 33 and 34 | 49, 55 |

17. *Stresses and Deformations in a Pull-out Specimen.*—As load is applied to the bar in a pull-out specimen of the form used in these tests, the tensile stress in the bar is gradually taken off along the embedded length by bond between the bar and the surrounding concrete. Thus the total tension in the bar, the total compression over the lower face of the concrete block and the total bond stress between the concrete and steel are equal. The principal stresses existing in a pull-out specimen of the form generally used are indicated in Fig. 4. If we consider the bond stress to be uniformly distributed along the length of the bar, the bond unit stress is:

$$u = \frac{P}{m\,h}$$

where $P$ is the total load on the bar, $m$ is the perimeter of the bar, and $h$ is the length of embedment. It is evident that owing to the elongation in the steel due to tensile stress and the shortening in the concrete block due to compressive stress, acting in opposite directions, the greatest bond stress during the early stages of the test and consequently the first slip between concrete and steel must occur at the point where the bar enters the block, and that slip will always be a little greater here than at points of greater embedment. Experimental verification of this statement will be found in the tests. A numerical example will assist in fixing ideas. Consider a 1-in. round bar embedded 8 in. in an 8-in. cylinder. Tests show that a specimen of these dimensions, of 1-2-4 concrete about 60 days old, will withstand a bond stress of about 260 lb. per sq. in. before slip begins. At this stress the total compression in the concrete and tension in the bar are each 6500 lb. The stresses in the concrete and in the steel are 132 and 8300 lb. per sq. in., respectively. If we assume that the deformations in the concrete and the steel are proportional to the stress and that the bond stress is uniformly distributed along the length of the bar we shall have 0.0002 in. and 0.0011 in. for the total deformation in the concrete and steel, using 2 500 000 and 30 000 000 lb. per sq. in., respectively, as the moduli of elasticity of the materials. This gives a total slip at the lower end of the block of 0.0013 in. when slip at the free end first becomes perceptible. These considerations show that for a 1-in. bar the relative movement at the bottom due to steel deformation is over five times as great as that due to concrete deformation, and indicate that as far as the bond resistance is concerned the compressive stress developed in the concrete block is of minor importance as compared with the steel stress. For smaller bars the influence of the concrete stress is negligible.

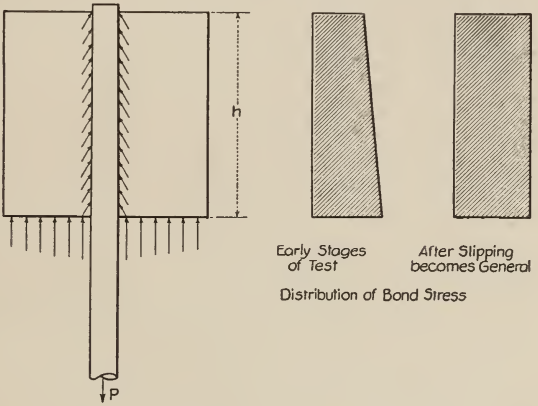


Fig. 4. Diagram Showing Principal Stresses in a Pull-out Specimen.

As an extreme case consider a 1¼-in. plain bar embedded 24 in. as in the tests in Table 9. Slipping at the free end of the bar began at an average bond stress of 278 lb. per sq. in. The average compressive stress over the lower face of the concrete block at this stage of the test was 540 lb. per sq. in. and the tensile stress in the bar, 21 300 lb. per sq. in. If the bond stress be considered uniformly distributed along the length of the bar, this gives a slip of 0.0112 in. at the bottom when slip at the top of the block begins. It will be found that for specimens of the form generally used in these tests, the bond resistance of plain bars increased as slip progressed and reached a maximum when the bar had slipped about 0.01 in. The tests on 1¼-in. plain bars in Table 9 and Fig. 11 bear out the conclusion that was reached from these computations, that with 24-in. embedment, the maximum bond resistance of the bar as a whole does not differ much from that corresponding to first slip at the free end.

In addition to the longitudinal stresses set up in the concrete and steel, stresses are developed normal to the surface of the bar. During the later stages of the test these stresses become considerable and may be sufficient to split the concrete surrounding the bar. This is especially evident with deformed bars, though the splitting action was found with plain bars.

18. *Phenomena of Pull-out Tests.*—Fig. 6 summarizes load-slip curves for plain round bars for a variety of conditions of age, mix, size of bar, length of embedment and storage. Each curve is a composite of from 5 to 10 tests. For ease of making comparisons all bond stresses have been plotted as a percentage of the maximum bond resistance. Only the portions of the curves preceding the maximum are plotted here; all of these tests are discussed in detail in the following pages. These curves are quite similar, considering the wide variations of conditions present. Attention should be called to the fact that in this figure the values of slip of bar were measured at the free end. Generally, slip at the free end began at a load between 60% and 80% of the maximum. In nearly all cases the maximum load occurred at an end slip of about 0.01 in. It seems that a movement of 0.01 in. between the concrete and steel was sufficient to destroy the irregularities which were most effective in increasing bond resistance after the adhesion was broken.

The curves in Fig. 5 represent the successive condition at each point along the length of the bar. The basis of these curves is given in Arts. 19, 22 and 23. The solid curve may be considered as typical of the load-slip relation found in pull-out tests with plain bars. This curve represents what may be considered as the bond-stress-slip history of each unit of the embedded length of a plain bar. It exhibits the following characteristics:

(1) There is no measurable slip of bar until a bond stress of about 260 lb. per sq. in. has been developed.

(2) Slipping begins at about 60% of the maximum bond resistance.

(3) When the bar has slipped 0.001 in. at any point the bond stress there is about 75% of the maximum bond resistance.

(4) When the slip at any point reaches 0.005 in. the bond stress there is 95% of the maximum.

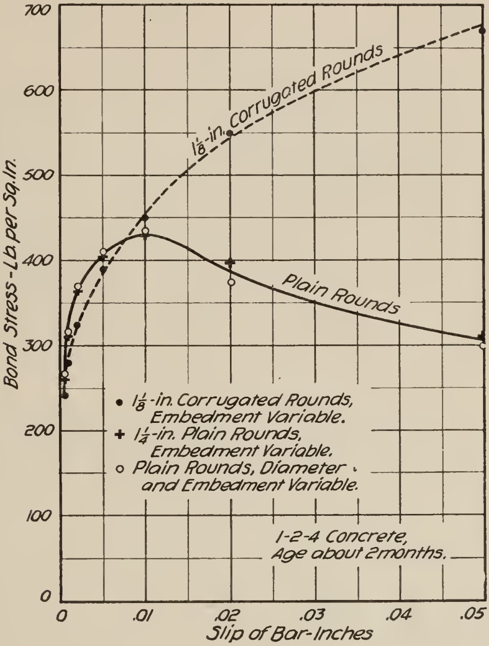


FIG. 5. LOAD-SLIP CURVES AFTER ELIMINATING SIZE OF BAR AND LENGTH OF EMBEDMENT.

(5) The maximum bond resistance occurs at a slip of about 0.01 in., and the bond resistance decreases with further movement of the bar.

(6) When the bar has slipped twice the amount which was measured at the maximum bond resistance the bond stress has decreased only 10%.

(7) When the bar has moved 0.05 in., five times the slip at the maximum bond resistance, the bond stress is still about 70% of the maximum.

The curve shows that the term "running friction" loses its significance in view of the data of these tests, since, properly speaking, we are dealing with "running friction" throughout the test after movement begins.

It should be borne in mind that these deductions apply only to test specimens of the form used, under a progressively applied load that does not exceed the yield point strength of the bar. It will be seen later that the continuation of a constant load after slipping has begun, and other variations materially modify the load-slip relation. The broken line in Fig. 5, from a series of tests on corrugated round bars, indicates the corresponding load-slip relations for this type of bar.

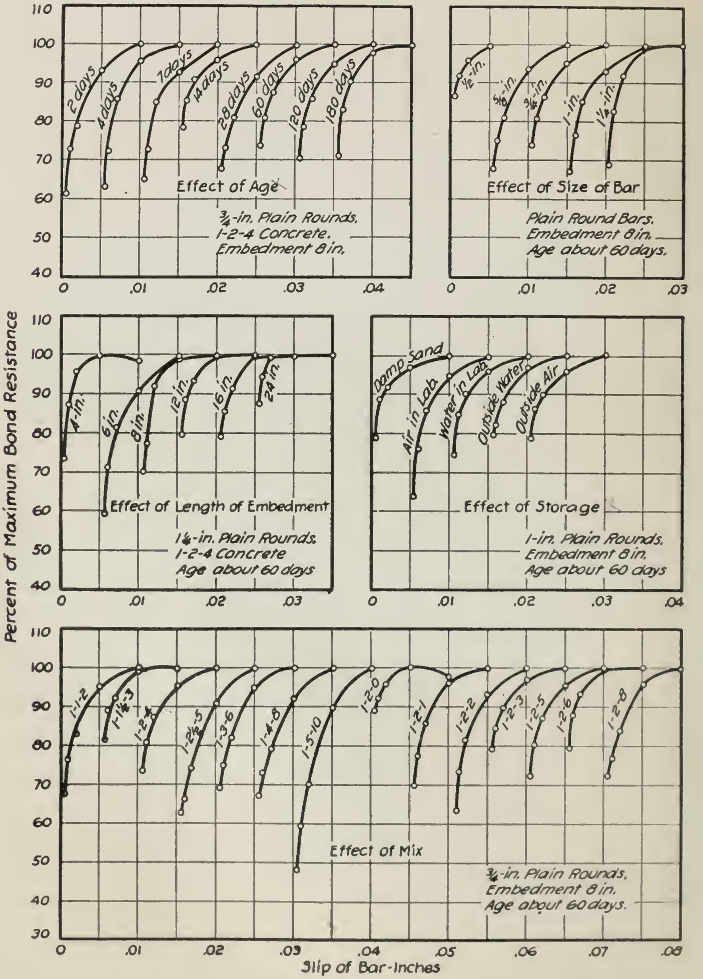


FIG. 6. LOAD-SLIP CURVES FROM PULL-OUT TESTS WITH PLAIN ROUND BARS.

## *a—Effect of Variations in the Dimensions of Pull-out Specimens.*

19. *Effect of Size of Plain Bars; Embedment Variable.*—In this group of tests the diameter of the concrete cylinders for all the specimens was 8 in., but the size of bar and length of embedment varied as shown in Table 7, between the limits ¼-in. plain round bar embedded 3 in. and 1¼-in. round bar embedded 16 in. It will be seen that these dimensions are such as to give a nearly constant ratio (about 50) be-

### TABLE 7.

### EFFECT OF SIZE OF BAR; EMBEDMENT VARIABLE.

Diameter of concrete cylinders 8 in. in all cases.

1-2-4 hand-mixed concrete from Batches 11, 22, 25, 31, and 36. (See Table 4.)

The average compressive strength of 18 6-in. cubes from this concrete, tested at age of about 60 days, was 1975 lb. per sq. in.

Stresses are given in pounds per square inch.

| Size of Bar | Length of Embedment | | Number of Tests | Age at Test days | Bond Stress at End of Slip of | | Maximum Bond Resistance |
|---|---|---|---|---|---|---|---|
| | inches | diameters | | | .0005 in. | .001 in. | |
| | | | Plain Round Bars. | | | | |
| ¼ in. | 3 | 12.0 | 4 | 71 | 376 | 389 | 476 |
| ⅜ in. | 4¾ | 12.6 | 4 | 73 | 335 | 371 | 423 |
| ½ in. | 6 | 12.0 | 5 | 71 | 263 | 316 | 408 |
| ⅝ in.° | 8 | 12.8 | 5° | 72 | 266 | 295 | 405 |
| ¾ in. | 9½ | 12.6 | 5 | 70 | 287 | 311 | 386 |
| 1 in. | 12¾ | 12.8 | 6 | 70 | 305 | 327 | 392 |
| 1¼ in.* | 16 | 12.8 | 9* | 75 | 275 | 298 | 359 |
| Average...... | .......... | .......... | ......... | 72 | 301 | 330 | 407 |
| | | | Corrugated Bars. | | | | |
| ¼ in. sq. | 3½ | 17.5 | 5 | 71 | 282 | 335 | 739† |
| ⅓ in. sq. | 4½ | 15.0 | 4 | 71 | 324 | 356 | ... |
| ½ in. sq. | 8 | 16.0 | 5 | 71 | 341 | 371 | 702† |
| ¾ in. sq. | 12 | 16.0 | 5 | 71 | 339 | 367 | 730† |
| 1⅛ in. rd.* | 24 | 19.2 | 9* | 77 | 268 | 301 | 688† |
| Average...... | .......... | .......... | ......... | 72 | 311 | 346 | 715† |

° The same tests are included in Table 8.

* Includes 4 tests made with group in Table 9.

† Blocks were reinforced against bursting by means of ¼-in. spirals. The maximum bond stress given in the tables for corrugated bars is the average stress developed at an end slip of 0.1 in.

tween the embedded area and the cross sectional area of the bar and corresponds to an embedment of about 12 diameters for each of the bars. It was felt that this was the proper basis for a series of tests to show the relation between the bond resistance and the size of bar. The load-slip curves are plotted in the upper portion of Fig. 7. In Fig. 8 the loads have been plotted for slips at the free end of the bar of 0.0005,

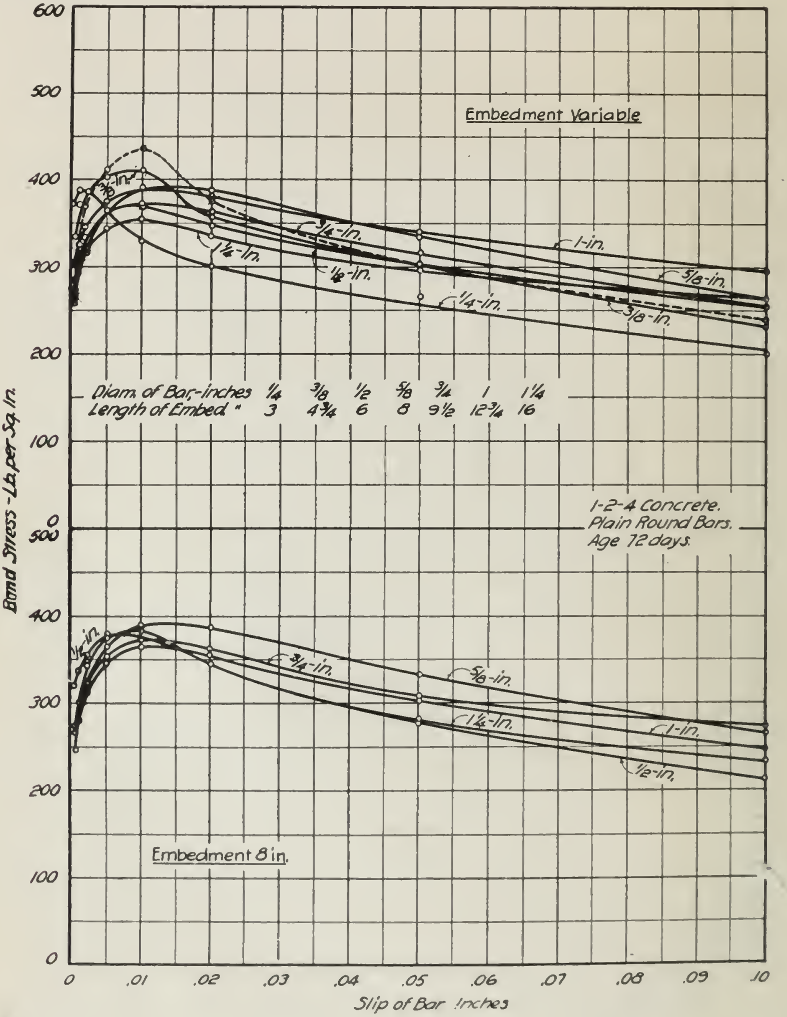


FIG. 7. LOAD-SLIP CURVES FOR PLAIN ROUND BARS OF DIFFERENT SIZE.

0.001, 0.002, 0.005 in. and for the maximum bond resistance, corresponding to a slip of about 0.01 in.; the corresponding points are connected by straight lines. A slip of 0.0005 in. may be considered as the beginning of slip.

In general the smaller bars gave a bond resistance slightly higher than the bars of larger size, but the relation is not well defined at all stages of the tests. The maximum bond resistance decreased as the diameter of the bar increased. The ⅜-in. bars gave values at the maximum load about 15% higher than the 1¼-in. bars. The ¼-in. bars embedded 3 in. were somewhat erratic in their behavior. Some of the concrete blocks split at the maximum load. The cause for this is somewhat problematical, but it probably was due to slight irregularities in the contact between the concrete block and the bearing plate and to lack of stiffness in the blocks which prevented them from distributing the compressive stress over their entire base.

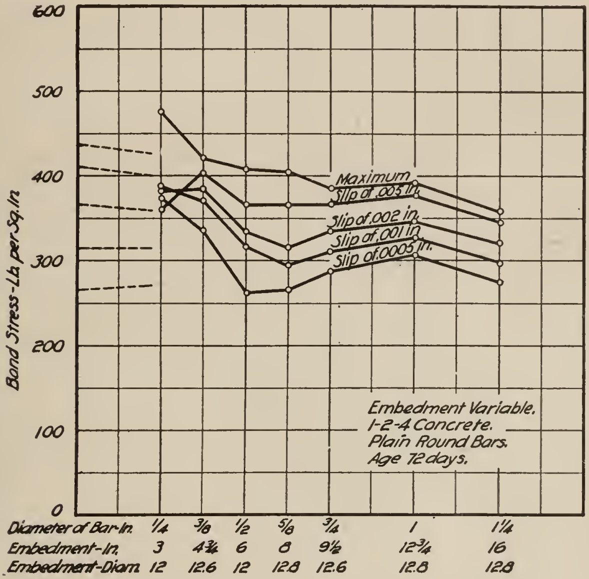


FIG. 8. BOND RESISTANCE OF PLAIN ROUND BARS; EMBEDMENT VARIABLE.

With a constant ratio between the average bond stress throughout the length of the bar and the tensile stress in the steel, that is, with the bars embedded a constant multiple of the diameter, there are several variables which may affect the bond stresses developed. Among these may be mentioned:

(1) Variation in the compressive stress in the concrete block;

(2) Contraction in the section of the bar due to the tensile stress;

(3) Variations in the section and alignment of bars of different sizes.

(1) The compressive stress in the concrete blocks in these tests varies directly with the section of the bar, if the bond unit stress is the same. The maximum compressive stress at the bottom of the concrete blocks in the tests on the ½-in. bars embedded 6 in. and on the 1¼-in. bars embedded 16 in. are 76 and 622 lb. per sq. in., respectively, using 400 lb. per sq. in. bond stress in both cases. It is evident from these tests and the tests discussed in Art. 21 and 22 that a wide variation in the amount of compressive stress in the concrete block has very little influence on the bond stresses developed in the tests.

(2) The total contraction of the section of the bar due to tensile stress will be proportional to the diameter of the bar for the same unit stress in the steel. For the 1¼-in. bars used in this group of tests the diameter of the bar is shortened as much as 0.0004 in. when it is carrying the highest stress. A portion of this shortening was counteracted by the corresponding expansion in the concrete, but since the distribution of the concrete stresses is not known and the amount of this expansion uncertain, the relation of these deformations cannot be determined. It seems that the contraction in the section of the bar may have a slight influence in reducing the bond resistance of the larger bars for loads near the maximum. This influence would not be important in the group of tests discussed in Art. 22, since the steel stresses were not high.

(3) The bars used presented surfaces which apparently were similar in all respects. It has frequently been observed, however, that small rolled bars are more irregular in section and alignment than larger bars. This fact may partially account for the higher stresses developed by the smaller bars in the later stages of the tests. The lower bond stress developed by the smaller bars after an end slip of 0.01 in. has occurred lends color to this opinion.

These tests indicate that small bars give a somewhat higher bond resistance than larger bars. Earlier discussions of this subject have been based on the maximum bond stresses developed by bars of different sizes embedded equally without regard to the load-slip relations present. A group of tests of this kind is discussed in the following article.

In Fig. 8 dotted lines have been drawn which indicate the general trend of the values of bond resistance in these tests for the various amounts of slip. In constructing these lines least weight has been given to the values for ¼ and ⅜-in. bars, since the load-slip curves in Fig. 7 show these tests to be abnormal. The points at which these lines intersect the vertical axis may be said to represent the bond stresses for a bar of infinitesimal diameter embedded an infinitesimal length; the embedded length is still equal to about 12 diameters. These values and values obtained in a similar way from Fig. 12 have been used to plot the load-slip curve shown by the solid line in Fig. 5. It will be seen that the values given in the figure for the two series of tests are nearly identical. This curve may be said to show the successive variations in bond resistance at each point along the length of the bar during the progress of the test after eliminating the size of bar and length of embedment. This consideration shows slipping to have begun at a somewhat lower load than was indicated by the measurements of slip at the free end of the bar and that the maximum bond resistance from point to point was a little higher than that found for the bar as a whole. This is as might have been expected, since, up to the load at which slip reached a considerable amount and became general, each point along the length of the embedded bar was in a different stage of its load-slip history. After the maximum load the curve for infinitesimal embedment follows the general course of the other curves, which indicates that after the adhesion was entirely broken and slip became general, the frictional resistance was about the same for all sizes of bars of the kind used in these tests. It will be seen in the discussion of Fig. 12 in Art. 22 that a curve of almost exactly the same form is found from a consideration of a group of tests on 1¼-in. plain rounds in which the length of embedment varied.

20. *Effect of Size of Corrugated Bars; Embedment Variable.*—The group of tests on corrugated bars given in Table 7 may be considered as forming a series parallel to the group of tests on plain rounds discussed in Art. 19; they were made from the same concrete. The specimens varied from a ¼-in. corrugated square bar embedded 3½ in. to a 1⅛-in. corrugated round bar embedded 24 in. As in the group of tests on plain rounds these dimensions were such as to give approximately a constant ratio (about 48) of bond area to the cross-sectional area of the bars, corresponding to an embedment of about 16 short diameters of the bars (21.4 diameters in the case of 1⅛-in. corrugated round bars). The bond resistances for various amounts of end slip are plotted in Fig. 9. The values for ⅓-in. bar embedded 4½ in. have

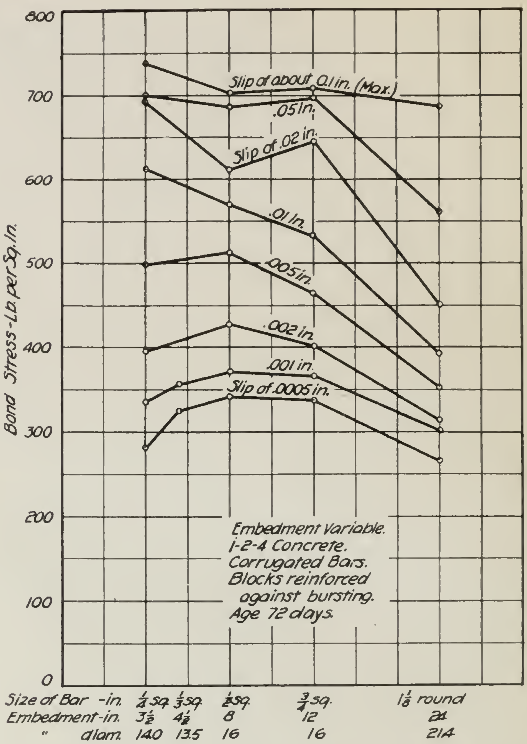


Fig 9. Bond Resistance of Corrugated Bars; Embedment Variable.

## TABLE 8.

## Effect of Size of Plain Round Bars; Embedment 8 in.

Made from the same concrete as the tests in Table 7.

Stresses are given in pounds per square inch.

| Size of Bar | Length of Embedment | | Number of Tests | Age at Test days | Bond Stress at End Slip of | | Maximum Bond Resistance |
|---|---|---|---|---|---|---|---|
| | inches | diameters | | | .0005 in. | .001 in. | |
| ½ in. | 8 | 16.0 | 4 | 72 | 323 | 339 | 381 |
| ⅝ in. | 8 | 12.8 | 5* | 72 | 266 | 295 | 405 |
| ¾ in. | 8 | 10.7 | 5† | 81 | 275 | 303 | 387 |
| 1 in. | 8 | 8.0 | 5 | 64 | 247 | 281 | 385 |
| 1¼ in. | 8 | 6.4 | 12‡ | 74 | 269 | 296 | 397 |

* The same tests as given in Table 7.

† Note the similarity between these values and those for 9½-in. embedment in Table 7; compare also tests in Table 10.

‡ The same tests as given in Table 9.

been omitted from the table and diagram after a slip of 0.001 in., since the data for some of the tests are not complete; the position of the two points in the figure indicate that the remaining points probably would have occupied their proper places in the diagram.

In general, slipping began at a bond unit-stress only a little greater than for the plain rounds. For end slips of 0.0005 and 0.001 in. the average bond stresses are 301 and 330 lb. per sq. in. for plain rounds and 311 and 336 lb. per sq. in. for the corrugated bars; at this stage of the tests the stresses developed by the plain rounds were about 97% of those given by the corrugated bars. The highest bond stresses reported (end slip of 0.1 in.) are not materially different for the bars of the sizes used in these tests, and average about 715 lb. per sq. in. The somewhat lower values given by the 1⅛-in. corrugated round bars are probably due to the design of this bar as compared with the square bars; the projections on this bar present a smaller area to take the bearing stress which replaces the bond resistance after the failure of the adhesion than the square bars of type B which were used in the other tests in this group.

21. *Effect of Size of Plain Round Bar; Embedment 8 in.*—In this group of tests the diameter of the concrete cylinders was 8 in. and the length of embedment 8 in., while the diameters of the plain round bars used varied from ½ to 1¼ in. The results are summarized in Table 8. The load-slip curves are plotted in the lower portion of Fig. 7. The loads for various amounts of slip of the free ends of the different bars have been plotted in Fig. 10. The broken lines drawn in Fig. 10 show the trend of the values for the different amounts of slip and for the maximum loads. The values for end slip of 0.0005 in. may be taken as the beginning of slip. It is seen that in the earlier stages of the tests, the smaller bars develop the higher bond stresses; the load at beginning of slip of the free end of the bars varies from 323 lb. per sq. in. for the ½-in. rounds to about 253 lb. per sq. in. for the 1 and 1¼-in. bars. The maximum bond resistance is nearly constant and averages 391 lb. per sq. in.

22. *Effect of Length of Embedment; 1¼-in. Plain Rounds.*—In this group of tests the length of embedment varied from 4 to 24 in., corresponding to 3.2 to 19.2 diameters. One and one-fourth-inch bars were used in order to secure a wide range of embedded lengths without overstressing any of the bars. A summary of the tests is given in

Table 9. The load-slip curves have been plotted in Fig. 11. The bond stresses corresponding to various amounts of end slip up to and including the maximum load are shown for the different lengths of embedment in the upper portion of Fig. 12. In the lower part of the figure the maximum resistance and the stresses after the maximum for slips of 0.02 in., 0.05 in. and 0.10 in. are plotted.

The load-slip curve for 4-in. embedment drops off sharply after the maximum load, as a result of the splitting of some of the concrete blocks. Fig. 12 shows that the average bond stress corresponding to beginning of slip at the free end of the bar varies but little with increased embedment. At a slip of about 0.001 in. the average bond stress is nearly the same for all lengths of embedment included in these tests and amounts to about 300 lb. per sq. in. It is evident that when a slip of 0.001 in. has occurred at the free end of the bar, slipping has become general throughout the length of the bar, but it will be seen from a consideration of the conditions present and from the load-slip curves that the amount of slip represents quite different stages of the test in the specimens of different lengths of embedment. For the 4 or 6-in. embedment the difference between the amount of slip at the two ends of the block for any given load is not great, and

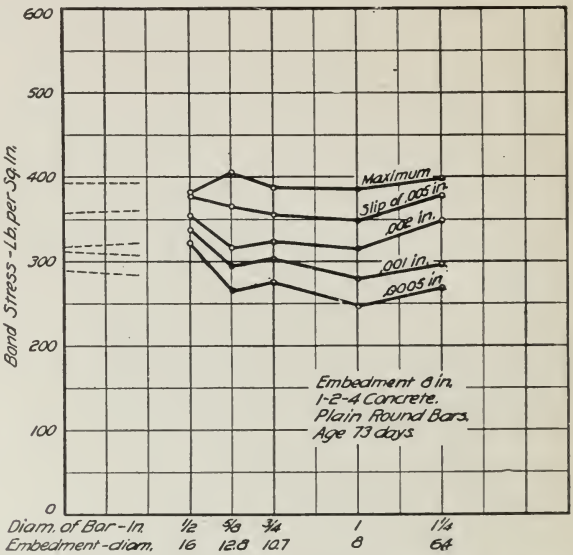


FIG 10. BOND RESISTANCE OF PLAIN ROUND BARS OF DIFFERENT SIZE; EMBEDMENT 8 IN.

## TABLE 9.

## Effect of Length of Embedment.

1-2-4 concrete from Batches 10, 17, 24, 27 and 30.

Diameter of concrete cylinder 8 in. in all tests.

These specimens were made from the same concrete as the specimens with deformed bars in Table 14.

The average compressive strength of 24 6-in. cubes from same concrete tested at about 60 days was 1760 lb. per sq. in.

Stresses are given in pounds per square inch.

| Length of Embedment | | Number of Tests | Age at Test days | Bond Stress at End Slip of | | Maximum Bond Resistance |
|---|---|---|---|---|---|---|
| inches | diameters | | | .0005 in. | .001 in. | |
| 1¼-in. Plain Rounds. | | | | | | |
| 4 | 3.2 | 5 | 74 | 265 | 314 | 375 |
| 6 | 4.8 | 5 | 74 | 243 | 391 | 420 |
| 8 | 6.4 | 12* | 74 | 269 | 296 | 397 |
| 12 | 9.6 | 5 | 75 | 284 | 312 | 390 |
| 16 | 12.8 | 9+ | 75 | 275 | 298 | 359 |
| 24 | 19.2 | 5 | 76 | 278 | 300 | 328 |
| 1⅛-in. Corrugated Rounds°. | | | | | | |
| 4 | 3.6 | 5 | 74 | 228 | 272 | 830† |
| 8 | 7.1 | 5 | 86 | 250 | 286 | 775† |
| 16 | 14.2 | 5 | 76 | 281 | 306 | 846† |
| 24 | 21.4 | 9+ | 77 | 268 | 301 | 688† |

* Includes five tests made with the group in Table 8 and two tests from Batch 37.

+ Includes five tests made with the group in Table 7.

° Blocks were reinforced against bursting by means of a ¼-in. spiral.

† Bond stress corresponding to end slip of 0.1 in.

we may expect that the load-slip relations given by these specimens (barring the premature splitting of the 4-in. blocks) are not much different from those for a unit of area at any point along the embedded length of the bar, while for the 24-in. embedment the load-slip relation at the free end of the block must be quite different from that at the other end.

If the straight lines which represent the general trend of the values of bond resistance for the amounts of slip shown in Fig. 12, be extended to the left to intersect the vertical axis of coordinates, the points of intersection may be said to represent the bond resistance per unit of area for a very short length of the embedded portion of the bar. These points have been plotted as shown by the dotted curve in Fig. 11. The load-slip curve which they form differs from the others in the same group in that slip begins at a somewhat lower proportional load and the bond resistance reaches a higher maximum value than indicated by the other curves, and the curve drops off a little more rapidly after the maximum. This curve may be said to represent the conditions for an infinitesimal embedment of a bar of this kind, and indicates what happens at each point along the embedded length of a bar during the progress of the pull-out tests in

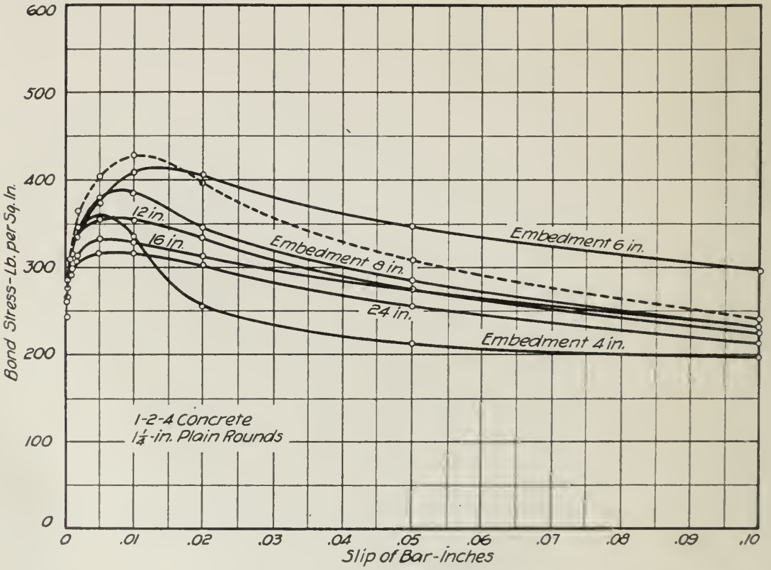


Fig. 11. Load-slip Curves for 1¼-in. Plain Round Bars; Embedment Variable.

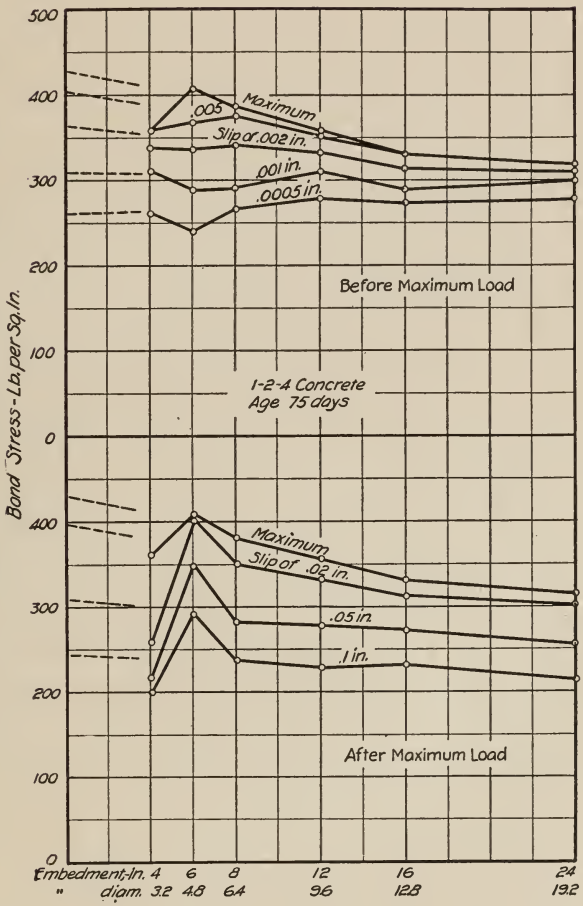


FIG. 12. BOND RESISTANCE OF 1¼-IN. PLAIN ROUND BARS; EMBEDMENT VARIABLE.

the same way as indicated in Art. 19 for another group of tests. The values derived in this way are as dependable as those given for other lengths of embedment, since they are based on the mean stresses from the entire group of tests. Data from these tests were used in plotting the solid curve in Fig. 5. The points in Fig. 5 for the plain round bars were derived from these tests and from the tests discussed in Art. 19. Only one curve has been drawn for the plain bars, although the values from the two series of tests are indicated by distinctive symbols. The similarity of the values obtained in this way is noteworthy, when we consider that these specimens were made from bars of different size and from different batches of concrete. The uniformity of these values gives considerable confidence in the tests and in the conclusions based on them. These values are significant in arriving at a proper conception of bond action for bars of this kind.

The maximum bond stress decreased as the length of embedment increased and varied quite uniformly from 410 lb. per sq. in. for 6 in. embedment to 320 lb. per sq. in. for 24 in. embedment. The lines in the lower division of Fig. 12 converge at a point about 290 lb. per sq. in. bond stress and 30 in. embedment. This indicates that with a longer embedment than 24 diameters the maximum average bond resistance would be less than that which gives first slip with shorter embedments. When a slip of 0.1 in. is reached, approximate equality of bond stress for all lengths of embedment is again found at about 235 lb. per sq. in.

23. *Effect of Length of Embedment;* 1⅛-*in. Corrugated Rounds; Blocks Reinforced against Bursting.*—The tests with corrugated bars embedded in reinforced blocks summarized in Table 9, may be considered as parallel to the group of tests on plain rounds made from the same concrete and included in the same table. In this group the concrete cylinders were all 8 in. in diameter, but the length of embedment varied from 4 to 24 in. The concrete blocks were reinforced against bursting by means of a ¼-in. wire in the form of a spiral as shown in Fig. 1 (b). The load-slip curves are given for the four lengths of embedment in Fig. 13. Each curve is the composite of all the tests in a set. According to the practice in all tests on deformed bars described in this bulletin, the highest bond stress which has been reported was that developed at an end slip of 0.1 in., although in most of the tests the load continued to rise beyond that point. In Fig. 14 the bond stress corresponding to various amounts of end

slip of the bar have been plotted against the length of embedment. The general direction of each of the zig-zag lines is indicated by the short dotted lines at the left margin of the figure. The points of intersection of these lines with the vertical axis of co-ordinates may be taken to represent the action of a similar bar of infinitesimal embedment, based on the general trend of the values from these tests. The load-slip curve corresponding to these points is shown by the dotted line in Fig. 13. As may be expected, it follows closely the curve for the 4-in. embedded length.

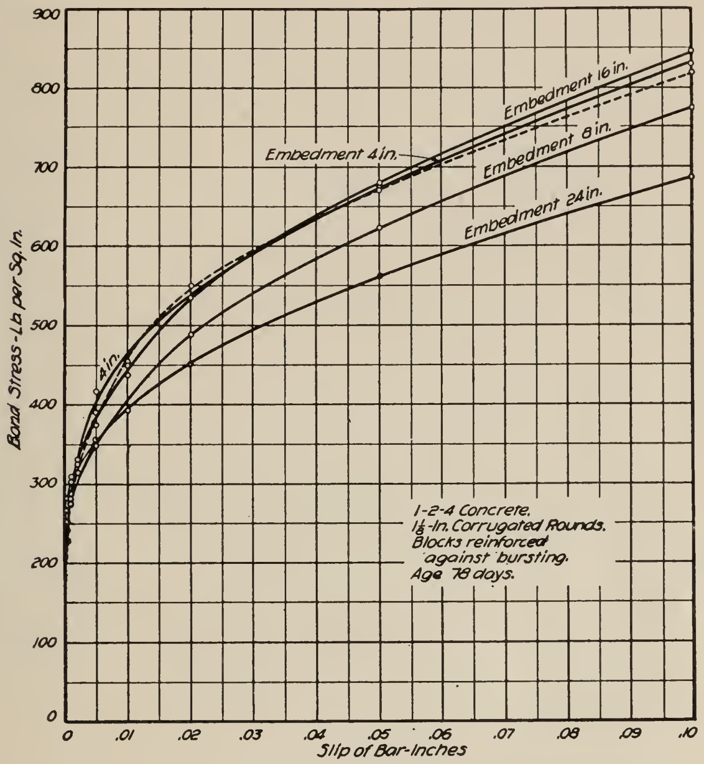


Fig. 13. Load-slip Curves for 1⅛-in. Corrugated Round Bars; Embedment Variable.

If the lines in Fig. 14 are projected to the right it will be found that they intersect at approximately the same point. The interpretation of this feature of the tests is probably the same as that suggested in the preceding article for plain round bars. For an embedment of more than 56 in. (50 diameters) the excessive deformation developed in the concrete and steel would so modify the distribution of bond stresses that an average stress greater than, say, 320 lb. per sq. in. could not be developed in a pull-out test with a bar of this kind.

In these tests slip began at a lower unit stress and the maximum stress was higher in the specimens of short embedment than in those of longer embedment. The bond stress at a slip of about 0.002 in. was about the same (320 lb. per sq. in.) for all lengths of embedment included in the tests.

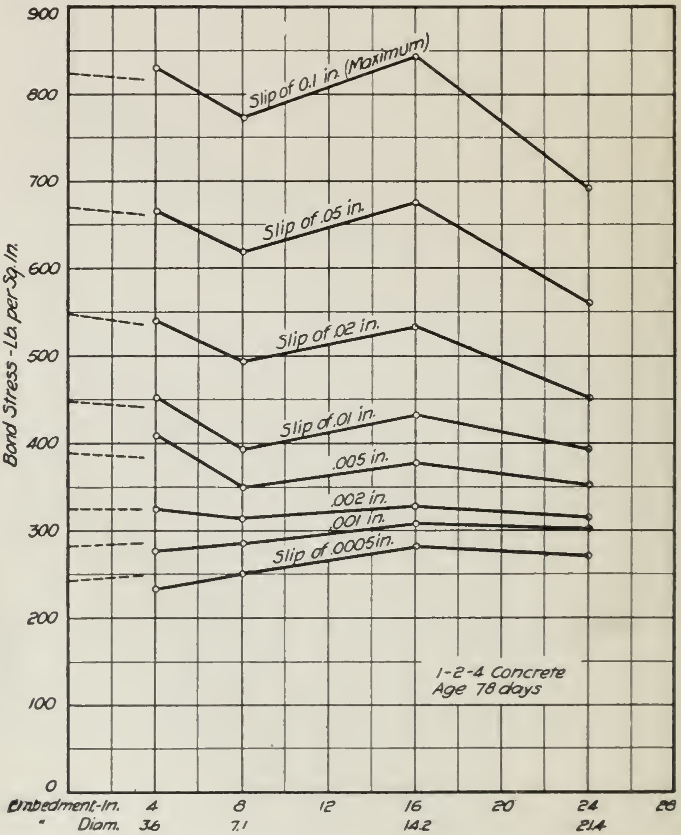


FIG. 14. BOND RESISTANCE OF 1⅛-IN. CORRUGATED ROUND BARS; EMBEDMENT VARIABLE.

There is a striking similarity between the bond stresses found for the beginning of slip of the 1⅛-in. corrugated round bars and those for the 1¼-in. plain rounds in the preceding article. Up to a slip of about 0.002 in. the plain rounds give slightly higher values at each stage of the tests. It is notable also that the line of equal bond stresses for all lengths of embedment for each of these groups came at a bond stress of about 300 lb. per sq. in. and at approximately the same amount of slip.

The load-slip curve for the 1⅛-in. corrugated rounds corresponding to an infinitesimal embedment has been plotted in Fig. 5. The interpretation of this curve is much the same as given above for the curve obtained in a similar manner for plain bars and included in the same figure. A comparison of the values for an infinitesimal embedment for this group of tests on 1⅛-in. corrugated rounds, with the similar curve from the two groups of plain bars discussed in Art. 19 and 22, is of interest. The plain rounds gave higher bond resistance than the corrugated bars for all amounts of end slip up to that corresponding to nearly the maximum resistance of the plain bars; after this point the corrugated bar steadily gained in bond resistance while the bond resistance of the plain bar decreased in the typical manner, after passing a slip of 0.01 in.

Pull-out tests with corrugated bars embedded in concrete blocks without spiral reinforcement are discussed in Art. 64.

24. *Effect of Variation in Diameter of the Concrete Block.*—Table 10 summarizes a group of pull-out tests using ¾-in. plain round bars in which the diameters of the concrete cylinders varied from 3 to 12 in. The embedment was 8 in. for all tests except the specimens with 8-in. cylinders which were embedded 9½ in. The load-slip curves for these tests are plotted in Fig. 15. In Fig. 16 the values of bond unit-stress for various amounts of slip and for maximum loads have been plotted. The values for the 3-in. cylinders do not show the usual increase of bond resistance after beginning of end slip. This is probably due to the high compressive stress developed in the concrete; at the maximum load this stress was about 930 lb. per. sq. in. In the 3-in. cylinders slip began at approximately 78% of the maximum load. If we disregard the 3-in. cylinder tests, the bond stresses showed a decided falling-off for all amounts of slip as the diameter of the cylinder increased. The maximum loads varied quite uniformly from 420 lb. per sq. in. for the 4 and 6-in. cylinders to 358 lb. per sq. in. for the 12-in. cylinders. The values for the 12-in. cylinders were about 84% of the average values for the corresponding amount of slip in the 4 and 6-in. cylinders. The difference may be due to variation in relative shrinkage in longitudinal and radial directions.

## TABLE 10.

### EFFECT OF VARIATION IN DIAMETER OF CONCRETE BLOCK.

All bars ¾-in. plain rounds.
Made from same batches as tests in Table 7.
Stresses are given in pounds per square inch.

| Diameter of Concrete Cylinder inches | Length of Embedment inches | Number of Tests | Age at Test days | Bond Stress at End Slip of | | Maximum Bond Resistance |
|---|---|---|---|---|---|---|
| | | | | .0005 in. | .001 in. | |
| 3 | 8 | 5 | 71 | 253 | 288 | 326 |
| 4 | 8 | 5 | 71 | 283 | 319 | 413 |
| 6 | 8* | 5 | 71 | 297 | 333 | 426* |
| 8 | 9½° | 5 | 70 | 287 | 311 | 386° |
| 12 | 8 | 5 | 72 | 245 | 272 | 358 |

* Compare the specimens with ¾-in. bars in Table 8.

° The same tests are given in Table 7.

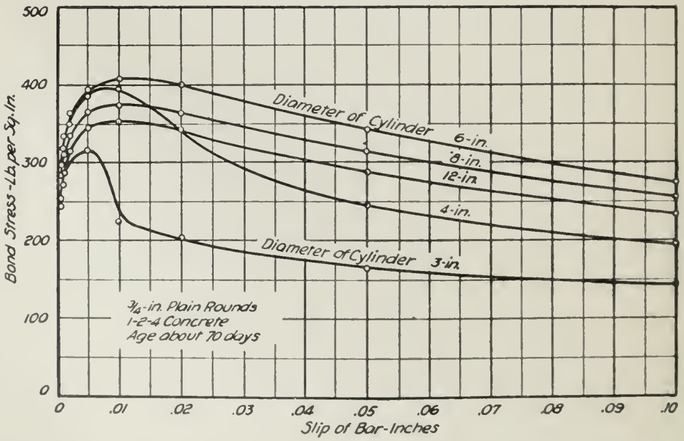


FIG. 15. LOAD-SLIP CURVES FOR ¾-IN. PLAIN ROUND BARS IN CYLINDERS OF DIFFERENT DIAMETERS.

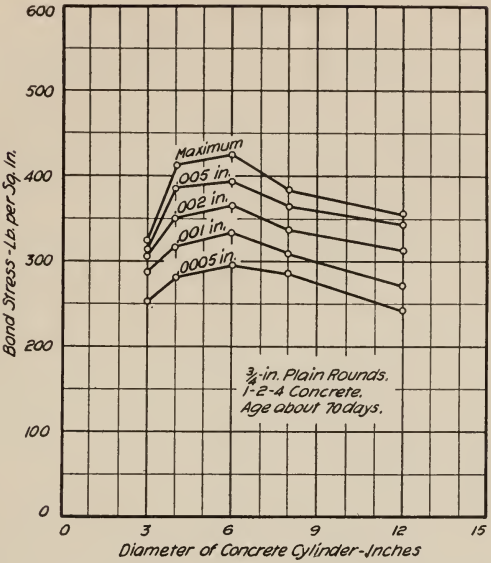


FIG. 16. BOND RESISTANCE OF ¾-IN. PLAIN ROUND BARS IN CYLINDERS OF DIFFERENT DIAMETERS.

### *b. Effect of Shape of Section and Condition of Surface of Bar.*

25. *Bond Resistance with Rusted Bars.*—Fig. 17 gives the load-slip curves for the specimens in Table 11 which have ordinary mill surfaces and rusted surfaces. For the bars having ordinary mill surfaces, end slip began at 267 lb. per sq. in.; the maximum bond resistance was 380 lb. per sq. in. and corresponded to an end slip of 0.01 in. The rusted bars had a heavy coat of firm rust caused by allowing them to remain below the surface in a pile of damp sand for five weeks previous to making the specimens. The tests on rusted bars gave a bond resistance higher than that developed with bars having ordinary mill surfaces. End slip began at 302 lb. per sq. in.; 13% higher than for round bars with ordinary mill surfaces. The maximum bond resistance of the rusted bars was 440 lb. per sq. in.; 16% higher than for ordinary rounds. The load-slip curve shows the maximum bond resistance to come at a somewhat greater slip than in the round bars of ordinary surface; this result is a natural consequence of the rougher surface, which is responsible also for the higher bond resistance developed by the rusted bars.

## TABLE 11.

### Effect of Shape of Section and Condition of Surface of Bar.

Embedment 8 in.

1-2-4 hand-mixed concrete from Batches 10, 11, 17, 22, 24, 25, 27, 30, 31 and 36.

The average compressive strength of 42 6-in. cubes made from 10 different batches of concrete, tested at about 60 days, was 1850 lb. per sq. in.

All stresses are given in pounds per square inch.

| Size and Kind of Bar | Number of Tests | Age at Test days | Bond Stress at End Slip of | | Maximum Bond Resistance |
|---|---|---|---|---|---|
| | | | .0005 in. | .001 in. | |
| **Plain Round Bars.** | | | | | |
| 1-in., ordinary mill surface | 5 | 69 | 267 | 302 | 380 |
| 1-in., rusted | 5 | 69 | 302 | 331 | 440 |
| **Flat Bars.** | | | | | |
| 1x½-in. | 6 | 69 | 359 | 395 | 459 |
| 2x¼-in. | 4 | 81 | 239 | 263 | 293 |
| **T-Bars.** | | | | | |
| 1x1x⅛-in. (area 0.27 sq. in.) | 5 | 64 | 282 | 295 | 310 |
| 1¼x1¼x $\frac{3}{16}$-in. (area 0.48 sq. in.) | 5 | 71 | 227 | 282 | 305 |
| 2x2x¼-in. (area 1.07 sq. in.) | 5 | 64 | 202 | 221 | 242 |
| **Polished Round Bars.** | | | | | |
| 1-in., polished | 5 | 69 | 149 | ... | 152 |
| ¾-in. tool steel, polished | 5 | 69 | 137 | 146 | 160 |
| ¾-in. tool steel, ordinary surface | 6 | 71 | 170 | 192 | 255 |
| **Bars of Wedging Taper*. Polished 1-in. Rounds.** | | | | | |
| Tapered 0.025 in. per ft | 4 | 66 | 171 | ... | 250† |
| Tapered 0.07 in. per ft | 5 | 69 | 162 | ... | 482† |
| Tapered 0.10 in. per ft | 5 | 69 | 173 | ... | 547† |
| Tapered 0.20 in. per ft | 5 | 69 | 163 | ... | 633† |
| Average | ........... | 68 | 167 | | |
| **Bars of Non-Wedging Taper. Polished 1-in. Rounds.** | | | | | |
| Tapered 0.025 in. per ft | 5 | 69 | 164 | ... | 170 |
| Tapered 0.07 in. per ft | 5 | 69 | 201 | ... | 222 |
| Tapered 0.10 in. per ft | 5 | 69 | 204 | ... | 210 |
| Tapered 0.20 in. per ft | 5 | 69 | 150 | ... | 155 |
| Average | ........... | 69 | 180 | ........... | 189 |

---

* The concrete blocks were reinforced against splitting by means of a ¼-in. wire in the form of a spiral. See Fig. 18 for sketch of specimen.

† Corresponding to an end slip of 0.1 in.

26. *Bond Resistance with Flat Bars.*—The flat bars presented about the same character of surface as the plain rounds. Through an error in making the test pieces, 6 specimens were made with 1 by ½-in. bars and 4 with 2 by ¼-in. bars. Direct comparison of the results of the tests is not as convincing as it might have been with an equal number of specimens of each size. The load-slip curves are plotted in Fig. 17. The 1 by ½-in. bars show a higher bond resistance and the 2 by ¼-in. bars a lower bond resistance than the corresponding plain rounds.

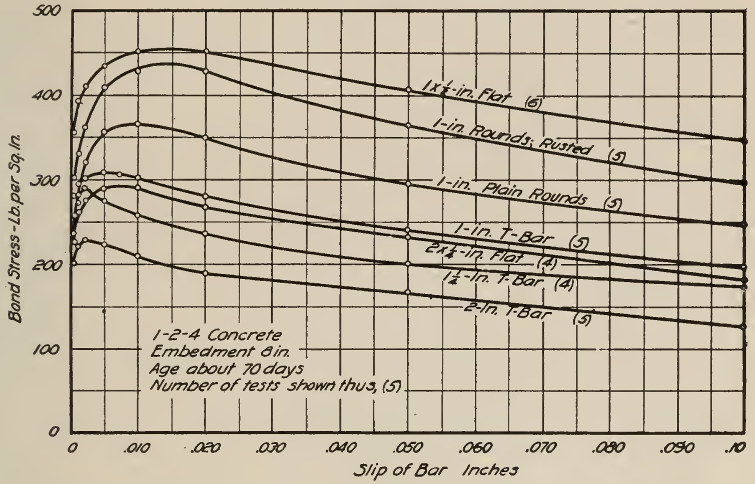


FIG. 17. LOAD-SLIP CURVES FOR PLAIN ROUND, FLAT AND T-BARS.

27. *Bond Resistance with T-bars.*—Pull-out tests were made on three sizes of T-bars, 1, 1¼ and 2-in., as shown in Table 11. The bond-slip curves in Fig. 17 show that the bond resistance for the T-bars reaches a maximum at a smaller amount of slip than for the rounds or flats; this is especially true of the larger sizes. The curves for the 1¼-in. and 2-in. T-bars drop off more sharply than usual immediately after the maximum. The smaller T-bars develop higher bond stresses than the larger ones, but all the values fall below those for the plain rounds. On the other hand the bond resistance per lineal inch of bar is twice as high for the 2-in. T-bars as for the 1-in. plain rounds, and the bond resistance per lineal inch for the 1-in. T-bar is about the same as for the 1-in. round.

28. *Bond Resistance with Polished Round Bars.*—A comparison of the bond resistance with polished bars and that with plain bars of the ordinary mill rolling may be expected to show how much of the bond is due to adhesive resistance and how much is due to what has been termed sliding resistance, resulting from irregularities of the surface of ordinary mill steel, and to inequalities in the section or alignment of the bars usually furnished for reinforced concrete construction. In other words, to what degree does the plain bar partake of the nature of a deformed bar? Two sets of tests were made on polished round bars; see Table 11. The 1-in. polished bars were round cold-rolled steel; they were polished by turning rapidly in a lathe and applying fine emery cloth. The ¾-in. tool steel was used as it came from the rolls; these bars had the smooth surface and uniform diameter which are usually found in bars of this kind.

It was characteristic of the tests of specimens with polished bars that as soon as an end slip amounting to between 0.0005 and 0.001 in. was developed, the adhesion was suddenly destroyed and the bar pulled out so rapidly that it was generally impossible to weigh the decreasing load on the testing machine. However, readings were obtained in one test, and the load-slip relation has been plotted in the lowest curve of Fig. 18. This curve differs from those of plain round bars in that the maximum load comes at the very beginning of slip and the curve drops off quite rapidly as soon as slip becomes appreciable.

The mean values of bond resistance from five tests each on ¾ and 1-in. polished bars are 143 lb. per sq. in. at slip of 0.0005 in. and 156 lb. per sq. in. at the maximum. The corresponding values for the 1-in. bars of ordinary surface are 267 and 380 lb. per sq. in. If special weight is given to the bond stresses developed for a small amount of slip, it may be said that the adhesive resistance of clean steel to concrete of this quality is about 150 lb. per sq. in., which amounts to about 55% of the bond resistance of bars of ordinary mill surface at a small amount of slip.

The results of the pull-out tests with tool steel bars having the original surface take a mean position between the values for the polished bars and ordinary plain round bars. Tests reported in Bulletin No. 1 (also in Table 25, Bulletin No. 8), University of Illinois Engineering Experiment Station, gave 136 and 147 lb. per sq. in. as the maximum bond resistance of 1-in. cold rolled bars, and ¾-in. round tool steel bars, respectively, embedded 6 in. in 1-3-6 concrete, and tested at 60 days. The pull-out tests on specimens with cold rolled round bars in which the concrete set under pressures of 100 lb. per sq. in. gave values about 10% higher than those in concrete setting without pressure (see Table 22.) The tests on the effect of loads reapplied after failure of bond on specimens in which smooth bars were used are discussed in Art. 60. Double pull-out specimens with cold rolled rounds are discussed in Art. 61.

29. *Tests with Tapered Bars.*—The tests on tapered bars were designed to throw light on the nature of bond resistance. Tapered bars of two distinct types were used. These types will be referred to as bars with wedging taper and bars with non-wedging taper. The form of the wedging-taper bar is shown in Fig. 18. Four different degrees of taper were used for each type. The taper varied from 0.025 to 0.2 in. per foot for each of the two forms. These bars were polished after machining so as to reduce them to a surface condition as nearly uniform as possible. The specimens with bars with wedging taper were reinforced against bursting by means of 6 or 7 turns of ¼-in. wire in the form of a spiral.

The results of the tests are given in Table 11. The bars with non-wedging taper developed an end slip of less than 0.001 in. and an average maximum bond resistance of 189 lb. per sq. in. before the adhesion was destroyed. Almost immediately the bond resistance fell to nothing. In only a few of the tests did the bar slip as much as 0.001 in. before the maximum resistance was reached. There was no apparent difference in the amount of slip developed before failure in the bars of the different degrees of taper, and there was no systematic relation between bond resistance and amount of taper. These bars gave approximately the same maximum bond resistance as the straight polished bars.

The tests on bars with wedging taper gave average loads for all the tests at the beginning of slip nearly the same as the average for the bars with non-wedging taper for the same amount of slip. The phenomena of these tests were quite different from those of other forms of polished bars or bars with mill surface, and exhibited some unusual features. The uniformity of the loads at first end slip is noteworthy. The load-slip curves for the bars with wedging taper are given in Fig. 18. Each curve is a composite of all the tests in a set. For comparison the load-slip curves for a single test on a straight polished bar and for the set of plain rounds with mill surface have been plotted on the same diagram.

It should be noted that the concrete blocks in this group of tests were reinforced against bursting and this allowed very high bond stresses to be developed. Up to the time that slip began, the bond stress developed was much the same as in the tests with the cylindrical polished bars; and it was not materially different from that found in the tests with the bars with non-wedging taper. In all the tests except those with a taper of 0.2 in. per ft., after a slip of about 0.005 in., corresponding to a bond stress of about 205 lb. per sq. in., the load dropped off as the bar was being withdrawn. With continued slip (the amount depending on the degree of taper), the load finally began to rise. During the first stage of the test it appears that the load was carried principally by adhesive resistance. After the breaking of the adhesion the load on the bar was taken by (a) frictional resistance developed by the compression and (b) the longitudinal component of the compression. The latter was small except with the largest tapers. That the break in the curves is a result of failure in adhesion is shown by the similarity of the stresses up to a slip of about 0.005 in., and by the fact that this bond stress is only a little higher than that found in the tests of polished straight bars and polished bars with non-wedging tapers. These load-slip curves indicate that after adhesion was destroyed the bar

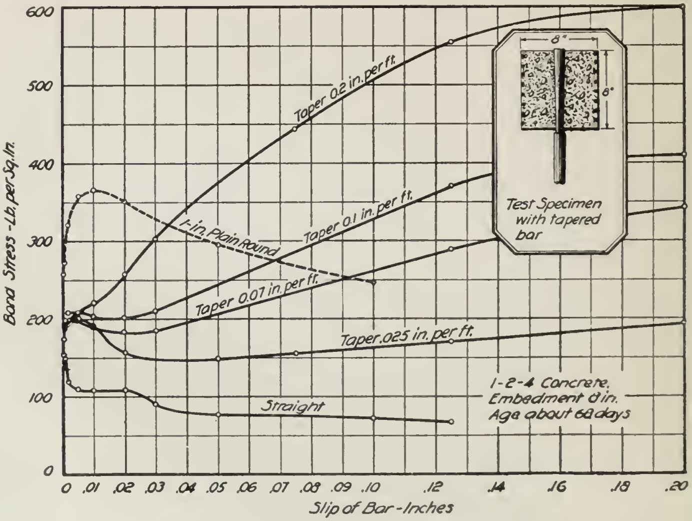


Fig. 18. Bond Resistance with Polished Bars of Wedging Taper.

slipped an amount which gave only a very small increase in diameter of section—less than 0.0005 in.—before the bond resistance again reached the maximum amount taken before adhesion was destroyed. As slip continued, the amount of increase in diameter at a given section of the concrete block, which corresponds to an additional bond resistance of 100 lb. per sq. in., was about 0.0005 in., being nearly the same for the several tapers. This change of diameter accompanied the compressive stress set up in the concrete. If the coefficient of friction between concrete and steel be taken at 0.25, the calculated normal compressive unit-stress between the concrete and steel will range from two to four times the bond unit-stress; the former figure being for the largest taper. It will be noted that for the larger tapers at the higher loads the load-slip curves change character and round off toward uniform bond resistance. It seems evident that this condition is due to the very high normal compressive stresses in the concrete.

The most notable feature of these tests is the sharp line of demarkation between the effect of adhesion and the effect of wedging action. The similarity of the behavior of the bars with wedging taper and the twisted square bars is discussed in Art. 41.

### c. *Effect of Condition of Storage.*

30. *Preliminary.*—It is a not uncommon belief that the permanency of submerged concrete work reinforced with plain bars may be seriously impaired by the ultimate failure of the bond between the concrete and steel. With a view to obtaining information on the effect of a variety of conditions of storage the tests summarized in Table 12 were made. The tests were generally made at age of about 60 days. In two of the groups the age at test varied from 7 days to about 3 years. All specimens, except those placed out-doors, were stored in the Hydraulic Laboratory. Thus the air-stored specimens were in a warm, damp atmosphere. The water-stored specimens were generally tested about 1 to 6 hours after removal from the water. Table 12 also contains the results of the compression tests of 6-in. cubes which were stored under the same conditions as the corresponding pull-out specimens.

31. *Batches 3 and 4.*—From batches 3 and 4 two nearly parallel groups of tests were made using different cements. Five specimens were made for each condition of storage and all tests were made at about 60 days. These specimens were made on January 12 and 13. Twenty-four hours after making, the forms were removed and the speci-

## TABLE 12.

### EFFECT OF CONDITION OF STORAGE.

1-2-4 hand-mixed concrete. Embedment 8 in.
Specimens were stored indoors unless otherwise noted.
Stresses are given in pounds per square inch.

| Size of Round Bar inches | Age at Test days | Number of Tests | Storage | Bond Stress at End Slip of | | Maximum Bond Resistance | Compressive Strength of 6-in. Cubes, Average of 3 Tests |
|---|---|---|---|---|---|---|---|
| | | | | 0.0005 inches | 0.001 inches | | |
| **Batch 3, Universal Cement.** | | | | | | | |
| ¾ | 62 | 5 | Damp sand | 556 | 620 | 702 | 2898 |
| ¾ | 60 | 5 | Air in laboratory | 313 | 375 | 498 | 1937 |
| ¾ | 62 | 5 | Water in laboratory | 493 | 562 | 670 | 3253 |
| ¾ | 62 | 5 | Water outdoors | 413 | 421 | 523 | 2558 |
| ¾ | 63 | 5 | Outdoors in air | 354 | 389 | 454 | 2022 |
| **Batch 4*, Chicago AA Cement.** | | | | | | | |
| ¾ | 61 | 5 | Damp sand | 387 | 421 | 538 | 1913 |
| ¾ | 50 | 5 | Air in laboratory | 246 | 278 | 398 | 1295 |
| ¾ | 62 | 5 | Water in laboratory | 359 | 405 | 522 | 1993 |
| ¾ | 61 | 5 | Water outdoors | 415 | 447 | 539 | 1677 |
| ¾ | 62 | 5 | Outdoors in air | 335 | 366 | 489 | 1642 |
| ¾ | 62 | 5 | Made outdoors in freezing weather | 64 | 67 | 80 | .... |
| **Batch 35, Universal Cement.** | | | | | | | |
| 1 | 7 | 3 | Air | 200 | 224 | 301 | .... |
| 1 | 7 | 3 | Water | 239 | 273 | 392 | .... |
| 1 | 41 | 3 | Air | 283 | 314 | 491 | .... |
| 1 | 41 | 3 | Water | 465 | 514 | 607 | .... |
| 1 | 41 | 3 | Damp sand | 429 | 482 | 606 | .... |
| 1 | 60 | 3 | Air | 221 | 254 | 416 | .... |
| 1 | 60 | 3 | Water | 415 | 492 | 601 | .... |
| 1 | 60 | 3 | Damp sand | 475 | 485 | 601 | 3040 |
| 1 | 14 mo. | 3 | Air | 332 | 389 | 562 | .... |
| 1 | 14 mo. | 3 | Water | 635 | 731 | 817 | .... |
| 1 | 26 mo. | 8 | Air | 445 | 513 | 668 | .... |
| 1 | 26 mo. | 3 | Water | 624 | 772 | 950 | .... |

* It should be noted that Batch 4 was a much leaner mix than Batch 3; the percentages of cement by weight are 12.7 and 15.4, respectively. The tests in Table 1 show that these two cements were of about equal strength.

## TABLE 12—CONTINUED.

### EFFECT OF CONDITION OF STORAGE.

1-2-4 hand-mixed concrete. Embedment 8 in.
Specimens were stored indoors unless otherwise noted.
Stresses are given in pounds per square inch.

| Size of Round Bar inches | Age at Test days | Number of Tests | Storage | Bond Stress at End Slip of | | Maximum Bond Resistance | Compressive Strength of 6-in. Cubes Average of 3 Tests |
|---|---|---|---|---|---|---|---|
| | | | | 0.0005 inches | 0.001 inches | | |
| Batch 41, Universal Cement. | | | | | | | |
| 1 | 7 | 2 | Air | 243 | 279 | 381 | .... |
| 1 | 7 | 2 | Water | 281 | 343 | 450 | .... |
| 1 | 7 | 5 | Water 4 days; air 3 days | 253 | 285 | 380 | .... |
| 1 | 31 | 2 | Damp sand | 456 | 528 | 694 | 2550 |
| 1 | 31 | 2 | Air | 309 | 426 | 632 | 2190 |
| 1 | 31 | 2 | Water | 383 | 437 | 640 | 2203 |
| 1 | 31 | 5 | Water 14 days; air 17 days | 417 | 465 | 562 | 2484 |
| 1 | 60 | 2 | Damp sand | 395 | 440 | 608 | 2560 |
| 1 | 60 | 2 | Air | 381 | 439 | 586 | 2650 |
| 1 | 60 | 2 | Water | 506 | 600 | 732 | 2420 |
| 1 | 60 | 5 | Water 30 days; air 30 days | 532 | 598 | 691 | 3160 |
| 1 | 14 mo. | 2 | Damp sand | 821 | 967 | 1060 | .... |
| 1 | 14 mo. | 2 | Air | 576 | 668 | 806 | .... |
| 1 | 14 mo. | 5 | Water 3 mo.; air 11 mo | 669 | 739 | 848 | .... |
| 1 | 26 mo. | 2 | Air | 531 | 656 | 815 | .... |
| 1 | 26 mo. | 2 | Water | 676 | 836 | 984 | .... |
| 1 | 37 mo. | 4 | Water | 750 | 780 | 936 | .... |
| Batch 39†, Universal Cement. | | | | | | | |
| ¾ | 61 | 4 | Water 50 days; air 11 days | 495 | 532 | 671 | 2360 |
| 1¼ | 61 | 4 | Water 50 days; air 11 days | 430 | 477 | 634 | .... |
| ¾ | 61 | 4 | Water 55 days; air 6 days | 476 | 517 | 641 | 2560 |
| 1¼ | 61 | 4 | Water 55 days; air 6 days | 431 | 483 | 572 | .... |
| ¾ | 61 | 4 | Water 58 days; air 3 days | 504 | 554 | 709 | 3007 |
| 1¼ | 61 | 4 | Water 58 days; air 3 days | 437 | 498 | 621 | .... |
| ¾ | 61 | 4 | Water 61 days | 387 | 559 | 658 | 3020 |
| 1¼ | 61 | 4 | Water 61 days | 318 | 449 | 471 | .... |
| ¾ | 70 | 4 | Water 60 days; air 10 days | 586 | 648 | 789 | 3223 |
| 1¼ | 70 | 4 | Water 60 days; air 10 days | 392 | 451 | 594 | .... |
| ¾ | 79 | 4 | Water 60 days; air 19 days | 561 | 673 | 766 | 2713 |
| 1¼ | 79 | 4 | Water 60 days; air 19 days | 460 | 536 | 619 | .... |

† The compressive strength of 6-in. cubes from Batch 39 which were stored in damp sand and tested at abou 60 days was 2190 lb. per sq. in.

mens stored as indicated in the table. The specimens stored out-doors in air were placed near the north wall of the building, where they were exposed to the weather during the greater part of an unusually severe winter. The specimens marked "made outdoors in freezing weather" were made at about noon with the thermometer at 15° F. The concrete was mixed indoors and carried to the forms in buckets. The specimens were left outdoors till the time of test. The concrete probably froze and thawed several times before finally hardening. When the specimens were broken up after testing, evidence of freezing could be seen in innumerable fossil-like crystal forms which were distributed throughout the concrete. Fig. 19 shows the load-slip curves for the tests from Batch 4; each curve is a composite of the curves for the five tests in a set.

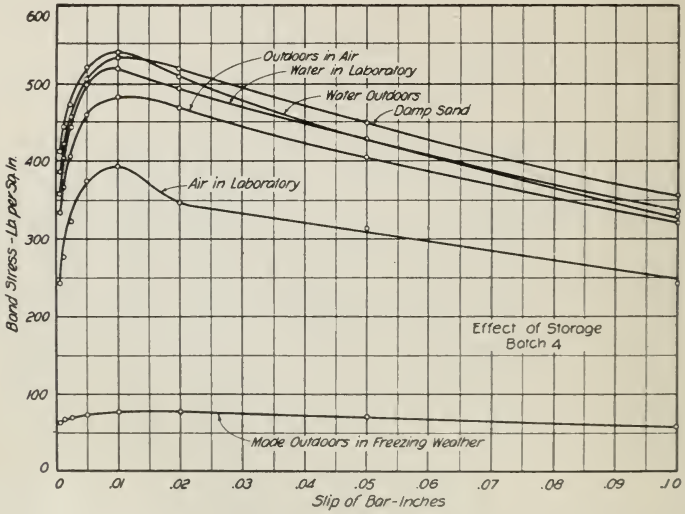


FIG. 19. LOAD-SLIP CURVES FOR DIFFERENT CONDITIONS OF STORAGE.

An examination of the values for maximum bond resistance given in the table shows that for both groups the damp sand storage gave the highest resistance, with water storage a close second. For the cube tests, these relations are reversed. There is little difference between the values for water storage indoors and outdoors—an average of 11% in favor of the indoor storage. In the case of air storage the outdoor specimens (average of batches 3 and 4) were 5% stronger than those stored indoors. The average of all water-stored specimens is 23% greater than for the corresponding air-stored specimens. For the cube tests the values are in the same order; the corresponding percentages are 21, 14 and 42, respectively. The similarity of the results for indoor and outdoor storage is surprising when the difference in temperatures is considered. The mean outdoor temperature during the season of storage was 31° F., while the temperature indoors probably seldom fell below 70° F. It seems probable that the loss of water due to evaporation from the specimens indoors in air had more effect in reducing the concrete strength than the low temperature outdoors. The excess of 11% for the water-stored specimens indoors over those stored in water outdoors may be considered to represent the difference in strength due to the higher indoor temperature.

The specimens made outdoors in freezing weather were almost devoid of bond strength. The cubes for this set were accidentally destroyed before the time of test.

The curves in Fig. 19 show considerable similarity in the bond-slip relations of the specimens stored differently. This similarity is maintained until the tests were discontinued at a slip of 0.1 in.

32. *Batch 35.*—In this group a comparison of sand, water and air storage was made for ages varying from 7 days to 26 months. The maximum bond resistances for the different ages for the air-stored and water-stored specimens are plotted in Fig. 20. The values for damp sand and water storage are nearly identical for both ages at which tests were made—41 and 60 days. The bond resistance for water storage is greater than for air storage for all ages at which tests were made; the maximum bond resistance for the water-stored specimens is greater by 30% and 23% at 7 and 41 days, and 77% and 42% at 14 mo. and 26 mo. The 60-day tests show an increase over the 7-day tests of 38% for the air-stored and 53% for the water-stored specimens. The 26-mo. tests show an increase over the 60-day tests of 50% for air storage and 64% for water storage. The specimens tested at 60 days show a slightly lower bond resistance than the specimens tested at 41 days; this, however, is probably an accidental variation.

The bond resistance at beginning of end slip shows about the same relation as at the maximum loads. The high bond resistance developed in the older specimens is noteworthy. For the water-stored specimens tested at 26 mo. the bond stress at beginning of slip of the free end of the bar was 624 lb. per sq. in., and the maximum bond resistance was 950 lb. per sq. in. The bond stresses developed by the specimens from Batch 35 are somewhat lower than those from Batch 3 for the same age and storage, although the 6-in. cube tests indicate that the concrete was of about the same quality.

33. *Batch 41.*—The tests from Batch 41 were in part duplicates of those in Batch 35. Four sets of specimens were placed in water during the first half of their storage period and in air for the remainder of the time. In this group the water-stored specimens gave a higher bond resistance at all stages of the tests than the air-stored. At 31 days the sand-stored specimens were stronger than the water-stored, while at 60 days the reverse was true. The highest values for bond on plain bars from the tests reported in this bulletin were found in the tests of the sand-stored specimens in this series, which were tested at 14 mo.; the maximum bond stress of 1060 lb. per sq. in. from these tests corresponds to a steel stress of 31 400 lb. per sq. in. in the 1-in. bar which was embedded only 8 in.

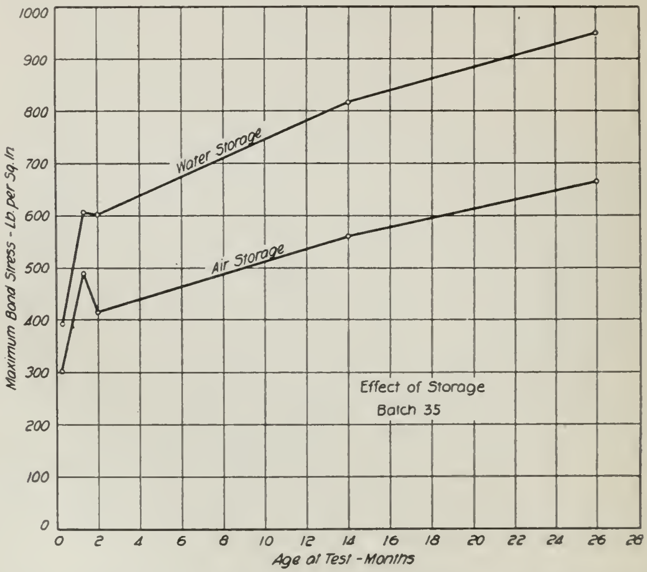


FIG. 20. MAXIMUM BOND RESISTANCE FOR SPECIMENS STORED IN AIR AND IN WATER.

The specimens stored 4 days in water and 3 in air gave about the same values as those stored in air for 7 days. Those stored 14 days in water and 17 days in air gave a somewhat lower maximum bond resistance than those stored 31 days in air. The specimens stored 30 days in water and 30 days in air gave values between the water and air stored specimens tested at 60 days. The specimens stored 3 mo. in water and 11 mo. in air gave values between the sand-stored and the air-stored specimens tested at 14 mo. In general these tests show a progressive increase of bond strength with age for all conditions of storage. The values at the older ages for all conditions of storage are high.

34. *Batch 39.*—In order to study the effect of storing specimens for a period in water followed by varying periods of air storage tests were made as given under Batch 39 in Table 12. The specimens stored in water 61 days were tested immediately upon removal from the water and were thoroughly wet when the load was applied. These tests indicate that the drying-out of the water-stored specimens before testing has an influence in increasing the bond resistance, although the evidence is not entirely conclusive.

35. *Discussion of Effect of Storage.*—The tests in Table 12 show about the same bond resistance for damp-sand storage as for water storage for ages up to 60 days; and the high stresses developed by the specimens in Batch 41 stored in damp sand for 14 mo. indicate that this relation may be expected to hold indefinitely unless affected by other agencies. The water-stored specimens gave from 10% to 45% higher bond resistance than the corresponding air-stored specimens, an average of 26% for 13 parallel sets of tests, based on the maximum bond stresses given in Table 12. This difference seems to increase with age; three parallel sets of tests made after 1 year show an average increase in bond resistance of 37% in favor of the water-stored concrete. The compression tests on 6-in. cubes show an average excess of strength of 25% for the water-stored specimens over the air-stored specimens. Since sufficient water was used in mixing the concrete, it is probable that evaporation of water from the concrete was the cause of the lower bond resistance in the air-stored specimens.

The presence of water not only does not injure the bond between the concrete and steel for ages up to 3 years, but it is an important factor in producing conditions which result in high bond resistance. Conditions of stress in concrete under load may modify these results, but it seems likely that the relative bond resistances in service will not be materially different from those found in the above tests with concrete of the same quality and with similar exposure.

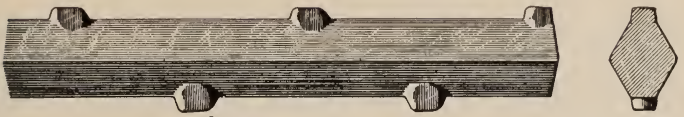
### *d. Bond Tests with Deformed Bars.*

36. *Preliminary.*—The term "deformed bar" is applied to a form of bar with projections on the surface or other frequent irregularities of section and designed and rolled specially for concrete reinforcement. These irregularities of section or projections are expected to increase the bond resistance of the bar. Some hundreds of patents have been issued on bars of this kind, but only a few forms have been used to any extent. Present building codes and engineering specifications generally permit the use of higher bond stresses in the design of structures in which deformed bars are to be used than are permitted in the case of plain bars.

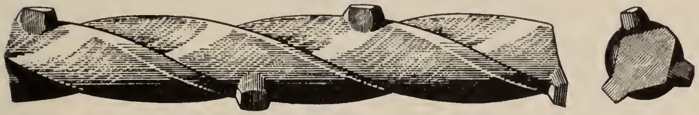
In the 1909 series, pull-out tests were made with seven commercial types of deformed bars, which number included most of the types of deformed bar in use at the time the tests were begun. Fig. 21 shows the forms of the bars used, and Table 13 gives the characteristics of these bars. For comparison, tests on specimens with plain round, twisted square, and threaded round bars were included. The size of the bars varied from ½ to 1⅛ in. The concrete cylinders were 8 in. in diameter with 8-in. embedment. The concrete was 1-2-4, hand-mixed. All specimens were stored in damp sand and tested at the age of about 2 months. In general, the tests were made in sets of five, one specimen of each kind having been made from each of five batches of concrete. A period of about 2 months elapsed between the making of the first and the last specimens. In all the specimens except those with plain round bars the concrete cylinders were reinforced against splitting by means of a wire spiral, since the load-slip relation was desired rather than values for ultimate strength. A summary of the average values from the tests is given in Table 14. The load-slip curves are plotted in Fig. 23 and 24.

Deformed bars were used in several other groups of pull-out tests which are not included in the present discussion. Tests with deformed bars in which the concrete blocks were not reinforced against splitting are discussed in Art. 64.

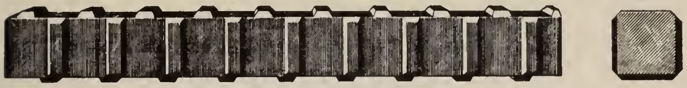
37. *Phenomena of Pull-out Tests with Deformed Bars.*—In the discussion of bond on plain bars it was seen that the adhesion between the concrete and steel was broken by a very small movement of the bar and as movement continued the bond was gradually taken by sliding resistance, which resulted from inequalities of the surface of the bar and from irregularities of its section and alignment. The projections on a



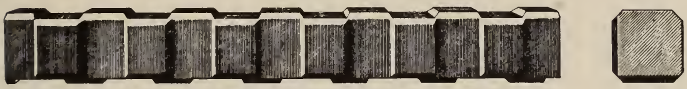
LUG BAR – STRAIGHT



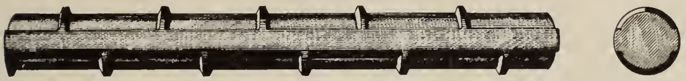
LUG BAR – TWISTED



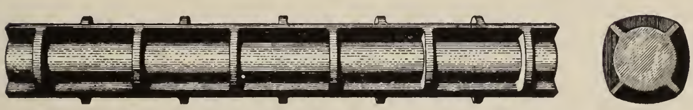
CORRUGATED SQUARE (TYPE A)



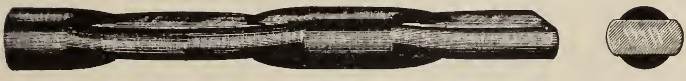
CORRUGATED SQUARE (TYPE B)



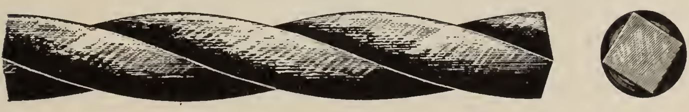
CORRUGATED ROUND (TYPE C)



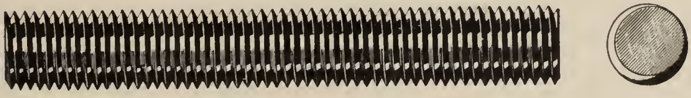
CUP BAR



THACHER BAR



TWISTED SQUARE BAR



ROUND BAR WITH STANDARD THREADS

FIG. 21. DEFORMED BARS USED IN PULL-OUT TESTS.

deformed bar give an exaggerated case of inequality of surface and irregularity of section. The tests described below indicate that the projections do not assist in resisting a force tending to withdraw the bar until a slip of bar has occurred approximating that corresponding to the maximum sliding resistance of plain bars. The sliding resistance of plain bars reaches its maximum value at a slip of about 0.01 in. It will be shown in Art. 51, in the discussion of tests on bars anchored by means of nuts and washers, that with the large bearing area present a distinct movement of the bar was necessary to bring this anchorage into action. This makes it clear that the adhesive resistance must be detroyed and sliding resistance largely overcome and that the concrete ahead of the projections must undergo an appreciable compression before the projections become effective. The action of anchored and deformed

## TABLE 13.

### Notes on Deformed Bars Used in 1909 Pull-out Tests.

The forms of these bars are shown in Fig. 21.

| Bar | Equivalent Section | Perimeter,[*] inches | Remarks |
|---|---|---|---|
| ½-in. twisted lug bar[°] | Hexagonal in section, equivalent to square bar of same nominal size. | 1.87 | Lugs 2.4 in. apart; 2.7 twists per ft. |
| ⅝-in. twisted lug bar[°] | | 2.32 | Lugs 2.4 in. apart; 2.0 twists per ft. |
| ¾-in. twisted lug bar[°] | | 2.78 | Lugs 2.4 in. apart; 1.7 twists per ft. |
| 1 -in. twisted lug bar[°] | | 3.78 | Lugs 2.4 in. apart; 1.0 twists per ft. |
| ½-in. cup bar | Round bar, equivalent to square bar of same nominal size. | 1.77 | Cups 1 in. long. |
| ¾-in. cup bar | | 2.65 | Cups 1⅛ in. long. |
| 1 -in. cup bar | | 3.56 | Cups 1¼ in. long. |
| ½-in. Corrugated sq. Type A | Square bar having the perimeter shown. | 1.32 | Corrugations .06 in. high 0.5 in. apart on all faces. |
| ¾-in. Corrugated sq. Type A | | 2.50 | Corrugations .08 in. high 0.7 in. apart on all faces. |
| 1 -in. Corrugated sq. Type A | | 3.56 | Corrugations .09 in. high 0.9 in. apart on all faces. |
| ½-in. Corrugated sq. Type B | Square bar of same size | 2.00 | Corrugations .06 in. high ¾ in. apart on all faces. |
| ¾-in. Corrugated sq. Type B | Square bar of same size | 3.00 | Corrugations .09 in. high 1⅛ in. apart on all faces. |
| 1 -in. Corrugated sq. Type B | Square bar of same size | 4.00 | Corrugations .11 in. high 1½ in. apart on all faces. |
| 9/16-in. Corrugated round Type C | Round bar | 1.77 | Circumferential corrugations 0.05 in. high, ⅞-in. apart. |
| 1⅛-in. Corrugated round Type C | Round bar | 3.34 | Circumferential corrugations 0.09 in. high, 1.6 in. apart. |
| ¾-in Thacher | ¾-in. round bar | 2.35 | Plain round bar flattened at intervals. |
| ½-in. sq. twisted | ½ in. square | 2.00 | Cold twisted; 2 twists per lineal foot. |
| 1 -in. sq. twisted | 1 in. square | 4.00 | Cold twisted; 1 twist per lineal foot. |
| 1-in. round bar, threaded | Round, diameter 0.89 in. | 2.80 | Standard V-shaped threads, 8 per inch. |

[°] The straight lug bars were like those given in the table, except for the twisting.

[*] The perimeters given are those of bars of the same weight and having the form of section indicated in the second column.

bars during the first stage of the tests does not differ much from that of plain bars. This conclusion might have been reached from our knowledge of the behavior of elastic bodies, but it is one which generally has been overlooked in discussions of the action of deformed bars.

After the adhesion is overcome and the sliding resistance of the smoother portions of the bar reaches its maximum value, the projections become effective and we may recognize a third stage in a pull-out test of a deformed bar in which the bond stress arises principally from the bearing stress between the projections and the concrete ahead. As slip continues, a larger and larger portion of the stress is taken in direct bearing. This bearing stress is opposed by a shearing stress over an area of concrete enveloping the projections. The exact stage of the test at which the projections become effective in taking stress and the amount of stress which finally may be taken in this way, depend, of course, on the design of the bar. The influence of the form of the bar and the secondary stresses developed in the concrete by deformed bars of different types are discussed in Art. 42.

Due to the elongation of the bar and to the yielding of the concrete ahead of the projections it is evident that the three stages mentioned above (adhesion, sliding resistance and bearing resistance) may be co-existent over a comparatively short embedded length of a deformed bar. In other words, the transition from the condition in which practically all the bond stress is taken by adhesion, only a little by sliding resistance and none by bearing, to that in which none is taken by adhesion, only a little by sliding resistance and practically all by bearing, is so gradual that we may not expect to find a sharp line of demarkation between them, but that the load-slip relation for deformed bars will be a continuous curve, as long as the concrete is intact.

Fig. 22 gives a typical load-slip curve for deformed bars. This curve is a composite from 55 tests, including all the tests given in Table 14 on bars of ¾ in. and larger sizes, except the plain round, twisted square and threaded bars. The twisted square and threaded bars are not considered in the present discussion of deformed bars.

The bond stresses for various amounts of end slip are given as percentages of the average bond stress which was developed at an end slip of 0.1 in. Since the value used for the highest bond resistance considered is purely arbitrary, it will be seen that the percentages given in Fig. 22 are only relative. However, the curve plotted in this form is useful in showing the relation of bond resistance to end slip of bar. It would have made little difference in the relative position of the curve if the average bond resistance corresponding to an end slip of, say, 0.05 in. or 0.2 in. had been used as 100%. End slip of bar began at about one-third the stress corresponding to an end slip of 0.1 in. After slipping began, the increase in bond resistance was nearly proportional to the amount of slip up to an end slip of about 0.01 in. After an end slip of 0.05 in. the bond resistance increased very slowly with further withdrawal of the bar. It should be borne in mind that spiral reinforcement was used to prevent splitting of the concrete blocks. It will be seen later that the presence of this reinforcement had a marked influence in increasing the bond resistance of the deformed bars even at small amounts of end slip.

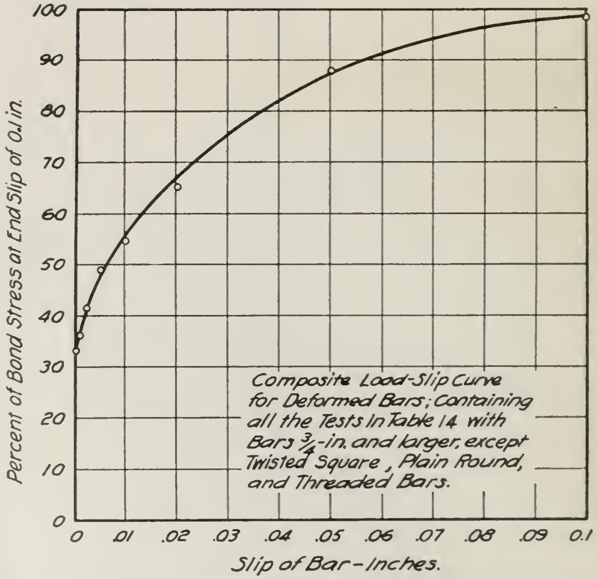


FIG. 22. COMPOSITE LOAD-SLIP CURVE FROM PULL-OUT TESTS WITH DEFORMED BARS.

A reference to Fig. 23 and 24, where the individual load-slip curves for these tests are plotted, will show that the form of the curves for deformed bars and their position on the diagram are not materially different from the curves for the plain bars up to an end slip of about 0.01 in., corresponding to the maximum bond resistance of the plain bars. This makes it apparent that up to this point the bond resistance of deformed bars is principally due to the same causes as in the plain bar tests, and that the projections have not appreciably come into action at this stage of the tests. All further increase in bond resistance is due entirely to the presence of the projections. The form of the load-slip curve after slip became general varied widely for bars of different types. The form assumed by any curve probably depends on the shape, size and spacing of the projections.

Some types of deformed bar developed very high bond stresses and it is apparent that the bearing stress between the concrete and the adjacent projections was abnormally high. Some notion of the importance of these stresses and their bearing on the proper design of a deformed bar which will best resist slipping through the concrete is given in Art. 42.

38. *Basis of Comparison of Bond Resistance of Deformed Bars.*—In computing the unit stresses given in Table 14 and for other tests with deformed bars, the perimeter used was that of a bar having the same sectional shape and the same weight per unit of length as the deformed bar under consideration. The perimeters given in Table 13 for these bars were determined from the weights of the bars in this way. In order to secure a uniform basis for comparison the bond stresses corresponding to given amounts of end slip will be used. Most of the deformed bars showed an increasing bond resistance even after a large amount of slip had occurred. The highest bond resistance reported for deformed bars was that corresponding to an end slip of 0.1 in., unless the maximum load came at a smaller amount of slip. A slip of 0.1 in. is very much larger than would be available or permissible in reinforced concrete construction.

It will be seen in the discussion of bond failures in beams reinforced with either plain or deformed bars that the amount of slip at the free end of the bar which may be developed before the maximum bond resistance in the beam is reached is much less than in the pull-out tests. It is evident that if we are to secure a proper criterion of the bond value of a deformed bar from the pull-out tests we must use a load which corresponds to a slip very much smaller than that at the maximum bond stress as defined above. Certain beam tests indicate that if the load which produces a slip of about 0.001 in. at the free end of the bar is repeated or maintained constant for a considerable period the beam will ultimately fail in bond at this load. This conclusion is based on tests of beams in which the longitudinal steel was not highly stressed—see Art. 90. In the case of plain round bars the pull-out tests show that the bond resistance at an end slip of 0.001 in. is about 75% of the maximum. Professor J. L. Van Ornum, Trans. Am. Soc. C. E., Vol. LVIII, 1907, p. 294, and Professor M. O. Withey, Bulletin No. 321, University of Wisconsin, have found that with plain bars a sufficient number of repetitions of a load 60 to 80% of what would be the maximum bond resistance under a load applied progressively to failure will cause bond failure in pull-out tests or in beam tests.

It will be seen also in the discussion of the beam tests that beginning of slip at the free end of the bar (slips amounting to 0.0002 in. or less) is almost simultaneous with the appearance of cracks in the outer region of the beam. This is an important reason for using for comparison bond stresses corresponding to small amounts of end slip.

These considerations show that it is the bar which longest resists beginning of slip that should be rated highest in comparison. In the following discussion of deformed bars, the bond resistance at a slip of 0.001 in. will be used as the principal basis of comparison of the bond resistance of bars of different forms. In the summary of the tests, Table 14, the relation of the bond resistance at an end slip of 0.001 in. has been given for each set of tests in terms of the resistance of plain round bars at the same slip. Similar ratios are given for the bond stresses at an end slip of 0.01 in. Mean values for 1 and 1¼-in. plain bars have been used as a basis of comparison in determining the ratios given in the table.

The high values found in the pull-out tests are much larger than would be obtained without the restraint of the spiral reinforcement. While such values may be available in the case of a bar simply anchored, it is evident that they are not available in beam or similar construction.

39. *Discussion of Deformed Bar Tests.*—The plain round bars used in this group of tests were of high-carbon steel with ordinary mill surface. The values of bond resistance found for the plain rounds are about the same as found in other groups of tests on plain bars. The average values for the two sizes of bar are 306 lb. per sq. in. for an end slip of 0.001 in. and 405 lb. per sq. in. for the maximum bond resistance. The blocks with plain round bars were not reinforced against bursting.

Using the above values from the plain bar tests as a basis, we may make some interesting comparisons. At an end slip of 0.001 in. the 12 sets of deformed bars of ¾-in. and larger sizes in Table 14 (the 55 tests which were used in plotting the curve in Fig. 22) developed an average bond resistance of 318 lb. per sq. in., or an average of 4% more than the plain rounds at the same end slip. At this stage of the tests, two sets of deformed bars gave practically the same bond resistance, five sets gave lower values, and five sets higher values than the plain bars. At an end slip of 0.01 in., corresponding to the maximum bond resistance of plain bars, the average bond resistance of the 12 sets of deformed bars was 445 lb. per sq. in., or 10% higher than the plain bars. At this stage, two sets gave about the same resistance, two sets gave lower values and eight sets gave higher values than the plain bars. The highest value (2 tests) was about 45% in excess of the plain bars; the average of the three next highest sets was about 23% in excess of the plain bars.

In the following discussion, little will be said about the tests with deformed bars smaller than ¾ in., but the values may be seen in Table 14. The tests with threaded round and twisted square bars are discussed in Art. 40 and 41.

Tests were made on three types of corrugated bars—types A, B, and C. Types A and B are square bars, type C is designated as round but has an oval contour. These bars were of high-carbon steel.

At a slip of 0.001 in. with bars of type A (formerly known as corrugated square, old style) the ¾-in. size gave 89% of the bond resistance of plain bars. The 1-in. bars gave 123%. The individual tests for the ¾-in. bars show a wide variation; the values ranged from 183 to 390 lb. per sq. in. at a slip of 0.001 in., with an average of 273 lb. per sq. in. The load-slip curves follow closely the composite curve for deformed bars given in Fig. 22. Type B (formerly known as corrugated square, new style) gave a percentage of 101 for both the ¾ and the 1-in. bars; the value for bond stress at slip of 0.001 in. averaged 308 lb. per sq. in. for each of these sizes. Type C (commonly known as corrugated round) gave a bond resistance of 256 lb. per sq. in., for the 1⅛-in. bar at an end slip of 0.001 in., a value below all the other deformed bars. However, in other groups of pull-out tests and in the beam tests in which this bar has been used the comparison is more favorable. In the tests on effect of length of embedment, Table 9, the 1⅛-in. corrugated rounds give a bond resistance of 291 lb. per sq. in. at a slip of 0.001 in., as the average of four lengths of embedment, ranging from 4 to 24 in., while the 1¼-in. plain rounds for the same lengths of embedment gave 302 lb. per sq. in. The bars of type A give the highest bond resistance both at beginning of slip and at maximum load found in the tests on corrugated bars. It is shown in Art. 42 that these differences may readily be accounted for by a consideration of the details of the design of the bars. The relative values for the larger sizes of corrugated bars may be seen by reference to Figs. 23, 24 and 25.

# TABLE 14.

## PULL-OUT TESTS WITH DEFORMED BARS (1909).

1-2-4 hand-mixed concrete from Batches 10, 17, 24, 27 and 30. Damp sand storage. Embedment 8 in.
All specimens except those with plain round bars were reinforced against bursting by means of 6 or 7 turns of ¼-in. wire in the form of a spiral.
The average compressive strength of 27 6-in. cubes at age of 60 days, was 1750 lb. per sq. in.
Stresses are given in pounds per square inch.

| Size and Kind of Bar | Number of Tests | Age at Test days | Bond Stress at End Slip of (inches) | | | | | | | Highest Bond Stress Considered* | Per cent at End Slip of 0.001 in.✚ | Per cent at End Slip of 0.01 in.✚ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | .0005 | .001 | .002 | .005 | .01 | .02 | .05 | | | |
| 1-in. plain round | 5 | 73 | 268 | 301 | 328 | 362 | 380 | 356 | 303 | 380 | 100 | 100 |
| 1¼-in. plain round | 9 | 70 | 272 | 311 | 352 | 401 | 418 | 390 | 317 | 418 | | |
| ½-in. cup | 5 | 76 | 306 | 425 | 453 | 497 | 545 | 631 | 828 | 891 | 139 | 134 |
| ¾-in. cup | 5 | 70 | 321 | 347 | 367 | 413 | 468 | 583 | 935 | 1146 | 113 | 116 |
| 1-in. cup | 5 | 73 | 290 | 323 | 365 | 426 | 504 | 630 | 916 | 1084 | 106 | 124 |
| ½-in. lug (straight) | 2 | 63 | 405 | 454 | 514 | 598 | 636 | 702 | 774 | 774 | 148 | 157 |
| ¾-in. lug (straight) | 5 | 69 | 306 | 332 | 356 | 415 | 454 | 521 | 647 | 760 | 108 | 112 |
| 1-in. lug (straight) | 5 | 73 | 324 | 371 | 408 | 455 | 490 | 503 | 562 | 644 | 121 | 121 |
| ½-in. lug (twisted) | 5 | 71 | 365 | 399 | 476 | 586 | 694 | 833 | 1072 | 1232 | 130 | 171 |
| ⅝-in. lug (twisted) | 3 | 62 | 263 | 291 | 330 | 425 | 535 | 655 | 842 | 1041 | 95 | 132 |
| ¾-in. lug (twisted) | 5 | 71 | 239 | 258 | 278 | 318 | 369 | 478 | 684 | 966 | 84 | 91 |
| 1-in. lug (twisted) | 5 | 73 | 251 | 288 | 326 | 374 | 403 | 445 | 556 | 724 | 94 | 99 |
| ½-in. corrugated square (type A) | 5 | 76 | 356 | 398 | 444 | 540 | 654 | 797 | ... | 903 | 130 | 161 |
| ¾-in. corrugated square (type A) | 5 | 70 | 245 | 273 | 310 | 361 | 469 | 577 | 820 | 861 | 89 | 116 |
| 1-in. corrugated square (type A) | 2 | 73 | 334 | 377 | 412 | 499 | 588 | 738 | 945 | 1025 | 123 | 145 |
| ½-in. corrugated square (type B) | 4 | 73 | 370 | 401 | 430 | 495 | 568 | 677 | 764 | 814 | 131 | 140 |
| ¾-in. corrugated square (type B) | 5 | 69 | 279 | 308 | 344 | 407 | 462 | 559 | 692 | 710 | 101 | 114 |
| 1-in. corrugated square (type B) | 3 | 68 | 247 | 308 | 355 | 428 | 497 | 590 | 741 | 797 | 101 | 123 |
| 9/16-in. corrugated round (type C) | 5 | 76 | 336 | 377 | 398 | 449 | 526 | 629 | 876 | 1048 | 123 | 130 |
| 1⅛-in. corrugated round (type C) | 5 | 74 | 236 | 256 | 271 | 299 | 334 | 411 | 624 | 824 | 84 | 82 |
| ¾-in. Thacher bar | 5 | 70 | 248 | 279 | 309 | 370 | 415 | 470 | 564 | 719 | 91 | 102 |
| ½-in. twisted square | 4 | 75 | 324 | 341 | 350 | 363 | 371 | 337 | 334 | 421 | 111 | 92 |
| 1-in. twisted square | 4 | 77 | 249 | 268 | 290 | 321 | 343 | 340 | 353 | 405 | 88 | 85 |
| 1-in. threaded round | 10 | 76 | 545 | 612 | 636 | 648 | 679 | 724 | 734 | 745 | 200 | 167 |

* For the deformed bars the highest bond stress considered corresponds to an end slip of 0.1 in. unless a maximum was reached at a smaller amount of slip.

✚ The bond resistance of deformed bars expressed in per cent of the average bond resistance of 1 and 1¼-in. plain rounds at an end slip of 0.001 in. and at an end slip of 0.01 in.

The general characteristics of the cup bar are somewhat similar to the corrugated rounds. These bars are round in section and have a sectional area equal to that of a square bar of the same nominal size. These bars were of high-carbon steel. At an end slip of 0.001 in. the ¾-in. cup bars show an efficiency of 113% and the 1-in. bars 106%, as compared with the plain rounds. The bond resistance developed by the cup bar at an end slip of 0.1 in. was the highest developed by any of the larger sizes of deformed bars; it averaged about 1100 lb. per sq. in. for the larger bars. However, it should be remembered that the concrete blocks were reinforced against bursting.

Tests were made on two different types of lug bar—straight and twisted. They were of high-carbon steel. The amount of twist and the spacing of the lugs are shown in Table 13. The ¾-in. straight lug bars show an efficiency of 108% and the 1-in. bars 121% as compared with the plain round bars at an end slip of 0.001 in. The ⅝-in. twisted lug bars show an efficiency of 95%, the ¾-in. bars 84%, and the 1-in. bars 94%. Considering only the ¾ and 1-in. bars, the values of bond resistance are from 10% to 30% higher for the straight bars than for the twisted bars. This is true for all stages of the tests except at the highest loads considered. It will be seen in the discussion of tests on square twisted bars (Art. 41) that this is a characteristic phenomenon of pull-out tests with twisted bars. It is noteworthy that at an end slip of 0.001 in. the straight lug bars had a bond resistance about 15% greater and the twisted lug bars about 10% less than plain rounds.

The Thacher bar consists of a round mild steel bar which is flattened at intervals. Only ¾-in. bars of this type were tested. This bar gave a bond resistance of 279 lb. per sq. in. at a slip of 0.001 in., 91% of the bond resistance of plain round bars at the same slip. The average load-slip curve for this bar follows almost exactly the composite curve for the larger sizes of deformed bars as given in Fig. 22.

In Fig. 25 the significant stresses from the pull-out tests on deformed bars have been plotted in a way that shows their relative values. The heavy solid line shows to scale the bond stress at an end slip of 0.001 in., the right end of the broken line indicates the bond stress at an end slip of 0.01 in., and the open line corresponds to the bond stress at an end slip of 0.1 in., which is the highest bond stress which has been considered in the discussion of the deformed-bar tests. The bars have been arranged in the figure in the order of the stress developed at an end slip of 0.001 in. For sake of comparison the tests on threaded bars,

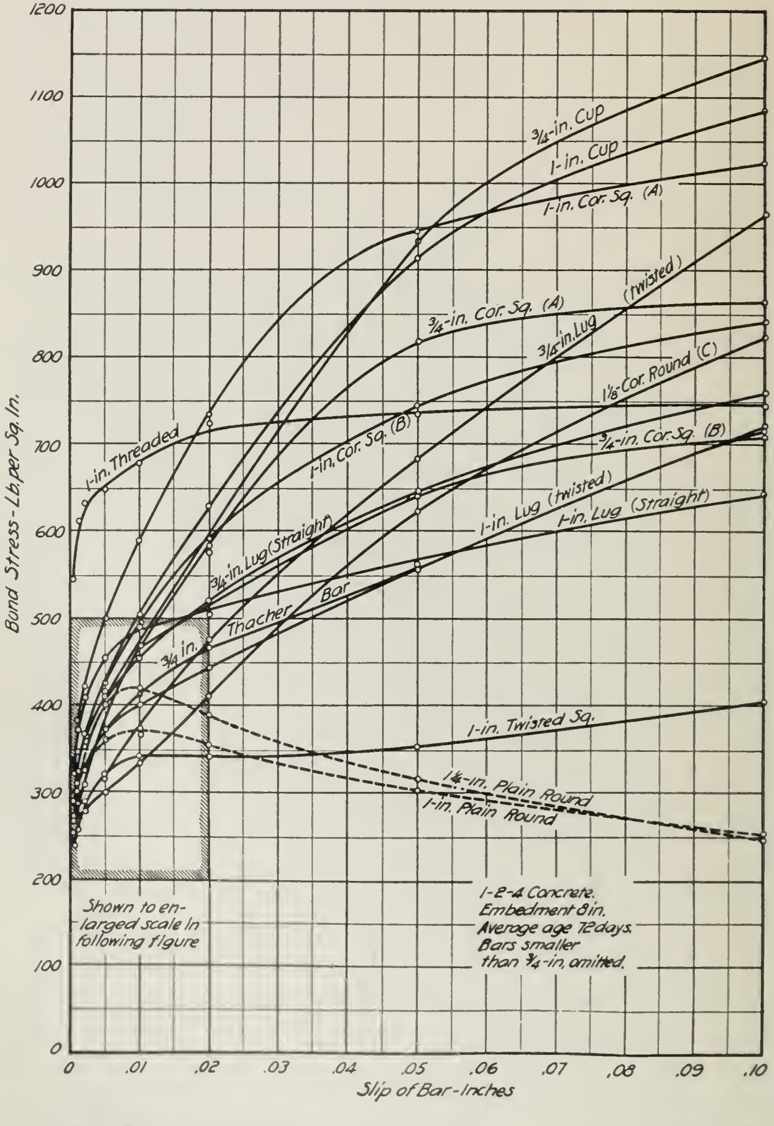


FIG. 23. LOAD-SLIP CURVES FROM PULL-OUT TESTS WITH DEFORMED BARS.

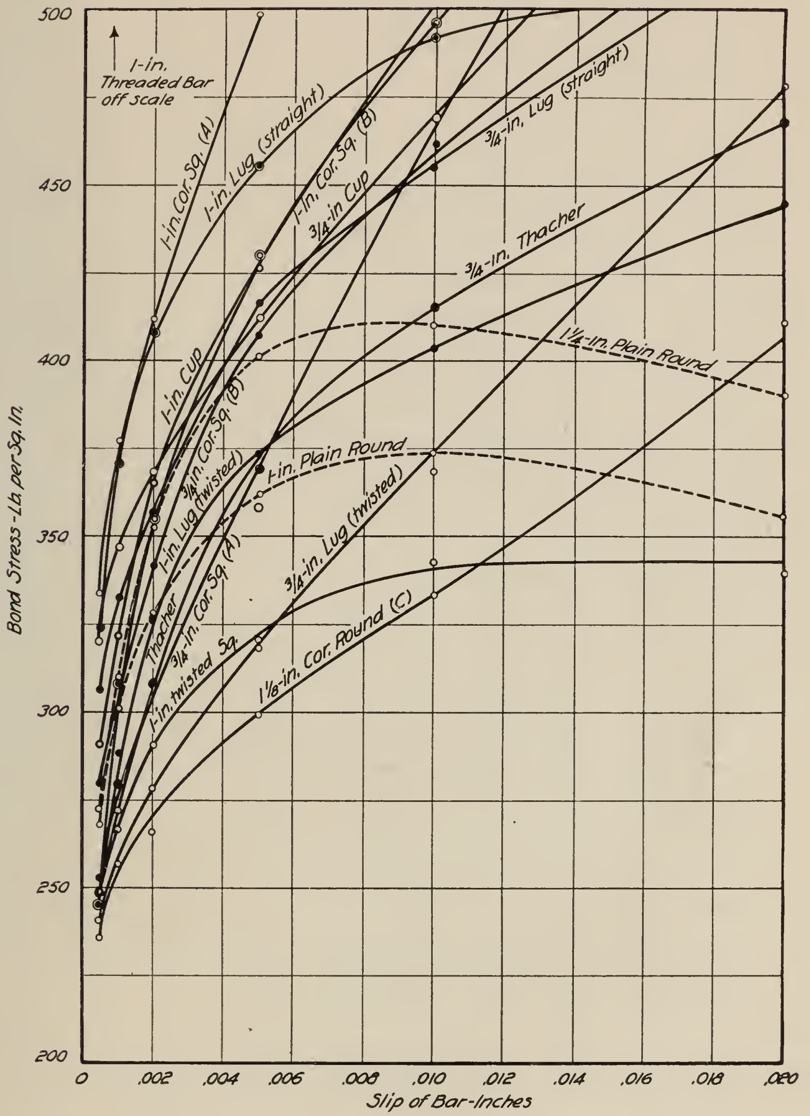


Fig. 24. Load-slip Curves from Pull-out Tests with Deformed Bars. (Enlargement of a Portion of Fig. 23.)

twisted square and plain rounds made from the same batches have been included. The average values for the 1 and 1¼-in. plain rounds have been used. With this arrangement it is of interest to note that the plain bars occupy about a mean position, with seven commercial bars above and six below.

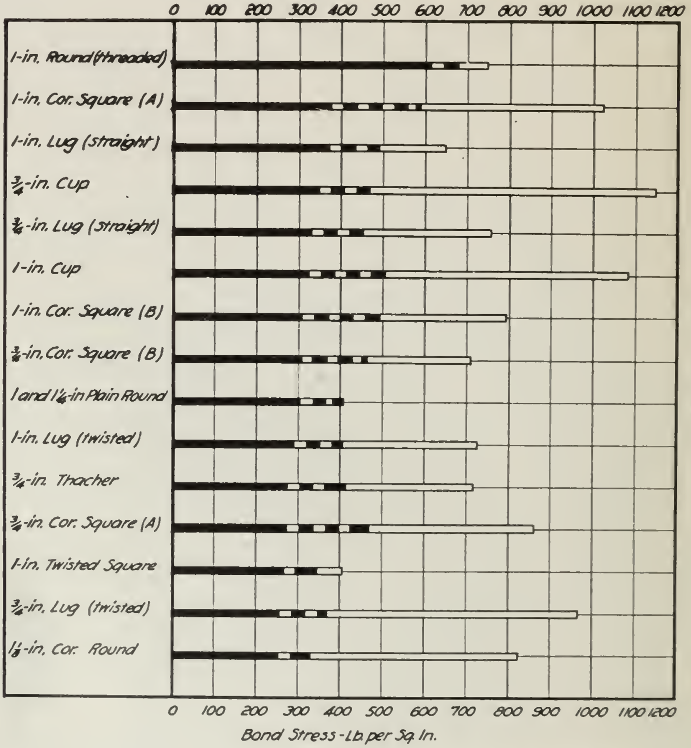


FIG. 25. RELATIVE VALUES OF BOND RESISTANCE OF DEFORMED BARS.

The heavy solid lines show to scale the bond stresses developed in the pull-out tests at an end slip of 0.001 in.; the right ends of the broken lines indicate the bond stresses at an end slip of 0.01 in.; the open lines indicate the bond stresses at an end slip of 0.10 in.

40. *Threaded Bars.*—In order to study the bond resistance of bars in which the projections were more numerous than in any of the available commercial bars, tests were made on 1-in. round bars on which a standard thread had been cut for the entire embedded length. The relation of this bar to the other deformed bars used is shown in Table 13. The results of 10 tests are given in Table 14 and the load-slip curves

in Fig. 23. This bar offers a much higher resistance to beginning of slip than any other form of bar used in these tests. The bond stress at an end slip of 0.001 in. varied from 500 to 742 lb. per sq. in.; average 612 lb. per sq. in. It will be seen that the lowest value for this bar is nearly as great as the highest for any of the commercial deformed bars of about the same size. This bar gave twice as much bond resistance as the plain round bars at a slip of 0.001 in. The maximum bond resistance came at a slip of about 0.1 in. and averaged 745 lb. per sq. in. The tests indicated that failure was produced by piecemeal shearing of the concrete surrounding the threads.

The pull-out tests on threaded bars made with the 1912 beams (see Table 34) gave somewhat lower values than those above, but the 1912 specimens were not reinforced against bursting.

41. *Twisted Square Bars.*—Pull-out tests were made on ½ and 1-in. cold twisted square bars. The characteristics of the bars may be found in Table 13. The load-slip curves for these tests have been included with the deformed bars in Fig. 23 and 24. Load-slip curves for the individual tests and for the plain rounds made from the same concrete are shown in Fig. 26. A mean curve has been drawn for each size of twisted square bars. The tests exhibit some peculiar phenomena. The load-slip curves are not unlike those for plain bars up to an end slip of about 0.01 in., except that they are lower on the scale. After a slip of about 0.01 in. has been reached there is a decided drop in the curves. This drop is found in all the curves in varying degree. It is more pronounced in the ½-in. than in the 1-in. bars. The mean curve for the 1-in. bars shows only a slight depression after a slip of 0.01 in., but the slip is increased to about 0.05 in. before there is any further increase in the bond resistance. From this point the curve continues to rise at an increasing rate. In the ½-in. bars the mean curve drops from about 373 lb. per sq. in. at a slip of 0.01 in. to 330 lb. per sq. in. at a slip of 0.04 in. and the end slip reaches about 0.10 in. before the bond resistance exceeds that corresponding to a slip of 0.01 in. After this point the course of the curve is similar to that of the 1-in. bars. The load required to pull out these bars would probably depend on the amount of restraint against the bursting of the concrete block. The maximum loads considered in the table were (as in the case of the deformed bars) those corresponding to an end slip of 0.1 in., although the load-slip curves show that a maximum point was reached at an end slip of about 0.01 in. Some of these bars were pulled out as much as

2 or 3 in. before reaching their highest resistance. The apparent bond stress at these large amounts of slip was very high, but of course such stresses and slips can not be developed in a structure, and are entirely meaningless under a rational interpretation of the tests. However, values obtained in this way have been commonly reported as the bond resistance of twisted square bars, and such tests have been generally used as the basis for the bond resistance of such bars.

The load-slip curves for twisted square bars are quite similar to those for polished bars with wedging taper given in Fig. 18. A little consideration will show the reason for this similarity and will also reveal important differences between the two cases. A longitudinal section of a twisted square bar presents an outline which consists of a series of equal arcs as shown in Fig. 26. The length of the radius and of the arcs will depend upon the amount of twist. The twisted bar is essentially a combination of a wedging taper and a non-wedging taper. As the bar begins to slip through the concrete the wedging tapers are drawn more firmly against the concrete, while at the same time the non-wedging tapers are separated from the concrete with which they were originally in contact. This separation of about one-half of the surface of the bar from its original contact is one important difference between the action of the twisted bar and that of the bar with wedging taper. Of course the separation of the surface of the twisted bars does not occur at once, on account of the curvature of the surfaces, and it may not be expected to occur as suddenly as with the polished bars on account of the difference in the texture of the surfaces. The load-slip curves for the twisted bars seem to indicate that this separation becomes pronounced at an end slip of about 0.01 in., which is about the slip at which plain bars have been found to reach their maximum bond resistance. The gradual drop in the curves suggests that the loss in bond resistance resulting from the separation of the non-wedging tapers and the continued sliding of the flatter portions of the longitudinal section have more influence in reducing the average bond resistance than the increased bearing of the wedging tapers against the concrete has in increasing the bond resistance. Continued slip of the bar finally allows the bond resistance due to the wedging action to rise and all further resistance must be due to this cause. It is evident that this action must produce a large twisting moment in the bar as well as a high splitting stress in the block. In the tests in which the bars were withdrawn, say ½ in. or more, this tendency to untwist was quite pronounced. This

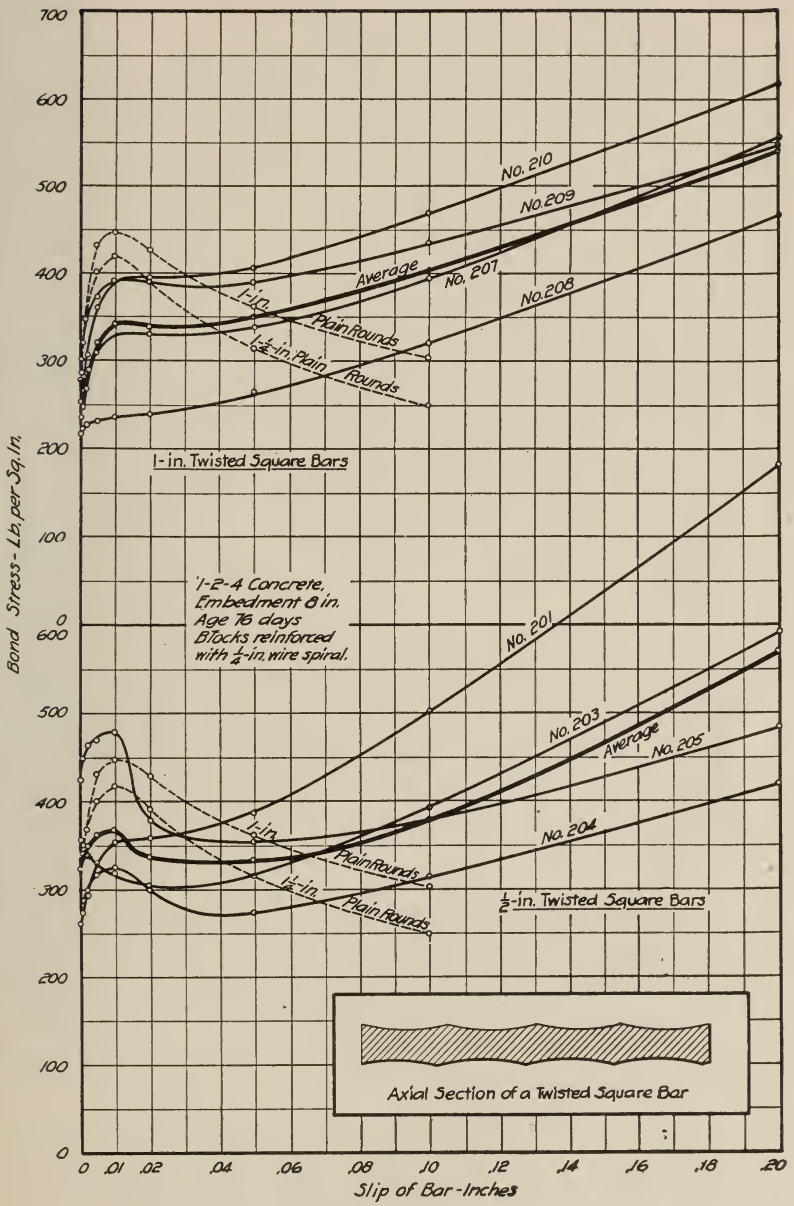


FIG. 26. LOAD-SLIP CURVES FOR TWISTED SQUARE BARS.

was shown in two ways: first, the cavity left in the top of the concrete block by the withdrawal of the bar presented the same spiral surfaces which were formed by the bar in its original position, the bar having to untwist somewhat during the movement; second, upon releasing the load at the completion of the test the blocks gave a sudden lurch in the direction of the twist in the bar. It should be borne in mind that this tendency to untwist was not pronounced until the bar had been withdrawn ½ in. or more, and that it had no effect at the loads given in the table, and may not be expected to have any influence at working stresses.

On the basis used in comparing deformed bars (the bond resistance at a slip of 0.001 in., as compared with the resistance of plain round bars) the rating of the ½-in. twisted square bars is 111% and the 1-in. bars 88%. At a slip of 0.01 in., which represents the first maximum in the load-slip curves for the twisted square bars, the bond stress for the 1-in. bars is 343 lb. per sq. in., or 80% of the value for plain rounds at the same slip. The bond stress on these bars at a slip of 0.1 in. is 93% of the maximum bond resistance for plain rounds.

It has been frequently stated that cold twisting is effective in raising the yield point of the bar by over-stressing a portion of the metal, and at the same time it furnishes a very severe test on the quality of the steel itself. However, it has been shown by tests that the elastic limit has been raised on only a portion of the section (the outside) and that for stresses above the original yield point the modulus of elasticity of the whole section is considerably smaller than the normal value for steel within the elastic limit. In other words, for stresses above the original yield point the metal in the interior of the section will be stressed beyond its elastic limit, and the rate of change in tensile deformations in the bar as a whole will be larger than at the lower stresses.

The tests here recorded show conclusively that the bond resistance of twisted square bars is inferior in characteristics to that of plain round bars of similar surface, and that these bars have little or no advantage in bond resistance within limits of slip which would be useful in structures. It seems strange that the twisted bar has gained such a wide popularity as a reinforcing material.

42. *Elements of the Bond Resistance of Deformed Bars.*—In Art. 37 and 39 the general phenomena of the tests of deformed bars were described. It seems desirable now to consider the secondary stresses which are developed in the concrete by deformed bars of the usual kind

and to learn the causes of the variations in the bond resistance of bars of different types. Such a study will indicate certain limitations of the deformed bar and furnish suggestions for the rational design of a bar which may be expected to give the best results.

We have seen that the projections do not become effective in taking bond stress until the bar has slipped an appreciable amount, and that during the later stages of the tests bond resistance is due principally to the bearing stress developed between the projections and the concrete in front of them. The pull-out tests showed that high bursting stresses were developed in the concrete blocks by the components of this bearing stress normal to the axis of the bar. This bursting action would be very destructive in a smaller mass of concrete which is under no restraint. This makes it apparent that if the bearing area of the projections is to be most effective in taking bond stress and least destructive in causing bursting, the planes of the bearing faces of the projections should be as nearly as possible at 90° to the longitudinal axis of the bar. The low bond resistance of the twisted square bar and certain deformed bars is due to the small angle which the so-called bearing faces make with the axis of the bar. In many of these tests the entire excess of bond resistance over that of plain bars was due to the restraint offered by the concrete block.

Having determined that the bearing faces should be as nearly perpendicular to the axis of the bar as practicable, the projections should be of such height and number that the proper relation between bearing stress on the concrete and shearing stress over the enveloping surface of concrete will be preserved. The ultimate bearing strength of concrete has not been studied, but these deformed-bar tests and the tests on anchored bars described in Art. 51 indicate that small areas which are restrained by a larger mass of concrete may be expected to carry a very high stress in bearing. These tests show that the concrete momentarily supported a bearing stress of from 8000 to 14 000 lb. per sq. in. Under these conditions the concrete was probably reduced to a powder as it yielded slowly under the stress.

The shearing value of concrete under these conditions has not been determined, but these deformed-bar tests show that it is higher than has commonly been assumed, and certain tests reported in Illinois Engineering Experiment Station Bulletin No. 8 indicate that it is nearly as great as the compressive strength. It may be noted that the values for the highest bond resistance given in Tables 14 and 15 were not the ultimate values of the bond resistance and that the shearing stresses finally developed over the surrounding concrete were larger than the values for bond stress which are given in the tables. However, the tests with threaded bars were the only ones in which failure occurred by shear in the concrete. In the threaded bars the bearing area was so large that the bearing stress was not great enough to produce an appreciable deformation in the concrete ahead of the threads and the elongation of the bar caused a piecemeal shearing of the concrete at an apparent stress much lower than was developed in the other tests. These considerations indicate that while high values of bearing and shearing stresses may be developed in such tests, the shearing resistance of the concrete will probably prove to be the limiting factor. The action of the threaded bars suggests that it is not desirable to keep the bearing stress too low in comparison with the shearing stress, since a slight yielding of the concrete due to a higher bearing stress will give a more uniform distribution of shearing stress and thus reduce the tendency to fail by shear. Practically all experiments in reinforced concrete in which the ultimate shearing stress was developed have emphasized the desirability of avoiding failures of this kind.

In order to study the bearing stresses developed in some of the deformed-bar tests, the values have been worked out for certain bars as shown in Table 15. The corrugated bars have been used since with the different types of bar they are instructive in making comparisons of bearing pressures, sliding resistance, and shear. For obvious reasons the lug bars and some other types could not be used in such a comparison.

It was seen above that, in general, the projections have little effect previous to an end slip of 0.01 in., and at a slip of 0.1 in. they may be considered to take all the bond stress except that taken by sliding resistance. If we consider the sliding resistance of a deformed bar to be the same as for a plain bar at the same slip, we may calculate what portion of the total bond stress is being carried by the projections at this stage of the test. It is appreciated that this method may be subject to error, since the secondary stresses in the concrete may be expected to cause an increased sliding resistance in the deformed bar, but on the other hand, this increase is probably largely counteracted by the bursting stresses developed in the concrete block. The sliding resistance of the plain bars in this series at an end slip of 0.1 in. was 250 lb. per sq. in. If we use 90% of this value, on account of the area lost by slip of bar, we may say that about 225 lb. per sq. in. of the highest bond stress of deformed bars was due to sliding resistance. The column in Table 15 headed "Bond Stress Carried by Projections at the Maximum Load Considered" shows the values obtained by subtracting 225 lb. per sq. in. from the bond stresses at an end slip of 0.1 in. for the bars shown. The bearing area of the projections per unit of area of the entire surface of the bar is shown in the fifth column of the table. The areas given are the areas of the normal projections of the bearing faces, without any reference to the angle at which they are placed. It will be seen that there is a wide variation in the ratio of the bearing area to the area of the surface of the bar. This value is about 11% in

### TABLE 15.

### Bearing Stresses Developed by Deformed Bars.

Stresses are given in pounds per square inch.

| Nominal Size of Bar inches. | Area of Section sq. in. | Height of Projections inches. | Spacing of Projections inches. | Area of Projections in Terms of the Area of the Bar, per cent. | Bond Stress at End Slip of | | Bond Stress Carried by Projections at the Maximum Load Considered. (At End Slip of 0.1 in.) | Bearing Stress at the Maximum Load Considered. (At End Slip of 0.1 in.) | Computed Bearing Stress with a Bond Stress of 100 lb. per sq. in., all Taken by the Projections. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 0.01 in. | 0.1 in. | | | |
| Corrugated Square (Type A). | | | | | | | | | |
| ¾ | .37 | .080 | .700 | 10.7 | 469 | 861 | 636 | 5850 | 940 |
| 1 | .70 | .105 | .875 | 11.4 | 588 | 1025 | 800 | 7000 | 880 |
| 1¼ | 1.07 | .125 | 1.185 | 10.0 | .......... | .......... | .......... | .......... | 1000 |
| Corrugated Square (Type B). | | | | | | | | | |
| ¾ | .56 | .078 | 1.10 | 6.0 | 462 | 710 | 485 | 8100 | 1660 |
| 1 | 1.00 | .112 | 1.50 | 6.5 | 497 | 797 | 572 | 8800 | 1540 |
| 1¼ | 1.56 | .114 | 1.75 | 5.5 | .......... | .......... | .......... | .......... | 1820 |
| Corrugated Round (Type C). | | | | | | | | | |
| ¾ | .44 | .082 | 1.30 | 5.5 | .......... | .......... | .......... | .......... | 1820 |
| 1 | .78 | .050 | 1.50 | 2.2 | .......... | .......... | .......... | .......... | 4550 |
| 1⅛ | .99 | .087 | 1.60 | 4.2 | 334 | 824 | 600 | 14000 | 2400 |
| Round Bar with Standard V-shaped Threads. | | | | | | | | | |
| 1 | .55 | .082 | .125 | 70.0 | 679 | 745 | 520 | 740 | 145 |

the type A bars, 6% in the type B bars and 2 to 5% in the type C bars; for the threaded bars the ratio is 70%. Considering that these bearing areas take all the stress in excess of that carried by adhesive resistance, the bearing stresses given in the table were computed. The bearing stresses for the corrugated bars, computed on this basis, vary from 5800 to 14 000 lb. per sq. in. The bearing stresses are seen to be inversely proportional to the bond stresses developed by these bars at an end slip of 0.01 in., when the projections were just beginning to take effect. These considerations show that the ratio of the normal projections of the bearing areas to the area of the surface of the bar is the proper criterion for judging the bond resistance of a deformed bar. The bearing stresses developed at this stage of the tests also show the absurdity of seriously considering the values that are usually reported as the bond resistance of many such bars.

The values in Table 15 also give some indication as to the proper ratio of the bearing area to the superficial area of the bar in order that the best results may be obtained. In order to obtain a notion of the bearing stress developed at ordinary bond stresses the values given in the last column of the table have been computed on the basis of a bond stress of 100 lb. per sq. in., all of which was considered to be taken in bearing by the projections. We may readily conclude that 11%, as in the type A bars, and the percentages found for the other corrugated bars are all too small, since the bearing stress is entirely too high; 70%, as in the threaded bars, is too high since the bearing stress is absurdly low and failure came by piecemeal shearing. The proper value lies between 11% and 70%; it seems probable that values of, say, 20% to 25% would give satisfactory results. Of course, a satisfactory value could readily be determined experimentally. With a ratio of 20% the bearing stress would be about four times the shearing stress over the enveloping surface if we disregard the sliding resistance; considering sliding resistance, the bearing stress would be about three times the shearing stress, which appear to be suitable values. A bar with projections extending around the circumference, practically perpendicular to the axis and having a height of, say, 1/10 of the diameter of the bar and spaced, say, ½ the diameter apart, would answer the requirements mentioned above. The spacing now ordinarily used would require so high a projection in order to secure the desired bearing area that the height would interfere with the practical requirements of manufacture and it would not give a good distribution of stress along the bar. A comparatively close spacing would give the necessary bearing area with the use of a minimum amount of metal. Advocates of deformed bars would do well to realize that a certain amount of metal must be sacrificed to the projections in a bar of proper design and to recognize the fact that if high bond resistance is desired, it must be paid for in much the same way as is done in dealing with tensile stress.

### *e. Effect of Age and Mix.*

43. *Preliminary.*—To determine the effect of age and mix, several series of pull-out tests were made, using ¾-in. plain round bars and ¾-in. corrugated square bars (type B), at ages varying from 2 days to 15 months and 3½ years, with the following mixes: 1-4-8, 1-3-6, 1-2-4, 1-1½-3 and 1-1-2. These tests are summarized in Table 16. All the bars were embedded 8 in. in 8-in. cylinders of concrete; they were all stored under the same conditions and tested in the same manner. All specimens tested after 4 days were stored in damp sand upon the removal of the forms; but the storage sand was allowed to dry out after about 2 years. The blocks with corrugated bars were reinforced against bursting by means of a spiral consisting of 6 or 7 turns of ¼-in. round wire. Generally the specimens were tested in sets of 5 at each age. The specimens of each mix were made from two batches of concrete, distributed as follows: two specimens with plain rounds and three of corrugated squares for each age were made from the first batch and the remainder from the second batch. In the case of the 1-1½-3 group a part of the tests with plain round bars was duplicated. The numbers of the batches used are given in the table; for further information regarding the concrete reference may be made to Table 4.

The "Highest Bond Stress Considered" for the corrugated bars is based on the stress at an end slip of 0.1 in., if a maximum had not been reached at a smaller slip. In only a few cases was the maximum load reached before a slip of 0.1 in. had occurred.

The values given in Table 16 are the averages for the number of tests noted. For convenience of reference, the values from the compression tests of 6-in. cubes for each mix have been included. As a ready method of comparing the effect of age on each mix, the percentage of the bond stress for the several ages at an end slip of 0.01 in. to the bond stress for the same slip at age of about 60 days is given in the table. It will be seen that it would not make any material difference in the percentages given if the bond stresses developed at another slip or at the

## TABLE 16.

## EFFECT OF AGE AND MIX.

Bars embedded in an 8 in. cylinder. Universal cement; hand-mixed concrete. Specimens tested after 4 days were stored in damp sand.
The specimens with corrugated bars were reinforced against bursting by means of 6 or 7 turns of ¼-in. wire in the form of a spiral (Fig. 1 (b)).
Stresses are given in pounds per square inch.

| Concrete | Age at Test days | ¾-in. Plain Round Bars | | | | | | | | ¾-in. Corrugated Square Bars | | | | | | | | | | Tests of 6-in. Cubes | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Number of Tests | Bond Stress at End Slip of (inches) | | | | | Maximum Bond Stress | Proportional Number† | Number of Tests | Bond Stress at End Slip of (inches) | | | | | | | Highest Bond Stress Considered* | Proportional Number† | Number of Tests | Average Crushing Strength | Proportional Number |
| | | | 0.0005 | .001 | .002 | .005 | .01 | | | | 0.0005 | .001 | .002 | .005 | .01 | .02 | .05 | | | | | |
| 1-4-8<br>Batches 21 and 26 | 2 | 5 | 16 | 17 | 19 | 23 | 25 | 27 | 14 | 4 | 12 | 13 | 17 | 21 | 28 | 37 | 51 | 64 | 11 | 6 | 191 | 18 |
| | 4 | 5 | 28 | 32 | 36 | 45 | 45 | 49 | 26 | 4 | 17 | 23 | 29 | 39 | 53 | 66 | 87 | 110 | 22 | 6 | 271 | 26 |
| | 7 | 5 | 27 | 32 | 38 | 47 | 52 | 54 | 29 | 5 | 30 | 35 | 41 | 56 | 70 | 85 | 115 | 133 | 28 | 6 | 375 | 36 |
| | 28 | 5 | 106 | 120 | 128 | 139 | 148 | 149 | 79 | 5 | 70 | 97 | 111 | 138 | 162 | 198 | 246 | 273 | 66 | 6 | 661 | 63 |
| | 65 | 4 | 124 | 135 | 150 | 171 | 183 | 190 | 100 | 5 | 109 | 139 | 158 | 203 | 246 | 302 | 365 | 391 | 100 | 6 | 1049 | 100 |
| | 122 | 3 | 160 | 172 | 176 | 179 | 187 | 210 | 111 | 5 | 141 | 159 | 189 | 232 | 276 | 343 | 445 | 470 | 112 | 6 | 1090 | 104 |
| | 15 mo. | 3 | 233 | 253 | 285 | 323 | 364 | 373 | 196 | 2 | 269 | 322 | 379 | 446 | 500 | 583 | 689 | 728 | 203 | .. | .... | ... |
| | 2 yr. | .. | ... | ... | ... | ... | ... | .... | .... | .... | .... | ... | ... | ... | ... | ... | ... | .... | .... | 3 | 2530 | 241 |
| 1-3-6<br>Batches 5 and 19 | 2 | 5 | 30 | 32 | 34 | 41 | 47 | 53 | 17 | 5 | 36 | 44 | 55 | 68 | 83 | 116 | 132 | 157 | 18 | 6 | 475 | 36 |
| | 4 | 5 | 37 | 43 | 52 | 62 | 69 | 77 | 25 | 5 | 69 | 74 | 88 | 111 | 134 | 165 | 213 | 239 | 30 | 6 | 641 | 48 |
| | 7 | 5 | 104 | 112 | 121 | 145 | 158 | 165 | 53 | 5 | 91 | 104 | 121 | 148 | 178 | 211 | 259 | 286 | 40 | 6 | 747 | 56 |
| | 28 | 5 | 118 | 130 | 159 | 206 | 230 | 241 | 77 | 5 | 155 | 179 | 211 | 269 | 312 | 363 | 436 | 462 | 69 | 6 | 1145 | 86 |
| | 60 | 5 | 208 | 227 | 248 | 287 | 302 | 311 | 100 | 5 | 241 | 280 | 317 | 398 | 450 | 502 | 597 | 623 | 100 | 6 | 1330 | 100 |
| | 132 | 5 | 352 | 398 | 441 | 498 | 525 | 536 | 173 | 5 | 266 | 326 | 363 | 436 | 508 | 586 | 693 | 746 | 113 | 3 | 1640 | 123 |
| | 6 mo. | .. | ... | ... | ... | ... | ... | .... | .... | 2 | 269 | 322 | 379 | 446 | 500 | 588 | 688 | 727 | 111 | 3 | 2250 | 169 |
| | 15 mo | 3 | 286 | 333 | 364 | 372 | 323 | 372 | 120 | .. | .... | ... | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... |
| 1-2-4<br>Batches 1 and 12 | 2 | 5 | 75 | 89 | 97 | 114 | 122 | 123 | 27 | 5 | 77 | 97 | 115 | 123 | 158 | 178 | 201 | 219 | 25 | 6 | 482 | 21 |
| | 4 | 5 | 97 | 110 | 131 | 146 | 152 | 153 | 34 | 5 | 108 | 129 | 145 | 170 | 196 | 222 | 271 | 305 | 31 | 6 | 726 | 32 |
| | 7 | 5 | 141 | 158 | 184 | 201 | 216 | 226 | 50 | 5 | 166 | 187 | 209 | 249 | 295 | 348 | 424 | 459 | 47 | 6 | 953 | 42 |
| | 14 | 5 | 227 | 247 | 261 | 278 | 289 | 300 | 66 | 5 | 200 | 216 | 240 | 290 | 342 | 401 | 449 | 477 | 55 | 6 | 1243 | 55 |
| | 28 | 5 | 266 | 288 | 318 | 360 | 392 | 404 | 90 | 5 | 274 | 306 | 347 | 414 | 457 | 510 | 607 | 641 | 73 | 6 | 1638 | 73 |
| | 60 | 5 | 332 | 363 | 394 | 432 | 450 | 452 | 100 | 5 | 390 | 434 | 479 | 559 | 627 | 682 | 820 | 854 | 100 | 6 | 2226 | 100 |
| | 120 | 5 | 419 | 469 | 511 | 562 | 596 | 603 | 134 | 5 | 537 | 576 | 628 | 707 | 767 | 871 | 1025 | 1079 | 120 | 3 | 2323 | 103 |
| | 6 mo. | 3 | 518 | 611 | 663 | 717 | 734 | 736 | 163 | 5 | 745 | 870 | 966 | 1070 | ... | ... | ... | 1426 | .... | 3 | 2370 | 105 |
| | 17 mo. | 4 | 705 | 800 | 818 | 829 | 785 | 841‡ | 186 | .. | ... | ... | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... |
| | 2½ yr. | .. | ... | ... | ... | ... | ... | .... | .... | 3 | 730 | 907 | 944 | 1080 | 1180 | 1383 | ... | 1549‡ | 189 | 6 | 4330 | 192 |
| | 3 yr. | 2 | 645 | 738 | ... | ... | ... | 848‡ | 188 | .. | .... | ... | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... |

* Corresponding to a slip of 0.1 in. at the free end of the bar, unless the maximum load was found at a smaller amount of slip.

† The per cent of the bond resistance at an end slip of 0.01 in. as compared with that of the 60-day tests with the same mix.

‡ Bars stressed to or beyond yield point.

## TABLE 16—CONTINUED.

### EFFECT OF AGE AND MIX.

| Concrete | Age at Test days | ¾-in. Plain Round Bars | | | | | | | | ¾-in. Corrugated Square Bars | | | | | | | | | | Tests of 6-in. Cubes | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Number of Tests | Bond Stress at End Slip of (inches) | | | | | Maximum Bond Stress | Proportional Number† | Number of Tests | Bond Stress at End Slip of (inches) | | | | | | | Highest Bond Stress Considered * | Proportional Number† | Number of Tests | Average Crushing Strength | Proportional Number |
| | | | 0.0005 | .001 | .002 | .005 | .01 | | | | 0.0005 | .001 | .002 | .005 | .01 | .02 | .05 | | | | | |
| 1-1½-3 Batches 2, 13 and 33 | 2 | 8 | 116 | 123 | 134 | 150 | 158 | 159 | 29 | 5 | 78 | 92 | 102 | 118 | 134 | 154 | 187 | 205 | 16 | 6 | 450 | 16 |
| | 4 | 7 | 176 | 195 | 210 | 226 | 224 | 231 | 42 | 5 | 100 | 115 | 124 | 145 | 167 | 190 | 231 | 258 | 20 | 6 | 713 | 25 |
| | 7 | 8 | 241 | 250 | 262 | 288 | 296 | 300 | 55 | 5 | 132 | 140 | 144 | 174 | 193 | 219 | 783 | 330 | 24 | 6 | 1044 | 36 |
| | 32 | 8 | 425 | 457 | 487 | 527 | 545 | 546 | 99 | 5 | 263 | 281 | 303 | 341 | 374 | 402 | 476 | 560 | 46 | 6 | 2288 | 80 |
| | 62 | 10 | 451 | 492 | 508 | 553 | 554 | 554 | 100 | 5 | 446 | 536 | 606 | 740 | 818 | 893 | ... | 1053+ | 100 | 6 | 2874 | 100 |
| | 120 | 3 | 484 | 538 | 585 | 653 | 663 | 667 | 120 | 2 | 474 | 564 | 640 | 770 | 894 | 1024 | ... | 1070 | 109 | 3 | 3928 | 137 |
| | 14 mo. | 5 | 768 | 875 | 892 | 891 | 875 | 896+ | 162 | .. | .... | ... | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... |
| | 2½ yr. | .. | .... | ... | ... | ... | ... | .... | .... | .. | .... | ... | ... | ... | ... | ... | ... | .... | .... | 3 | 4583 | 160 |
| | 3 yr. | 6 | 668 | 754 | 807 | ... | ... | 842 | 152 | .. | .... | ... | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... |
| | 3½ yr. | .. | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... | ... | ... | ... | ... | .. | .... | .... | 5 | 4804 | 167 |
| 1-1-2 Batches 20 and 33 | 2 | 5 | 99 | 107 | 118 | 133 | 141 | 141 | 27 | 5 | 87 | 96 | 104 | 123 | 145 | 162 | 202 | 231 | 17 | 6 | 490 | 13 |
| | 4 | 5 | 139 | 156 | 171 | 190 | 190 | 197 | 37 | 5 | 162 | 176 | 195 | 227 | 261 | 294 | 314 | 368 | 32 | 6 | 778 | 21 |
| | 7 | 4 | 177 | 202 | 221 | 240 | 245 | 246 | 47 | 5 | 158 | 171 | 194 | 238 | 276 | 314 | 379 | 419 | 33 | 6 | 1159 | 32 |
| | 28 | 5 | 272 | 300 | 328 | 365 | 378 | 393 | 74 | 5 | 313 | 344 | 383 | 461 | 537 | 633 | 750 | 828 | 65 | 6 | 2125 | 59 |
| | 61 | 5 | 354 | 399 | 436 | 498 | 526 | 530 | 100 | 5 | 445 | 498 | 555 | 678 | 829 | 989 | 1095 | 1132 | 100 | 12 | 3653 | 100 |
| | 121 | 2 | 437 | 479 | 551 | 613 | 645 | 666 | 126 | 3 | 540 | 599 | 668 | 776 | 920 | 1084 | 1142 | 1153 | 112 | .. | .... | .... |
| | 15 mo. | 3 | 454 | 656 | 694 | 738 | 769 | 779 | 147 | 2 | 710 | 892 | 1000 | 1168 | 1275 | 1405 | 1525 | 1535 | 136 | .. | .... | .... |
| | 2½ yr. | .. | ... | ... | ... | ... | ... | .... | .... | .. | .... | ... | ... | ... | ... | ... | ... | .... | .... | 3 | 5200 | 142 |

* Corresponding to a slip of 0.1 in. at the free end of the bar, unless the maximum load was found at a smaller amount of slip.

+ Bars stressed to or beyond the yeild point.

† The per cent of the bond resistance at an end slip of 0.01 in. as compared with that of the 60-day tests with the same mix.

maximum load were used as the basis of these values. Two groups of tests were made on mixes of the forms 1-2-*m* and 1-*n*-2*n*, where *m* varied from 0 to 8 and *n* from 1 to 5. The tests were made with ¾-in. plain rounds embedded 8 in. in 8-in. cylinders.

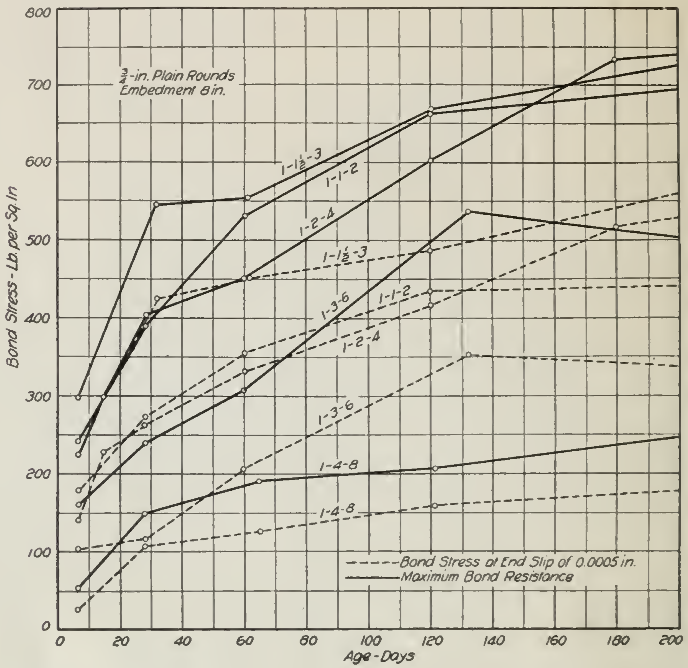


FIG. 27. EFFECT OF AGE AND MIX ON BOND RESISTANCE OF PLAIN ROUND BARS.

44. *Effect of Age on Bond Resistance.*—Fig. 27 shows the bond stresses developed in the pull-out tests with ¾-in. plain round bars with the different mixes of concrete at ages up to 6 months, at an end slip of 0.0005 in. and at the maximum. Load-slip curves for the 1-2-4 concrete at the various ages using plain bars are given in Fig. 28 and for the corrugated bars in Fig. 30. The load-slip curves for other mixes are omitted, but the values plotted for the 1-2-4 mix may be considered representative of the effect of age in the entire series.

The tests at early ages gave surprisingly high values. In the 1-2-4 concrete tests with plain bars at 2 days, slip did not become appreciable until a bond stress of 75 lb. per sq. in. was developed. The maximum bond resistance at this age was 123 lb. per sq. in—27% of the maximum at 60 days. At 4 days slip became appreciable at 97 lb. per sq. in.; maximum bond resistance 153 lb. per sq. in.—34% of the maximum at 60 days. The rate of growth of bond resistance was greatest from 2 to 7 days of age; for the older tests the rate of growth gradually became less. From 4 days to 28 days the growth in bond resistance for the various amounts of slip is nearly proportional to the age, as shown by the straight lines in Fig. 27 and 29; for 28 days to 6 months the growth in resistance is again proportional to the age. The rate of growth of bond resistance at early ages will probably depend largely on the quality of the cement used.

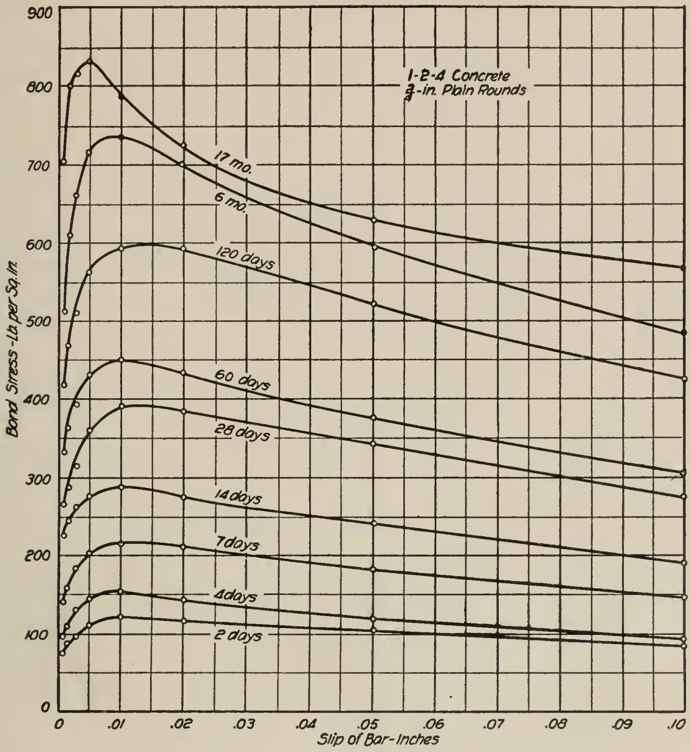


FIG. 28. LOAD-SLIP CURVES FOR PLAIN ROUND BARS EMBEDDED IN 1-2-4 CONCRETE.

At 60 days slipping began in the 1-2-4 concrete at a bond stress of 332 lb. per sq. in.; the maximum bond stress at this age was 452 lb. per sq. in. These values correspond closely to those found for 1-2-4 concrete in the tests reported in University of Illinois Engineering Experiment Station Bulletin No. 8. The bond stresses developed are about the same as found in the 1912 beam and pull-out tests for 1-2-4 concrete with 1-in. plain round bars. The bond stresses at a slip of 0.001 in. for ages 120 days, 6 mo. and 17 mo. are 129%, 168% and 220%, respectively, as compared to that at age of 60 days. It seems probable that the bond resistance for small slips, such as 0.001 in., for concrete of the kind used in these tests may be expected ultimately to reach a value as much as twice that developed at 60 days.

For mixes leaner and richer than 1-2-4 the relation between the bond resistance developed and slip of bar for the plain round bars is much the same as that found with the 1-2-4 concrete. The maximum bond resistance for the plain bars was found at a slip of about 0.01 in. and the bond stress at larger slips drops off in a manner similar to that shown in Fig. 28. In all cases the bond resistance for a given end slip increased with age, except for certain irregularities at the older ages. The values in Table 16 will enable the reader to make a further study of these tests.

The tests on ¾-in. corrugated squares formed a group parallel to those with ¾-in. plain rounds discussed above. The load-slip curves for the 1-2-4 concrete are given in Fig. 29. The load-age curves for all the mixes used are given in Fig. 30. During the interval between 2 days and 7 days the bond resistance of the 1-2-4 concrete increased about 100%. From 7 days to 6 mo. there is a nearly uniform increase in bond resistance with age. During this interval the rate of growth in bond resistance is about 110 lb. per sq. in. per month, for the beginning of slip, and 175 lb. per sq. in. per month at the maximum. The values for the 1-2-4 concrete at 180 days seem abnormally high as compared with those at earlier and later ages and as compared with the other mixes. For the smaller amounts of slip at ages under 6 days the bond resistance developed by the corrugated bars is not materially different from that found in the tests of plain round bars. For an end slip of 0.001 in. in the tests at 2 to 28 days, inclusive, the corrugated bars average 8% higher than the plain rounds; for ages of 60 days and over the average is about 25% higher. As the specimens were made from the same concrete at the same time, the results give comparative values for the two forms of bar. Absolute values will vary, of course, with the concrete used. At an end slip of 0.01 in., corresponding to the maximum bond resistance of the plain round bars, the corrugated bars developed from 10% to 25% more bond resistance than the plain rounds.

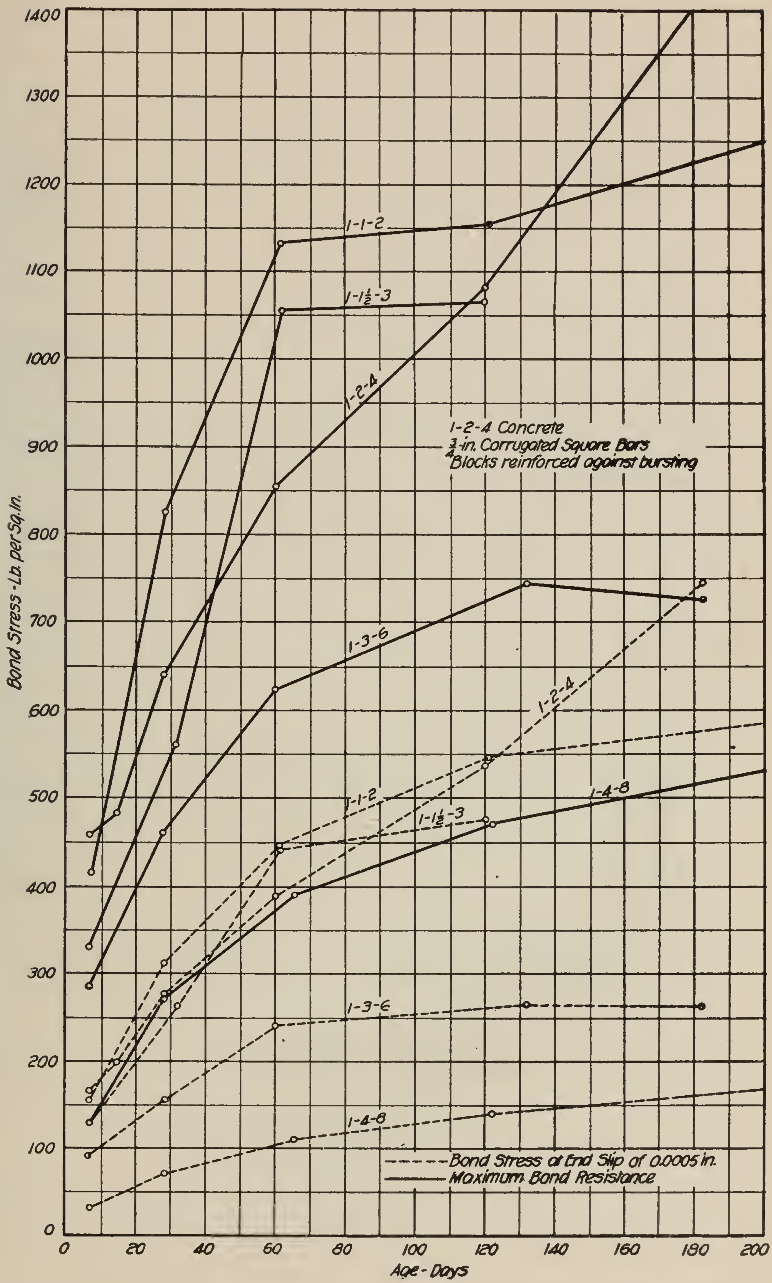


FIG. 29. EFFECT OF AGE AND MIX ON BOND RESISTANCE OF CORRUGATED BARS.

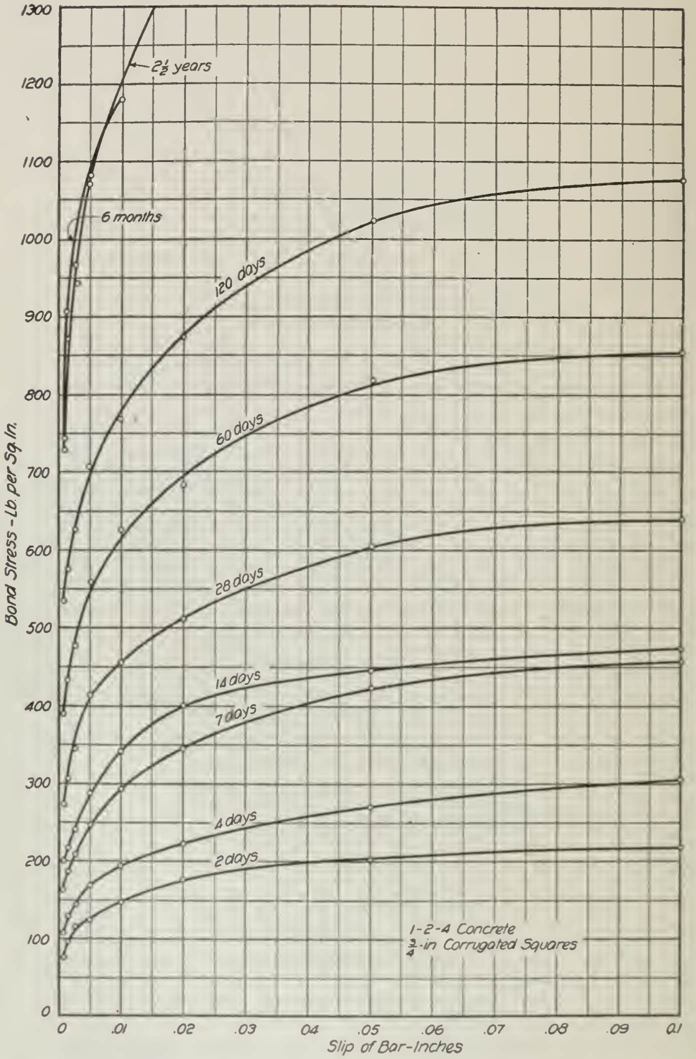


FIG. 30. LOAD-SLIP CURVES FOR CORRUGATED BARS EMBEDDED IN 1-2-4 CONCRETE.

With 1-4-8 concrete, for the smaller amounts of slip, the corrugated squares give lower values than the plain rounds for all ages under 15 months. This is true for slips up to and including 0.001 in., and in case of the 2 and 28 day tests the values for corrugated bars were lower until a slip of nearly 0.01 in. was produced, a slip corresponding to the maximum bond resistance of the plain rounds. The bond resistance for the 15-month tests is about twice that of the 60-day tests.

## TABLE 17.

### Comparative Values for Bond on Plain Round Bars.

**Compiled from the pull-out tests with ¾-in. plain rounds given in Table 16.**

**The values in each of the two principal divisions of the table are given as percentages of the corresponding value for the 60-day tests with 1-2-4 concrete.**

| Age at Test | At End Slip of 0.001 in. | | | | | At Maximum Bond Resistance | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1-4-8 | 1-3-6 | 1-2-4 | 1-1½-3 | 1-1-2 | 1-4-8 | 1-3-6 | 1-2-4 | 1-1½-3 | 1-1-2 |
| 2 days | 5 | 9 | 25 | 34 | 29 | 6 | 12 | 27 | 35 | 31 |
| 4 days | 9 | 12 | 30 | 54 | 43 | 11 | 17 | 34 | 51 | 44 |
| 7 days | 9 | 31 | 44 | 69 | 56 | 12 | 37 | 50 | 67 | 55 |
| 14 days | .. | .. | 68 | ... | ... | ... | ... | 66 | ... | ... |
| 28 days | 33 | 36 | 80 | 126 | 82 | 33 | 53 | 90 | 121 | 87 |
| 60 days | 37 | 63 | 100 | 135 | 110 | 42 | 69 | 100 | 123 | 118 |
| 4 mo. | 47 | 110 | 129 | 148 | 132 | 47 | 119 | 134 | 148 | 148 |
| 6 mo. | .. | ... | 168 | ... | ... | .. | ... | 163 | ... | ... |
| 13 to 17 mo. | 70 | 92 | 220 | 241 | 181 | 83 | 83 | 186 | ... | 177 |
| 2 to 3½ yrs. | .. | ... | 204 | 222 | ... | .. | ... | 188 | 198 | ... |

With 1-3-6 concrete, the values of bond resistance for the corrugated bars at a slip of 0.001 in. average about 25% higher than for the plain rounds at the same slip.

With 1-1½-3 concrete at ages 32 days and less the corrugated bars must slip 0.05 in. or more to develop the same bond unit stress as is developed by the corresponding plain rounds at a slip of about 0.01 in. For the 62-day and older tests slip began at about the same unit stresses in the two forms of bars.

The relative values of bond resistance of plain bars for the different ages and mixes at an end slip of 0.001 in. and at the maximum are given in Table 17.

45. *Effect of Mix on Bond Resistance.*—In Table 16 the comparative values of bond resistance at a slip of 0.001 in. and at the maximum for ¾-in. plain round bars and ¾-in. corrugated square bars are given for five mixes of concrete. In computing the proportional numbers in each of the two divisions of the table, the bond resistance for 1-2-4 concrete tested at 60 days has been taken as 100%. At 2 days and 7 days the leaner mixes show bond resistances relatively lower than the richer mixes. This is probably due to the slower hardening of the leaner mixes. The relative strength at ages of 15 months and over are about the same for the leaner as for the richer mixes. It will be noted that for all the tests on plain rounds the bond resistance at a slip of 0.1 in. is about the same as that causing beginning of slip.

Load-slip curves for plain bars embedded in concrete of different mixes, tested at about 60 days are shown in Fig. 31. The effect of mix will be further considered in the following articles.

46. *Mixes of the Form 1-2-m.*—This group included pull-out tests with eight mixes of the form 1-2-*m*, where *m* varied from 0 to 8; in other words the concrete consisted of a 1-2 mortar with varying quantities of coarse aggregate. ¾-in. plain round bars were used. The values from the tests are given in Table 18. Load-slip curves are plotted in the upper portion of Fig. 31. The relation of bond resistance to the per cent of cement in the mix is shown in the upper portion of Fig. 32. The percentage of cement is expressed in terms of the total weight of the aggregates in the batch. The values for 1-2-3 concrete seem erratic. If these values be disregarded, as indicated by the dotted lines in the figure, the adjacent points line up with the others. The maximum bond resistance varied from 341 lb. per sq. in. for the 1-2-8 concrete to 757 lb. per sq. in. for the 1-2-1 mix. We may say then that bond resistance in mixes of this kind increases roughly with the amount of cement used up to about 32% of cement. This relation does not hold when we omit the coarse aggregate, as was done in the 1-2-0 mix.

47. *Mixes of the Form 1-n-2n.*—The results of these tests are given in Table 18. The lines in the lower portion of Fig. 32 show the relation of bond resistance to per cent of cement used in the batch for various amounts of slip. While the 1-5-10 concrete gives very low values, it is seen that the bond resistance increases sensibly as a direct function of the amount of cement from the 1-5-10 to the 1-1½-3 concrete, or from about 6% to 20% of cement by weight. The values for the 1-1-2 mix do not differ much from the 1-1½-3 mix. It may be that

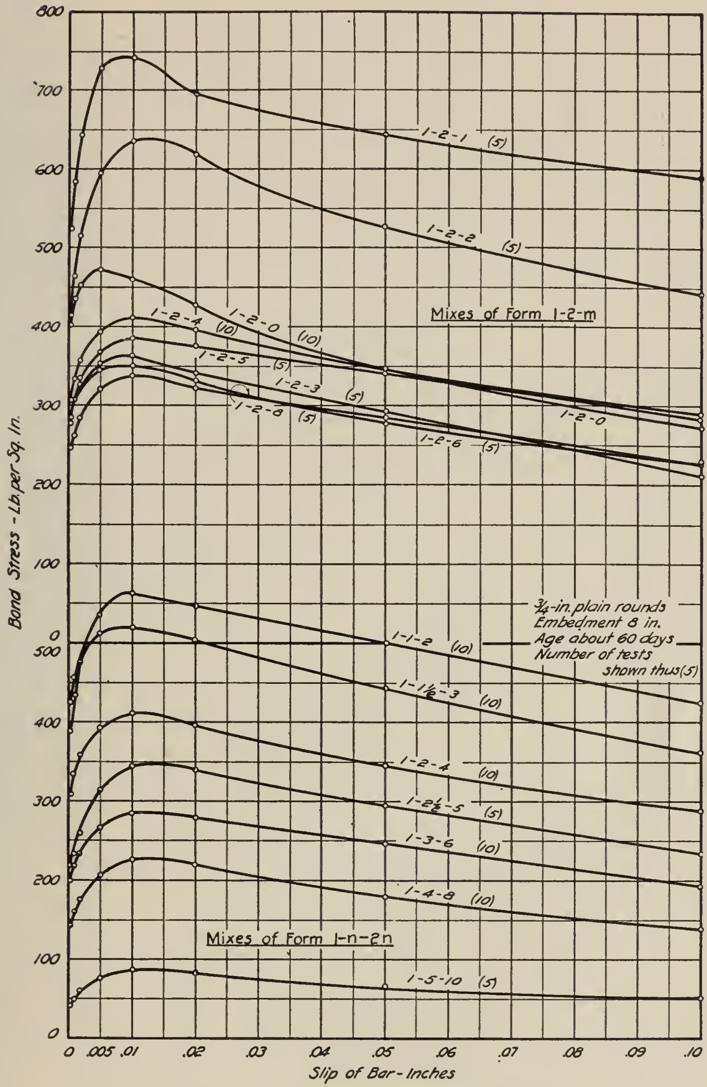


FIG. 31. LOAD-SLIP CURVES FOR PLAIN ROUND BARS.

the values for the 1-1½-3 concrete are abnormally high and those for 1-1-2 concrete somewhat low. The cube tests show a consistent relation for these mixes. However, it is evident that the general slope of these lines must decrease at the higher percentages, for it would be inconceivable for the bond resistance to continue to increase at the rate of, say, 100 lb. per sq. in. for each 5% of cement as it does for the interval between 5% and 20% in the figure. A comparison of the broken lines in the two diagrams of Fig. 32 shows that for the smaller percentages of cement the mixes of the form 1-2-*m* give somewhat higher bond resistances than those of the form 1-*n*-2*n*; but as the percentage of cement is increased (up to about 25%), the mixes of the latter form show a

## TABLE 18.

### EFFECT OF VARYING QUANTITY OF CEMENT AND MORTAR.

3¾-in. plain round bars; embedment 8 in.
Hand-mixed concrete; stored in damp sand.
Stresses are given in pounds per square inch.

| Mix | Batch No. | Per cent of Cement+ | Age at Test days | Number of Tests | Bond Stress at an End Slip of | | Maximum Bond Resistance | Test of 6-in. Cubes | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 0.0005 in. | 0.001 in. | | Number of Tests | Compressive Strength |
| | | | | Mixes of Form 1-2-m. | | | | | |
| 1-2-0 | 29 | 41.6 | 72 | 10 | 435 | 452 | 472 | .. | .... |
| 1-2-1 | 18 | 31.3 | 60 | 5 | 525 | 584 | 757 | 3 | 2870 |
| 1-2-2 | 8 | 25.0 | 60 | 5 | 401 | 464 | 639 | 6 | 2696 |
| 1-2-3 | 15 | 23.8 | 61 | 5 | 284 | 306 | 367 | 3 | 1733 |
| 1-2-4° | 1, 12, 24 | 16.3 | 65 | 10 | 304 | 333 | 418 | 12 | 1847 |
| 1-2-5 | 16 | 14.0 | 61 | 5 | 278 | 309 | 393 | 3 | 1685 |
| 1-2-6 | 28 | 11.1 | 65 | 5 | 277 | 307 | 355 | 3 | 1108 |
| 1-2-8 | 32 | 9.7 | 73 | 5 | 244 | 260 | 341 | 3 | 1240 |
| | | | | Mixes of Form 1-n-2n. | | | | | |
| 1-1-2° | 6, 20, 23 | 30.5 | 61 | 10 | 389 | 433 | 569 | 12 | 3653 |
| 1-1½-3° | 2, 13, 33 | 20.1 | 62 | 15 | 425 | 454 | 521 | 6 | 2874 |
| 1-2-4° | 1, 12, 24 | 16.3 | 65 | 10 | 304 | 333 | 418 | 12 | 1847 |
| 1-2½-5 | 14 | 12.7 | 61 | 5 | 218 | 230 | 352 | 3 | 1555 |
| 1-3-6° | 5, 19, 38 | 10.1 | 62 | 10 | 200 | 217 | 287 | 9 | 1372 |
| 1-4-8° | 7, 21, 26 | 7.4 | 60 | 9 | 144 | 160 | 242 | 12 | 1102 |
| 1-5-10 | 9 | 5.9 | 60 | 5 | 40 | 49 | 87 | 6 | 533 |

+ The per cent of cement is the ratio of the weight of the cement to the combined weights of the sand and stone.

° These sets of tests result from combining the 60-day tests in Table 16 with additional sets of 5 specimens each from separate batches.

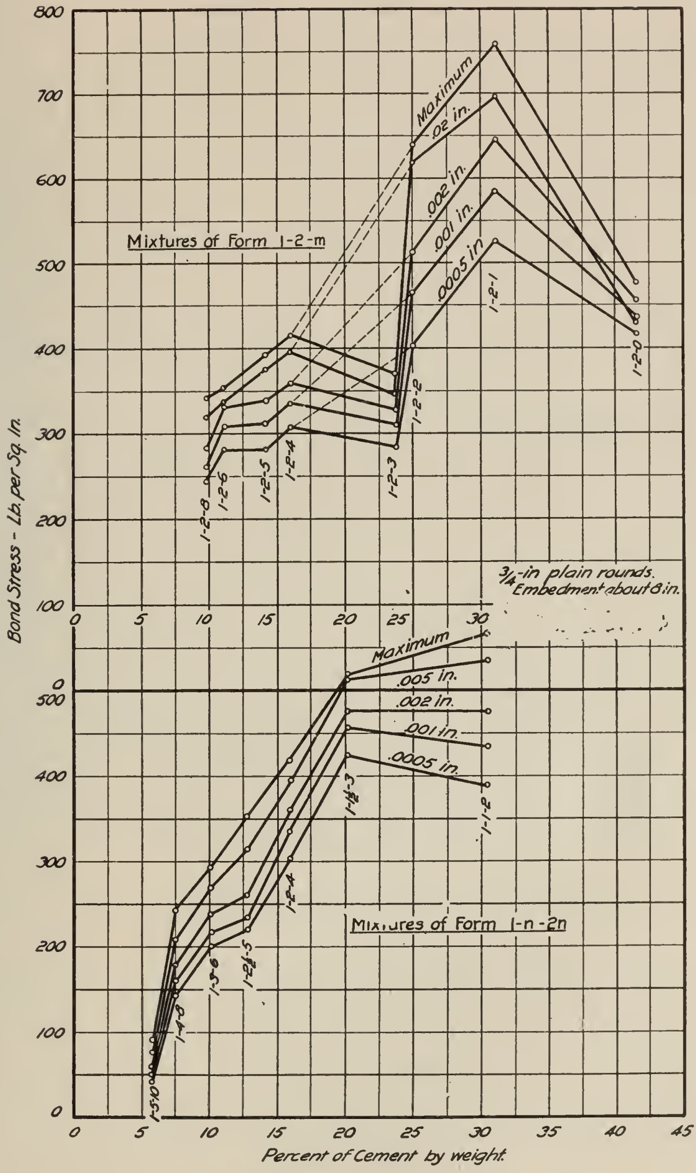


FIG. 32. BOND RESISTANCE FOR DIFFERENT MIXES.

more rapid increase. The load-slip curves given in Fig. 31 show the values of bond resistance as slipping of the bar progresses. The similarity of the curves will be noted. The values are the same as those in the group of curves given in the lower portion of Fig. 6.

48. *Relation of Bond Resistance to the Compressive Strength of Concrete.*—In Fig. 33 bond stresses corresponding to an end slip of bar of 0.0005 in. have been plotted as ordinates and the compressive strength of the corresponding 6-in. cubes have been plotted as abscissas. In Fig. 34 the maximum bond resistances have been used as ordinates. These figures include all sets of tests in Tables 12, 16 and 18, for which both values are given. The figures include tests at ages of 2 days to over 2½ years and from mixes of 1-5-10 to 1-1-2, and specimens stored under several different conditions. The pull-out tests were made with ¾-in. or 1-in. plain rounds embedded 8 in. Each point represents the average of from 3 to 10 pull-out tests and 3 to 12 cube tests. With a few exceptions the points fall within the regions defined by the dotted lines drawn upward and to the right from the origin. The heavy mean lines have been constructed in such a way that approximately one-half the points lie above and one-half below the lines. These figures show that end slip began in test specimens of this form at a bond stress per unit of area equal to 1/6 of the compressive strength of 6-in. cubes from the same concrete, and the maximum bond resistance is equal to about

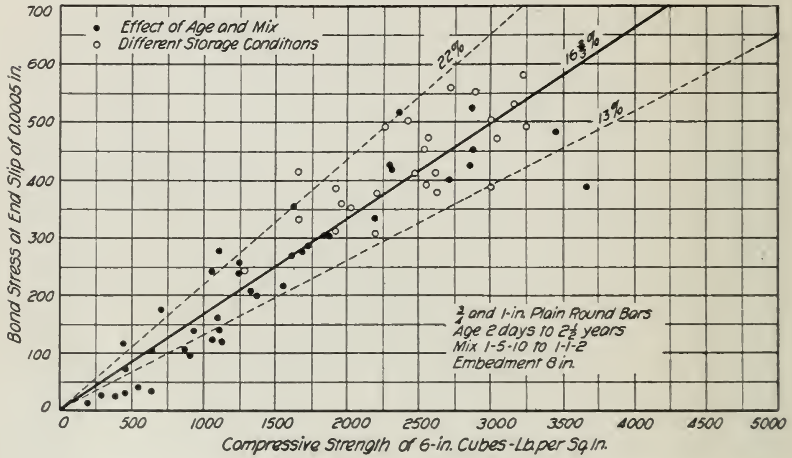


FIG. 33. RELATION OF BOND STRESS AT END SLIP OF 0.0005 IN. TO THE COMPRESSIVE STRENGTH OF 6-IN. CONCRETE CUBES.

¼ of the compressive strength of 6-in. cubes. The individual sets of tests show a variation of about 30% each way from the mean values.

49. *Effect of Age on the Compressive Strength of Concrete.*—The results of the tests on 6-in. concrete cubes made from the same concrete as the pull-out specimens are given in Table 16. These values have been plotted in Fig. 35. It will be seen that the curves for cube strength follow about the same courses as the corresponding curves for the pull-out tests with plain bars given in Fig. 27. The cube strength at 2 days is from 13% to 36% of the 60-day strength,—average 21%. The tests made at ages of 2 to 3½ years gave values which vary from 142% to 241% of the corresponding values at 60 days; the average for 5 mixes (no tests for 1-3-6) is 193%. The lowest value of this ratio is given by the 1-1-2 concrete and the highest by the 1-4-8 mix. This indicates only that the concretes rich in cement harden more rapidly and consequently obtain a higher proportion of their ultimate compressive strength at 60 days than do the leaner mixes. We may conclude, then, that, in general, concrete made and stored under the conditions present in these tests may be expected to finally develop a compressive strength about twice that at 60 days.

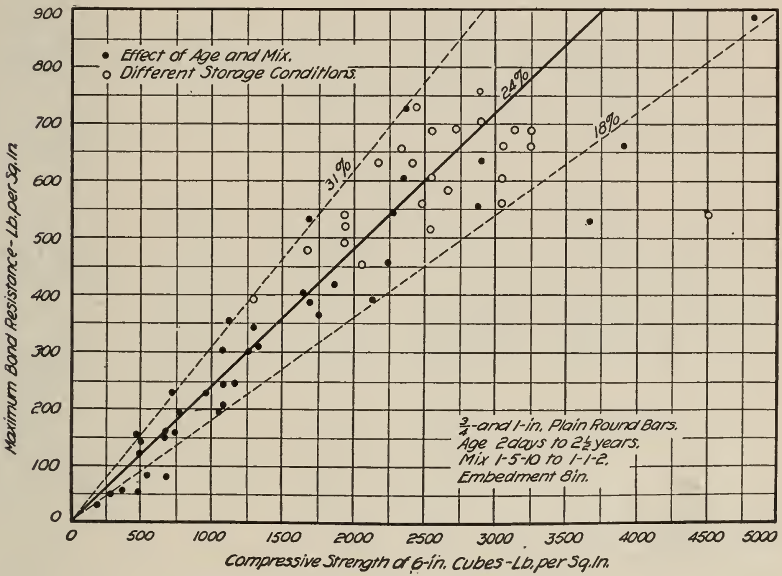


FIG. 34. RELATION OF MAXIMUM BOND RESISTANCE TO THE COMPRESSIVE STRENGTH OF 6-IN. CONCRETE CUBES.

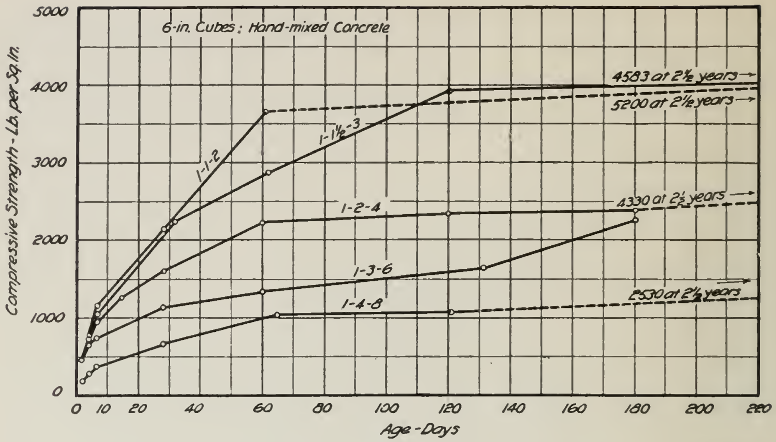


FIG. 35. EFFECT OF AGE ON THE COMPRESSIVE STRENGTH OF CONCRETE.

## TABLE 19.

### EFFECT OF ANCHORING ENDS OF BARS.

Embedment 8 in. 1-2-4 concrete from Batches 31, 34 and 37.

All specimens with curved or bent bars were reinforced against splitting by means of a ¼-in. wire spiral. See Fig. 36.

The average compressive strength of 6-in. cubes was 2240 lb. per sq. in.

Stresses are given in pounds per square inch.

| Size of Round Bar and Manner of Anchoring | Number of Tests | Age at Test days | Bond Stress at End Slip of | | Maximum Bond Resistance* |
|---|---|---|---|---|---|
| | | | 0.0005 in. | 0.001 in. | |
| ¾-in., no anchorage | 15 | 78 | 290 | 367 | 454 |
| 1-in., no anchorage | 11 | 66 | 324 | 370 | 478 |
| ¾-in., anchored with nut only | 5 | 62 | 292 | 339 | 925 |
| ¾-in., anchored with nuts and 2½-in. cut washer | 6 | 75 | 320 | 373 | 1020 |
| ¾-in., ¼ circumference of 3-in. circle | 5 | 69 | Values for slip of bar could not be determined, since the curved or bent portions of the bars were entirely embedded in the concrete. | | 736 |
| ¾-in., ½ circumference of 3-in. circle | 5 | 71 | | | 607 |
| ¾-in., 45° bend, 2 in. long | 5 | 70 | | | 829 |
| ¾-in., 90° bend, 2 in. long | 5 | 69 | | | 747 |
| ¾-in. 135° bend, 2 in. long | 5 | 69 | | | 672 |
| ¾-in., 180° bend, 2 in. long | 5 | 69 | | | 894 |
| 1-in., ¼ circumference of 3-in. circle | 5 | 62 | | | 858 |
| 1-in., ½ circumference of 3-in. circle | 5 | 62 | | | 653 |
| 1-in., 45° bend, 2 in. long | 5 | 62 | | | 776 |
| 1-in., 90° bend, 2 in. long | 5 | 62 | | | 863 |
| 1-in., 135° bend, 2 in. long | 5 | 62 | | | 866 |
| 1-in., 180° bend, 2 in. long | 5 | 62 | | | 1005 |

* The values for maximum bond resistance for the curved and bent bars were computed by dividing the total load by the superficial area of the part of the bar embedded in the concrete, without making allowance for the load taken in direct bearing.

### *f. Effect of Anchoring Ends of Bars.*

50. *Preliminary.*—Many designers have sought to increase bond resistance by anchoring the ends of the reinforcing bars in beams and other members. Hooks or bends at the ends of the bars are anchorages commonly employed; nuts with washers or bearing plates are also used. It seemed desirable to supplement the information gained as to the load-slip relation in other pull-out tests with tests in which the bars were anchored at the ends. The results of these tests are given in Table 19. Four methods of anchorage were used. The forms of these specimens are shown in Fig. 36. All specimens with curved or bent bars were reinforced against splitting by means of a ¼-in. wire spiral; the blocks for the specimens with bars anchored by means of nuts and washers were not reinforced against splitting. For comparison, tests were made from the same concrete using ¾ and 1-in. plain round bars without anchorage.

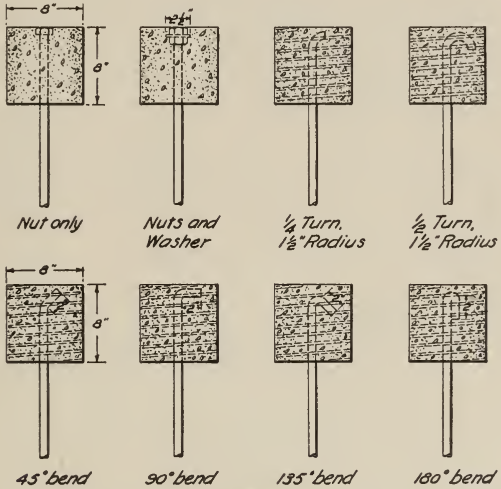


FIG. 36. FORMS OF PULL-OUT SPECIMENS WITH ANCHORED BARS.

51. *Bars Anchored with Nuts and Washer.*—One set of tests was made using ¾-in. plain round bars having their free ends anchored by means of a standard hexagonal nut, and another set with ends anchored by means of nuts and heavy cut washers 2½ in. outside diameter. The load-slip curves for these tests and for the tests on plain bars without anchorage have been plotted to the same scale in Fig. 37. The bond stresses for the anchored bars were computed on the basis of the em-

bedded area of bar in the usual way. It was found that the bars anchored in this way gave high values of bond, but only after a considerable end slip had occurred. For the bars anchored with nuts only, slipping began at the same bond stress as in the unanchored bars, and the load-slip curves show that there was no appreciable difference in their action until after an end slip of about 0.005 in. At an end slip of 0.01 in., corresponding to the maximum bond resistance for the bars without anchorage, the bars with nuts only showed a slight gain in bond resistance. Beyond this point the anchorage was more effective and the apparent bond stress finally reached 925 lb. per sq. in. at an end slip of 0.1 in.; the specimens failed by splitting of the blocks at a somewhat higher stress. The high bearing stress which was carried by the concrete under the

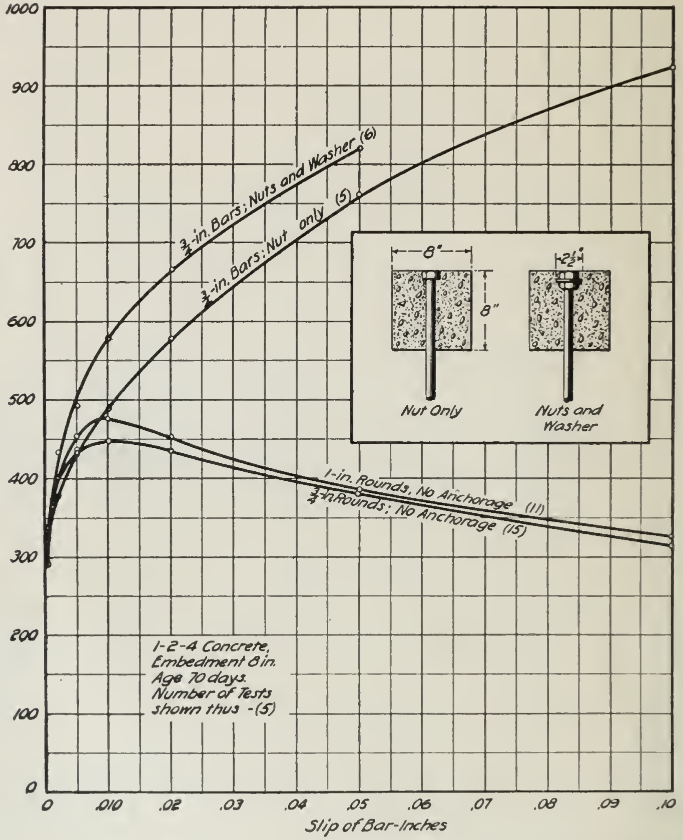


FIG. 37. LOAD-SLIP CURVES FOR BARS ANCHORED WITH NUTS AND WASHERS.

nuts was a noteworthy feature of these tests, and indicates that enormous pressures may be transmitted by concrete under such restraint. If we consider that the anchored bar was developing a sliding resistance of 325 lb. per sq. in., as in the tests of bars without anchorage, we must conclude that the remaining stress was being taken by bearing on the concrete under the nut. As shown in the figure, this bearing stress must account for a bond stress of 600 lb. per sq. in. over the entire embedded area, or a total stress of 11 300 lb. This stress was taken in bearing by an area of about 0.87 sq. in., equivalent to a bearing stress of 13 000 lb. per sq. in.

The bars anchored with nuts and washers gave values much the same as the bars without anchorage up to an end slip of about 0.001 in. After this point the bond resistance increased more rapidly than in the case of bars anchored with nuts only. These specimens also failed by splitting the concrete blocks.

52. *Bars Anchored by Means of Hooks and Bends.*—Two groups of tests were made with bars anchored by means of hooks and bends, as shown in Table 19. One group consisted of ¾ and 1-in. plain rounds having the free end bent to the form of ¼ or ½ the circumference of a circle 3 in. in diameter. The other group was anchored by means of 2-in. lengths of bar bent at angles of 45°, 90°, 135° and 180° with the projected axis of the bar. The forms of these specimens are shown in Fig. 36. The specimens were reinforced against splitting by means of a spiral of ¼-in. wire as in the tests with deformed bars. In these tests, values for slip of bar could not be determined, since the curved or bent portions of the bars were entirely embedded in the concrete. The values for maximum bond resistance were computed by dividing the total load on the bar by the superficial area of the length of bar embedded in the concrete, without making any allowance for the load taken in direct bearing. The maximum bond resistance computed in this way varied from about 600 to 1000 lb. per sq. in. The bars anchored with ¼ circumference gave higher resistance than those with ½ circumferences for both sizes of bar. This is probably due to the fact that in the latter tests a larger embedded area was used in computing the unit stresses, and indicates that the added length of the hook does not have a proportional effect in resisting a load tending to withdraw the bar. As may have been expected, the bars with ¼ circumference anchorage gave results almost identical with those of the bars with 45° bends. The ¾-in. bars with different angles of bend

did not give very consistent results, but in the tests with 1-in. bars the values increase with the angle, being greatest for the 180° bend. The high resistance found for the bars with 180° bend is doubtless due to direct bearing of the end of the bar against the concrete.

53. *Discussion of Tests with Anchored Bars.*—The tests with bars anchored with nuts and washers show that movement of the bar begins at the same load as in tests with bars without anchorage. As was pointed out in the discussion of tests with deformed bars these tests show that a small movement of the bar was necessary to bring the comparatively large area of the 2½-in. washer to a bearing and the usefulness due to the adhesion and sliding resistance of the bar itself has been largely destroyed. Tests of reinforced concrete beams made at the University of Illinois (not reported in this bulletin) in which the longitudinal reinforcing bars were anchored at their ends by means of nuts and bearing plates, show that this form of anchorage has little effect in increasing the resistance of beams to web failure. It will be seen later that in tests of simple reinforced concrete beams without web reinforcement, web failures may be expected to follow immediately after the appearance of a very small amount of end slip in the longitudinal reinforcement.

In a certain sense the designers of anchorages of this kind have attempted to accomplish the impossible, since it is generally assumed that in this way slip of bar is entirely prevented, whereas a certain amount of slip is essential to bring such an anchorage into action. However, end anchorage of this form may be effective in preventing total collapse or as a safeguard against defective workmanship which results in inferior bond resistance, but it may not be expected to be effective under ordinary working conditions, since under working loads stresses which cause a general movement of the bar are not permissible. The amount of movement necessary to bring this form of anchorage into action can be minimized by proper design of the details. The use of washers or bearing plates of ample area and stiffness which are rigidly attached to the reinforcing bar will accomplish much in increasing the usefulness of this form of anchorage.

It should be pointed out that the remarks above are not intended to apply to the form of anchorage used in certain buildings, in which the longitudinal beam reinforcement is anchored to the structural members of the supporting columns by means of nuts and bearing plates and subjected to a tensile stress before the concrete is placed. In this case the beam has certain advanatages in addition to those arising from beam action.

It is impossible to interpret the tests with bars anchored by means of hooks and bends in a way that will indicate the true value of this form of anchorage. In all the tests except those with 180° bend, evidence of the straightening-out of the bars was observed at loads from 70% to 90% of the maximum. Although the blocks were reinforced against splitting, the concrete of many of the specimens was badly shattered at the maximum load. It is apparent that very high bearing and bursting stresses were produced in the concrete. The ill effects of these stresses can be reduced by the use of circular bends of longer radii than those employed in these tests.

### *g. Miscellaneous Tests.*

54. *Preliminary.*—During the season of 1912 numerous pull-out tests were made which are of interest in indicating the influence of methods of loading which are different from that used in the tests described above and which indicate the effect of varying other details in the method of making and testing the specimens. These include the effect on bond resistance of different positions of the bar during molding, effect of distribution of load over lower face of block, effect of method of applying load in pull-out tests, double pull-out tests, repeated loads on pull-out specimens, effect of pressure during setting on bond resistance and on the compressive strength and the effect of loads reapplied after failure in bond and in compression. The results of these tests are given in Tables 20 to 24, inclusive.

55. *Repeated Load on Pull-out Specimens.*—We have seen in the foregoing discussion that it was desirable to use the bond resistance developed at a small amount of slip as the basis of comparison. This made it desirable to determine the effect of reapplying the load which caused first slip of the bar. Three tests of this kind were made on pull-out specimens. In Art. 90 similar tests on reinforced concrete beams are referred to. Fig. 38 gives the load-slip relation for a ¾-in. plain round bar embedded in 1-2-4 concrete. Load was applied until a slip of 0.001 in. was produced at the free end of the bar. It will be seen that slipping continues after this point during the time of releasing the load. At the next application the load was increased a small

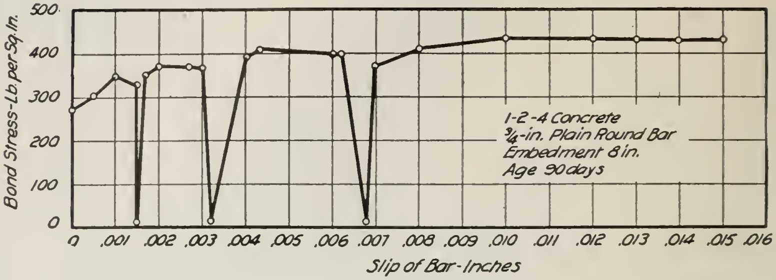


FIG. 38. LOAD-SLIP CURVE FOR REPEATED LOADS ON PULL-OUT SPECIMEN.

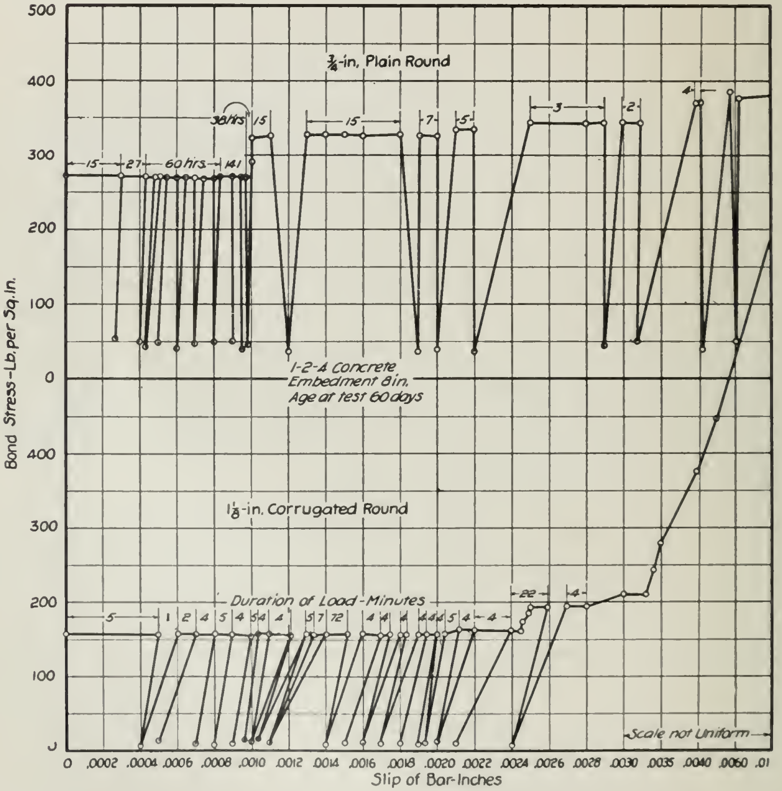


FIG. 39. LOAD-SLIP CURVES FOR REPEATED LOADS ON PULL-OUT SPECIMENS.

amount and slipping continued under constant load. The figure shows the subsequent action. The stress at a slip of 0.001 in. was 83% of the maximum bond resistance developed.

Fig. 39 shows the load-slip curves for two tests under repeated loads. The specimens were made from the same batch of concrete. In these tests the load was repeated which produced the smallest measurable amount of slip; about 0.0001 in. In the test of the ¾-in. round bar time intervals between loads are given in minutes unless otherwise noted. The slight recovery of slip upon release of load in the corru-

## TABLE 20.

### MISCELLANEOUS PULL-OUT TESTS.

1-in. plain rounds. 1-2-4 machine-mixed concrete. Age 80 days.
Embedment 8 in. unless otherwise noted.
Stresses are given in pounds per square inch.

| Ref. No. | Number of Tests | Characteristics | Unit Bond Stress at End Slip of | | Maximum Bond Resistance |
|---|---|---|---|---|---|
| | | | 0.0005 in. | 0.001 in. | |
| 17 | 5 | Bearing over entire base | 280 | 304 | 364 |
| 21 | 6 | Bearing over ring 1 in. wide as shown in Fig. 40 | 304 | 330 | 370 |
| 28 | 5 | Bearing over 2½-in. circle, Fig. 40 | 315 | 336 | 385 |
| 45 | 3 | Base plate adhering to block | 240 | 268 | 318 |
| 49 | 4 | Blocks directly on rubber cushion | 245 | 267 | 295 |
| 53 | 4 | Blocks cast with long end of bar upward (Fig. 1 (c)) | 278 | 315 | 372 |
| 57 | 4 | Blocks cast with bars horizontal and free to settle with concrete | 234 | 248 | 288 |
| 61 | 4 | Blocks cast with bars horizontal and held rigidly in place | 191 | ... | 211 |
| 33-36 | 4 | Specimens of form shown in Fig. 41 | 143 | 159 | 229 |
| 67, 68 | 2 | Specimens of form shown in Fig. 42, embedment 12 in. | 218 | 250 | 295 |
| 37* | 4 | Double pull-out specimens with plain bars as in Fig. 1 (d) | ... | ... | 179 |
| 37* | 4 | Second test on bars which did not pull out in first test | ... | ... | 268 |
| 41* | 2 | Double pull-out specimen with cold-rolled bars | ... | ... | 79 |
| 41* | 2 | Second test on bars which did not pull out in first test | ... | ... | 90 |

* The load was applied to these specimens through nuts at the free ends of the bars. Measurements of slip of bar were not taken. In each specimen of this form, failure in the first test was due to the pulling out of the bar which was embedded in the top portion of the specimen as it was molded. In the first tests the concrete blocks were subjected to a tensile stress. In making the second tests the specimens were set in the machine in the usual way for pull-out tests, and load applied to the bars which remained intact after the first tests.

gated bar test is noteworthy; there is no indication of such elastic recovery in the plain bar tests. These tests show that the repetition of the load which produces first slip of bar will cause a continued slipping, and if repeated a sufficient number of times would probably produce a slipping sufficient to endanger a structure. Not enough data are available upon which to base conclusions as to the values of the bond stress which may be indefinitely repeated without producing failure.

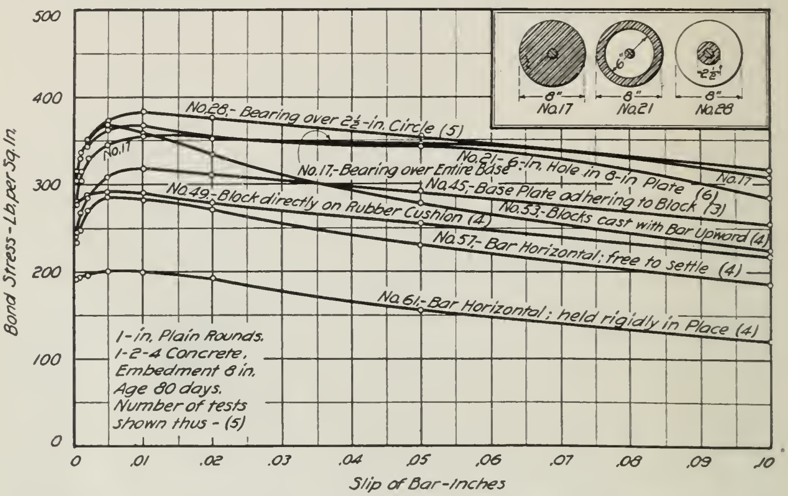


FIG. 40. LOAD-SLIP CURVES FOR MISCELLANEOUS PULL-OUT SPECIMENS.

56. *Effect of Distribution of Load on Face of Concrete Block.*—In Table 20 the results of tests which were made to study the effect on the bond resistance of a wide variation in the distribution of the load over the lower face of the concrete block in pull-out tests are given. It makes little difference whether the load is distributed uniformly over the entire base of the block or concentrated over an annular ring covering only the outer or the inner portion of the block. Whether the block rests directly on a rubber cushion or adheres to the base plate, has little effect on the bond resistance. The load-slip curves for these tests are given in Fig. 40.

57. *Effect of Position of Bar During Molding.*—In Table 20 the results of tests on pull-out specimens which were molded with the bars in different positions are given. In one set of tests the bar was supported by the form in such a way that settlement with the concrete was

prevented. Specimens were also made in which the long end of the bar was upward as the specimen was molded. Load-slip curves are given in Fig. 40. These tests show the effect of the settlement and shrinkage of concrete during the setting and hardening. For plain rounds it makes no difference whether the concrete settles in the same or in the opposite direction to that of the withdrawal of the bar. The effect on deformed bars is discussed in Art. 64. The specimens molded in a horizontal position gave distinctly lower bond resistances than those with bars vertical; and if the settlement of the bar with the concrete was entirely prevented, sliding resistance was very low and the maximum bond resistance was reduced to about 60% of that found for pull-out specimens from the same batch made with the bars in a vertical position. This shows one disadvantage of fixing the horizontal bars too rigidly in place before placing the concrete in a reinforced concrete member.

58. *Effect of Method of Applying Load in Pull-out Tests.*—Specimens of the form shown in Fig. 41 were made in order that measurements might be made on slip of bar at the point where the bar enters the concrete block, as well as at the free end. The load-slip curves for points A and B show this relation. Slipping begins at B at an average bond stress of about 70 lb. per sq. in. on the bar and reaches 0.001 in. at this point at an average stress of 136 lb. per sq. in. Slip at B reaches about 0.001 in. before slipping becomes appreciable at A. As may have been predicted, the curves indicate that slipping becomes general as soon as movement begins at A, and thereafter there is a nearly constant difference in the amount of slip measured at A and B. Part of the movement at A which, in the above statement, has been assumed to be slip at early loads, may be due largely to deformation in the lower face of the block.

Specimens of the form shown in Fig. 42 were made in order to measure the slip at several points along the embedded length. Measurements were made at points A to E at intervals of 3 in. It was desired to reproduce in a pull-out specimen the conditions of bond stress that exist near the end of a reinforced concrete beam, but the conditions in a beam were only imperfectly reproduced. The curves arrange themselves in two distinct groups. D and E show slipping to begin at a low bond stress; a much higher stress is required to produce slip at C. These specimens were made with the bar in a horizontal position; it was seen above that this has the effect of giving a lower bond resistance than was found in specimens molded with the bar vertical.

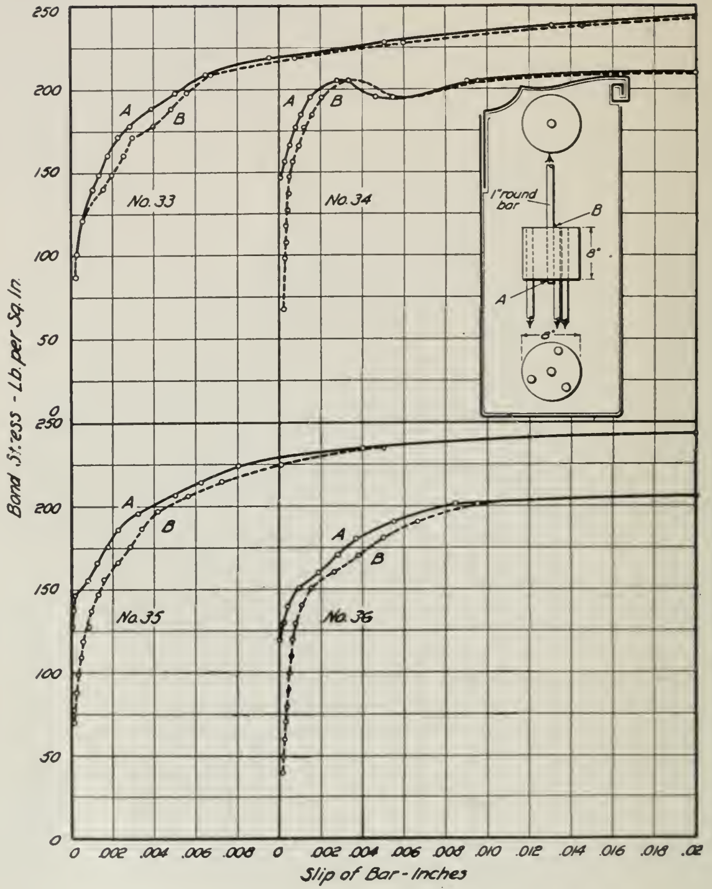


FIG. 41. LOAD-SLIP CURVES FOR SPECIMENS OF THE FORM SHOWN.

Results of double pull-out tests of the form shown in Fig. 1 (d) with ordinary round and cold-rolled bars are given in Table 20. Each bar was embedded 8 in. in a 16-in. cylinder of concrete. The maximum bond resistance in the double pull-out tests on ordinary bars was 179 lb. per sq. in., as compared with 375 for the usual pull-out specimens. It is of interest to note that in all the double pull-out tests the bar which was embedded in the upper portion of the cylinder was the one to pull out in the first test. The bar which did not pull out in the first test was later pulled out by supporting the block in the testing machine in the usual way for pull-out tests. The higher bond resistance given by the lower bar may be due in a measure to the concrete being protected against drying-out, as well as to the influence of the slight pressure under which setting occurred. The difference between the values for the first and second tests may be expected to show the influence of the different conditions of loading, but the entire difference in bond resistance cannot be attributed to this cause, since the bar with the lower bond resistance would be the one to pull out in the first test.

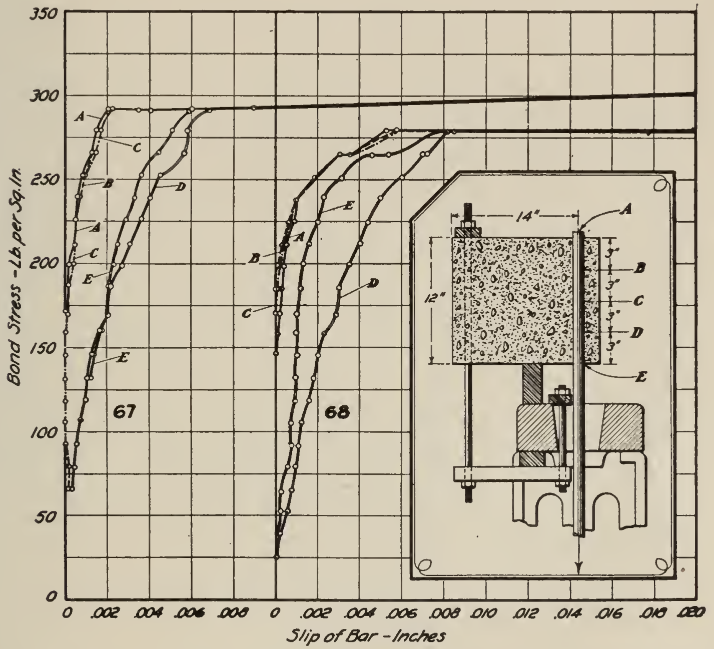


FIG. 42. LOAD-SLIP CURVES FOR SPECIMENS OF THE FORM SHOWN.

59. *Autogenous Healing in Concrete.*—The term "autogenous healing" may be applied to an interesting group of phenomena which were brought out in the tests on pull-out specimens loaded one or more times after having been previously tested to their maximum resistance, and in the compression tests on concrete cylinders which had been loaded to failure at an earlier age. These tests were suggested by certain observations made by the writer during the summer of 1911, while con-

ducting a load test on a 40-ft. through girder reinforced concrete bridge for the Illinois Highway Commission.* This test bridge was built by convict labor in December, 1907, inside the prison yard at the Southern Illinois Penitentiary near Chester, under the direction of Mr. A. N. Johnson, State Highway Engineer. In May, 1908, a test load amounting to 106 tons was placed on the bridge floor and removed. This load was sufficient to cause numerous diagonal tension cracks in the web near the ends of the girders. These cracks were very minute and were not observed until subsequent weathering brought them out. Before beginning the 1911 test all such cracks were carefully mapped, and the girders whitewashed. At this time crushed stone and pig iron were piled on the bridge floor and on the girders sufficient to give a live load of 309 tons on one girder. During the progress of the loading, some of the original cracks re-opened at loads much higher than that applied in the first test. Many of the original cracks, however, did not re-open under the final load, although new cracks sometimes formed in the immediate vicinity. The bridge was about four months old when first loaded, but it is probable that the concrete strength was not greater than would have been found at, say, two months, under more favorable temperature conditions. The behavior of the girders could be partially accounted for in the increased strength and stiffness of the concrete acquired during a period of more than three years since the first test; but this would scarcely account for the concrete of the girders being able to develop, without re-opening some of the cracks, a diagonal stress which was equivalent to about 6 times the stress which originally caused them. The only reasonable explanation was that the fractured surfaces of the concrete at the cracks had "knit" together and entirely healed during the interval between the tests, giving a joint which was in many instances stronger than the unbroken concrete. The tests described in the following articles on the effect of loads reapplied after failure in bond and compression were planned to throw further light on this subject. The phenomena discussed in Art. 60 and 63 are only other manifestations of the effect of retarded or interrupted hydraulicity, which have frequently been observed in tests on concrete setting under low temperatures, retempered concrete, cement reground after setting, etc.

60. *Effect of Loads Reapplied after Failure of Bond.*—Tests were made on 63 pull-out specimens to determine the effect of reapplying

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* "Test of a 40-ft. Reinforced Concrete Highway Bridge," by D. A. Abrams. Proceedings of the American Society for Testing Materials, 1913.

loads to specimens which had been tested to failure some time before the retest was made. Both plain and cold-rolled round bars were used. The specimens were stored under various conditions. See Table 21. Some of these specimens have been loaded as many as five times, and the tests will be continued.

The load-slip curves for two of the groups of tests in Table 21 are given in Fig. 43 and 44. The numbers in parentheses adjacent to the curves correspond to the number given to the tests in the table and indicate the order of loading. All specimens in a group were made from the same batch of concrete.

Group (a) was a preliminary series. At the age of 5 days part of the bars were pulled out 0.001 in. at the free end and the remainder to the maximum load, which came at an end slip of about 0.01 in. Some of the specimens were then stored in water and some in air. In the subsequent tests on these specimens all bars were pulled out 0.1 in.

It makes little difference in the subsequent tests whether the bars are pulled out 0.001 in. or 0.01 in. at the first test; in fact those pulled to 0.01 in. gave somewhat higher values in the second tests. With one exception (No. 3) the specimens stored in water during the interval between the first and second tests show considerably higher values than those stored in air. For all the specimens in this group the maximum bond resistances for the third test average 24% greater than those for the second tests made 4 or 5 months earlier when the concrete was 1 or 2 months old. The fourth tests, after another interval of 4 months, when the concrete was 10 months old and the bars had previously been pulled out over 0.2 in. gave maximum bond resistances which average the same as the second test at 1 or 2 months—about 290 lb. per sq. in. It is interesting to note that the period of 1 or 2 months in air following the first tests had the effect of temporarily retarding the increase of bond resistance which was largely overcome by the subsequent period of water storage.

Group (b) consisted of two parallel series of tests on 1-in. plain round bars stored in air and in damp sand. In all these tests the bars were pulled to the maximum load, corresponding to an end slip of about 0.01 in. Two specimens for each condition of storage were tested at 5 days, 30 days, 3 months, and 1 year. At each test period, all specimens which had been tested previously were loaded to the maximum

## TABLE 21.

### Effect of Loads Reapplied after Failure of Bond.

Pull-out tests; embedment 8 in. in an 8-in. cylinder of concrete. Load-slip curves are given in Fig. 43 and 44. Stresses are given in pounds per square inch.

| Ref. No. | No. of Tests | Age at Test | Remarks | Bond Stress at End Slip of (inches) | | | | | | | | Max. Bond Resistance |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | .0005 | .001 | .002 | .005 | .01 | .02 | .05 | .10 | |
| (a) 1-in. Plain Rounds; 1-2-4 Hand-mixed Concrete (1912). Bar pulled out 0.1 in. in each test unless otherwise noted. | | | | | | | | | | | | |
| 1 | 1 | 5 days | (1) Pulled to .001 in. | 143 | 147. | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 40 days | (2) Stored in air | 199 | 231 | 263 | 267 | 269 | 249 | 211 | 179 | 269 |
| | | 6 mo. | (3) Stored in water after second test | 278 | 310 | 329 | 340 | 354 | 306 | 258 | 215 | 354 |
| | | 10 mo. | (4) | 233 | 254 | 286 | 298 | 300 | 278 | 247 | 217 | 305 |
| | | 21 mo. | (5) | 223 | 239 | 243 | 260 | 265 | 250 | 229 | 203 | 265 |
| 2 | 2 | 5 days | (1) Pulled to .001 in. | 169 | 182 | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 72 days | (2) Stored in air | 179 | 203 | 229 | 260 | 268 | 260 | 232 | 189 | 268 |
| | | 5 mo. | (3) Stored in water after second test | 171 | 195 | 233 | 237 | 222 | 206 | 182 | 160 | 237 |
| | | 10 mo. | (4) | 161 | 183 | 214 | 227 | 241 | 220 | 205 | 171 | 241 |
| | | 21 mo. | (5) | 185 | 209 | 224 | 246 | 251 | 241 | 216 | 191 | 253 |
| 3 | 3 | 5 days | (1) Pulled to .001 in. | 142 | 151 | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | | 72 days | (2) Stored in water | 211 | 236 | 251 | 265 | 240 | 230 | 225 | 182 | 278 |
| | | 5 mo. | (3) Stored in water | 260 | 290 | 326 | 342 | 343 | 295 | 256 | 215 | 343 |
| | | 10 mo. | (4) Stored in water | 182 | 209 | 237 | 266 | 276 | 261 | 230 | 204 | 276 |
| | | 21 mo. | (5) Stored in water | 187 | 197 | 214 | 231 | 236 | 230 | 214 | 191 | 238 |
| 4 | 1 | 5 days | (1) Pulled to maximum | 154 | 161 | 167 | 179 | 186 | ..... | ..... | ..... | 186 |
| | | 40 days | (2) Stored in air | 139 | 199 | 207 | 230 | 225 | 215 | 187 | 163 | 230 |
| | | 6 mo. | (3) Stored in water after second test | 171 | 187 | 207 | 238 | 240 | 203 | 183 | 151 | 240 |
| | | 10 mo. | (4) | 145 | 173 | 200 | 235 | 243 | 221 | 195 | 172 | 243 |
| | | 21 mo. | (5) | 159 | 175 | 191 | 211 | 223 | 217 | 198 | 179 | 223 |
| 5 | 2 | 5 days | (1) Pulled to maximum | 189 | 202 | 214 | 226 | 232 | ..... | ..... | ..... | 232 |
| | | 40 days | (2) Stored in water | 243 | 282 | 289 | 307 | 301 | 273 | 235 | 191 | 307 |
| | | 6 mo. | (3) Stored in water | 277 | 328 | 366 | 398 | 404 | 384 | 307 | 273 | 404 |
| | | 10 mo. | (4) Stored in water | 230 | 256 | 292 | 326 | 336 | 326 | 298 | 263 | 340 |
| | | 21 mo. | (5) Stored in water | 222 | 234 | 252 | 272 | 290 | 285 | 266 | 233 | 292 |
| 6 | 3 | 5 days | (1) Pulled to maximum | 180 | 189 | 199 | 214 | 218 | ..... | ..... | ..... | 218 |
| | | 72 days | (2) Stored in water | 266 | 295 | 313 | 330 | 327 | 308 | 262 | 223 | 330 |
| | | 5 mo. | (3) Stored in water | 229 | 275 | 308 | 326 | 330 | 302 | 271 | 235 | 330 |
| | | 10 mo. | (4) Stored in water | 215 | 229 | 246 | 272 | 285 | 271 | 257 | 215 | 285 |
| | | 21 mo. | (5) Stored in water | 206 | 212 | 219 | 242 | 254 | 250 | 228 | 207 | 254 |
| (b) 1-in. Plain Rounds; 1-2-4 Machine-mixed Concrete. (1912). | | | | | | | | | | | | |
| 113 | 2 | 5 days | (1) Stored in air<br>Pulled to maximum | 113 | 124 | 132 | 149 | 162 | ..... | ..... | ..... | 162 |
| | | 1 mo. | (2) | 192 | 216 | 237 | 246 | 250 | ..... | ..... | ..... | 250 |
| | | 3 mo. | (3) | 177 | 220 | 243 | 259 | ..... | ..... | ..... | ..... | 269 |
| | | 1 yr. | (4) | 39 | 55 | 73 | 90 | 170 | 161 | ..... | ..... | 208 |
| 117 | 2 | 1 mo. | (1) do. | 162 | 173 | 188 | 215 | 248 | ..... | ..... | ..... | 263 |
| | | 3 mo. | (2) | 165 | 208 | 234 | 258 | 265 | ..... | ..... | ..... | 288 |
| | | 1 yr. | (3) | 41 | 69 | 92 | 162 | 190 | 207 | 212 | ..... | 213 |
| 121 | 2 | 3 mo. | (1) do. | 175 | 194 | 215 | 247 | 265 | ..... | ..... | ..... | 286 |
| | | 1 yr. | (2) | 38 | 67 | 113 | 159 | 178 | 190 | 198 | ..... | 198 |
| 125 | 2 | 1 yr. | (1) do. | 81 | 119 | 147 | 173 | 188 | 217 | ..... | ..... | 218 |
| 115 | 2 | 5 days | (1) Stored in damp sand<br>Pulled to maximum | 134 | 154 | 165 | 170 | 177 | ..... | ..... | ..... | 177 |
| | | 1 mo. | (2) | 265 | 301 | 318 | 320 | ..... | ..... | ..... | ..... | 320 |
| | | 3 mo. | (3) | 272 | 310 | 376 | 384 | ..... | ..... | ..... | ..... | 384 |
| | | 1 yr. | (4) | 407 | 425 | 452 | 457 | 457 | ..... | ..... | ..... | 457 |
| 119 | 2 | 30 days | (1) do. | 230 | 238 | 256 | 275 | 289 | ..... | ..... | ..... | 293 |
| | | 3 mo. | (2) | 307 | 348 | 376 | 384 | ..... | ..... | ..... | ..... | 384 |
| | | 1 yr. | (3) | 390 | 430 | 443 | 458 | 462 | ..... | ..... | ..... | 462 |
| 123 | 2 | 3 mo. | (1) do. | 354 | 378 | 395 | 397 | ..... | ..... | ..... | ..... | 400 |
| | | 1 yr. | (2) | 407 | 453 | 478 | 481 | ..... | ..... | ..... | ..... | 485 |
| 127 | 2 | 1 yr. | (1) do. | 424 | 443 | 446 | 450 | ..... | ..... | ..... | ..... | 451 |

## TABLE 21—CONTINUED.

(c) 1-in. Polished Rounds; 1-2-4 Machine-mixed Concrete.* (1912).

| Ref. No. | No. of Tests | Age at Test | Remarks | Bond Stress at End Slip of (inches) .0005 | .001 | .002 | .005 | .01 | .02 | .05 | .10 | Max. Bond Resistance |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 101 | 2 | 5 days | (1) Stored in air<br>Pulled to maximum | 119 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 119 |
| | | 1 mo. | (2) | 108 | 111 | ..... | ..... | ..... | ..... | ..... | ..... | 111 |
| | | 3 mo. | (3) | 80 | 91 | ..... | ..... | ..... | ..... | ..... | ..... | 95 |
| | | 1 yr. | (4) | 31 | 32 | ..... | ..... | ..... | ..... | ..... | ..... | 32 |
| 105 | 2 | 1 mo. | (1) do. | 189 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 192 |
| | | 3 mo. | (2) | 112 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 120 |
| | | 1 yr. | (3) | 23 | 23 | ..... | ..... | ..... | ..... | ..... | ..... | 23 |
| 109 | 2 | 3 mo. | (1) do. | 152 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 152 |
| | | 1 yr. | (2) | 29 | 33 | ..... | ..... | ..... | ..... | ..... | ..... | 47 |
| 103 | 2 | 5 days | (1) Stored in damp sand<br>Pulled to maximum | 119 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 122 |
| | | 1 mo. | (2) | 151 | 168 | ..... | ..... | ..... | ..... | ..... | ..... | 170 |
| | | 3 mo. | (3) | 158 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 164 |
| | | 1 yr. | (4) | 210 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 210 |
| 107 | 2 | 1 mo. | (1) do. | 148 | 157 | ..... | ..... | ..... | ..... | ..... | ..... | 157 |
| | | 3 mo. | (2) | 147 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 147 |
| | | 1 yr. | (3) | 178 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 178 |
| 111 | 2 | 3 mo. | (1) do. | 212 | 224 | ..... | ..... | ..... | ..... | ..... | ..... | 224 |
| | | 1 yr. | (2) | 133 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 133 |

(d) 1-in Plain Rounds from 1912 Beam Series; 1-2-4 Hand-mixed Concrete.°
Bar pulled out 0.1 in each test.

| Ref. No. | No. of Tests | Age at Test | Remarks | Bond Stress at End Slip of (inches) .0005 | .001 | .002 | .005 | .01 | .02 | .05 | .10 | Max. Bond Resistance |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1054.1 | 3 | 2 mo. | (1) Stored in air | 329 | 373 | 398 | 433 | 450 | 447 | 400 | 348 | 459 |
| | | 7 mo. | (2) Stored in water after first test | 259 | 298 | 339 | 373 | 389 | 370 | 343 | 394 | 389 |
| | | 11 mo. | (3) | 215 | 240 | 264 | 296 | 318 | 312 | 295 | 265 | 319 |
| | | 21 mo. | (4) | 251 | 271 | 288 | 307 | 323 | 338 | 314 | 292 | 338 |
| 1051.1 | 3 | 2 mo. | (1) Stored in air | 300 | 331 | 361 | 375 | 382 | 368 | 334 | 284 | 386 |
| | | 7 mo. | (2) Stored in water after first test | 220 | 252 | 284 | 316 | 337 | 309 | 275 | 243 | 337 |
| | | 11 mo. | (3) | 196 | 220 | 245 | 275 | 287 | 287 | 267 | 231 | 291 |
| | | 21 mo. | (4) | 203 | 212 | 220 | 247 | 262 | 266 | 246 | 227 | 266 |
| 1056.1 | 3 | 2 mo. | (1) Stored in air | 332 | 405 | 427 | 447 | 454 | 417 | 364 | 304 | 454 |
| | | 7 mo. | (2) Stored in water after first test | 198 | 233 | 265 | 292 | 306 | 296 | 274 | 249 | 306 |
| | | 11 mo. | (3) | 171 | 184 | 206 | 234 | 252 | 257 | 240 | 221 | 257 |
| | | 21 mo. | (4) | 191 | 199 | 208 | 229 | 241 | 248 | 234 | 219 | 248 |
| 1055.1 | 3 | 2 mo. | (1) Stored in air | 344 | 397 | 417 | 441 | 417 | 379 | 323 | 262 | 443 |
| | | 7 mo. | (2) Stored in water after first test | 168 | 194 | 220 | 254 | 265 | 264 | 236 | 209 | 267 |
| | | 11 mo. | (3) Stored in air after second test | 188 | 218 | 233 | 247 | 251 | 243 | 229 | 205 | 248 |
| | | 21 mo. | (4) | 59 | 71 | 83 | 110 | 132 | 140 | 144 | 136 | 147 |
| 1059.1 | 3 | 2 mo. | (1) Stored in air | 421 | 525 | 564 | 586 | 584 | 547 | 473 | 409 | 591 |
| | | 7 mo. | (2) Stored in water after first test | 205 | 252 | 278 | 315 | 334 | 339 | 321 | 296 | 339 |
| | | 11 mo. | (3) Stored in air after second test | 200 | 236 | 255 | 273 | 282 | 288 | 277 | 259 | 290 |
| | | 21 mo. | (4) | 69 | 83 | 96 | 117 | 134 | 150 | 183 | 186 | 187 |
| 1046.1+ | 3 | 2 mo. | (1) Stored in air | 233 | 258 | 275 | 306 | 326 | 330 | 316 | 289 | 336+ |
| | | 7 mo. | (2) Stored in water after first test | 185 | 220 | 266 | 301 | 316 | 295 | 270 | 238 | 316+ |
| | | 11 mo. | (3) | 166 | 183 | 202 | 233 | 245 | 250 | 234 | 213 | 252+ |
| | | 21 mo. | (4) | 179 | 190 | 198 | 211 | 225 | 232 | 215 | 202 | 232+ |

---

* Groups (b) and (c) were made from the same batch of concrete. Compressive strength of 6 6-in. cubes, tested at 3 mo., 2720 lb. per sq. in.

° Compressive strength of 18 6-in. cubes, tested at 63 days, 2360 lb. per sq. in.

+ 1-in. plain square bars.

again. Thus, as indicated in the table, some of the specimens have been loaded to their maximum as many as four times. The load-slip curves are given in Fig. 43. The notable feature of these tests is that the water-stored specimens which had been previously loaded to a maximum once, twice, or three times, gave values greater than or equal to those found in the specimens, which at the same age, were loaded for the first time. The same statement is true of the air-stored specimen, except that all the 1-year tests, both on specimens which had been previously loaded and those loaded for the first time, gave lower values. The two pairs of specimens tested at 5 days gave nearly identical results, as may have been expected, since the difference in storage conditions would not affect the relative strengths at that age. During the interval between the first and second tests of these specimens (5th to 30th day) the relative increase in maximum bond resistance is about the same for air storage and damp sand storage—37% and 30%; during the interval between the first and second tests of specimens No. 117 and 119 (1st to 3rd month) the increase was 9% and 31%, respectively.

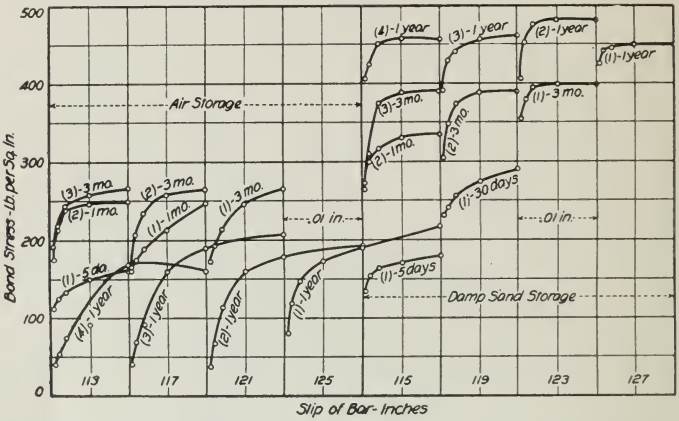


FIG. 43. LOAD-SLIP CURVES FOR LOADS REAPPLIED AFTER FAILURE OF BOND.

Group (c) was similar in every way to group (b) except that polished bars were used. It was thought that by comparing the behavior of plain and polished bars in these tests, additional information as to the components of bond resistance would be obtained. Generally, only one observation on slip of the end of the polished bar within the maximum load could be obtained. The maximum bond resistance came in

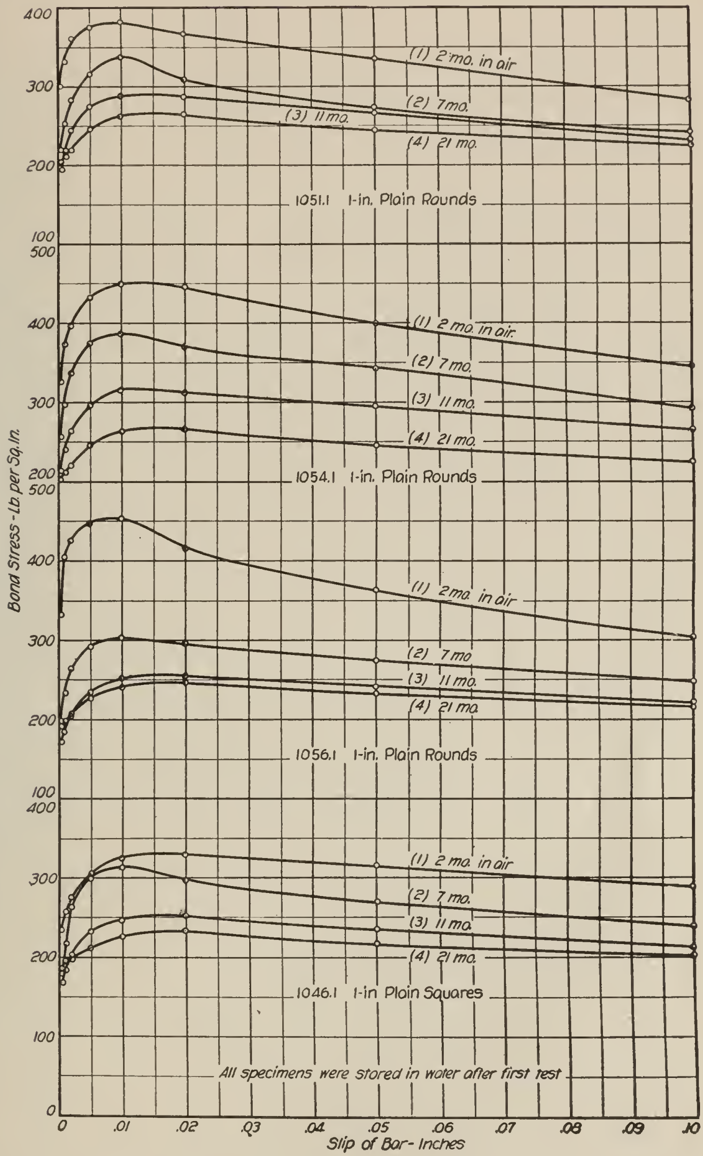


FIG. 44. LOAD-SLIP CURVES FOR LOADS REAPPLIED AFTER FAILURE OF BOND.

all the tests at a slip of about 0.001 in. The results of these tests are not as consistent as those on plain bars, but some interesting facts are brought out.

A study of the tests in groups (b) and (c) indicates that the bond resistance which is developed before the free end of the bar of the size used in these tests has slipped, say, 0.001 in. is due to adhesion. The value of this adhesion is dependent on the amount of moisture present and the age of the concrete. In the case of the polishd bars the original bond resistance was generally regained in the specimens stored in damp sand, but the increase with age was small. The air-stored specimens show about the same bond resistance as the sand-stored specimens in the first tests, but upon reapplication of load there is a material falling-off in strength. With the round bars of ordinary surface the adhesive resistance was entirely restored, except in the 1-year tests on air-stored specimens. The amount of increase in bond resistance apparently depends upon the presence of the water necessary for the continuation of the hydraulic action of the cement. The drying-out of a specimen after a certain period interferes with the phenomenon here discussed.

Group (d) included 6 sets of pull-out specimens from the 1912 beam series. All of these specimens were stored in air up to the time of the first test at the age of 2 months. In each of the tests the bar was pulled to an end slip of 0.1 in. The load-slip curves are given in Fig. 44. These tests are of interest in showing what may be expected when the specimens have attained a considerable age before the first failure of bond. In no instance were the values from the second to the fourth tests as high as found in the first tests. The average values for all the specimens with plain round bars (disregarding minor variations in storage conditions) are 406, 246, 220 and 167 lb. per sq. in. for the first, second, third and fourth tests, respectively, at a slip of 0.001 in.; and 467, 327, 281 and 237 lb. per sq. in. for maximum bond resistance. It will be recognized that the conditions in this group were the most adverse of any of the groups in which the load was reapplied after failure. The bond resistance did not subsequently reach the amount found in the first tests, but it is noteworthy that even at the fourth loading comparatively high values of bond resistance were found.

The foregoing tests are of value in showing that a considerable displacement of the bar, and a frequent disturbance of the bond between the concrete and steel, even several days or weeks after placing the concrete, does not necessarily produce permanent weakness. If disturbance occurs after a longer period the final effect will probably depend upon whether sufficient moisture is present in the concrete. These results give added confidence to the permanency of bond and indicate that plain bars may properly be used in certain classes of work in which they have sometimes been considered unsuitable. It should be noted that the specimens were unstressed in the interval between loadings.

61. *Bond Resistance of Concrete which Set under Pressure.*—As a means of gaining further knowledge as to the nature of bond resistance, a few tests were made on specimens in which the concrete was caused to set under pressure. 1-in. plain rounds, 1-in. cold-rolled rounds and 1⅛-in. corrugated rounds were used. The pressures used were 0, 6, and 100 lb. per sq. in. The specimens were molded in the usual way, except that in the specimens which set under a pressure of 100 lb. per sq. in. the long end of the bar was upward. The pressure was applied immediately after placing the concrete in the form. A circular plate with a central hole which allowed the rod to pass through was placed inside the form on the fresh concrete. The 6-lb. per sq. in. pressure was obtained by piling weights on the cover plate; the 100-lb. per sq. in. pressure was obtained by setting the form in a pan on the bed of a testing machine and running the head down onto a nest of springs, which transmitted the load to the cover plate. These loads remained on the specimens for 5 days. After removal from the forms, the specimens were stored in damp sand. The tests are summarized in Table 22. The load-slip curves are given in Fig. 45. The increase of maximum bond resistance and other properties with the pressure under which the concrete set are shown in Fig. 47. The maximum bond resistance for plain bars in concrete which set under pressures of 6 and 100 lb. per sq. in. are 9% and 91% higher, respectively, than the corresponding values for concrete setting under no pressure. In the case of the cold-rolled rounds, the increase due to the pressure was slight. The value for 6 lb. per sq. in. pressure seems abnormally high.

The corrugated bars show a large increase in bond resistance due to the pressure. These values cannot be compared directly, since some of the specimens were tested without removing the steel-pipe forms in which they were made. However, the corrugated bars show an increase in bond resistance of about 100% as compared with specimens tested under similar conditions which had set without pressure. The specimens which set under a pressure of 6 lb. per sq. in. (No. 91) show an increase of 66% to 90% as compared with the values for the same amount of slip in the specimens setting without pressure. The concrete blocks split in these two sets of tests at the same slip—0.02 in. Specimens No. 93 made and tested in a steel form gave very high values. No. 89 shows the effect of leaving the steel form in place during the test of a specimen which set without pressure; No. 95 shows the effect of casting the specimen with the long end of the bar upward. For a discussion of other tests with corrugated bars showing the effect of reinforcing the block against bursting, see Art. 64.

## TABLE 22.

### BOND RESISTANCE OF CONCRETE WHICH SET UNDER PRESSURE.

1-2-4 concrete, machine-mixed; embedment 8 in. Age at test 80 days.
Stresses are given in pounds per square inch.

| Ref. No. | Number of Tests | Characteristics | Bond Stress at End Slip of—inches | | | | | | | | Maximum Bond Resistance |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | .0005 | .001 | .002 | .005 | .01 | .02 | .05 | .10 | |
| (a) 1-in. Plain Rounds. | | | | | | | | | | | |
| 71 | 2 | Without pressure............ | 273 | 302 | 328 | 345 | 351 | 350 | 320 | 290 | 353 |
| 73 | 6 | Concrete set 5 days under pressure of 6 lb. per sq. in....... | 221 | 256 | 288 | 332 | 363 | 378 | 351 | 297 | 378 |
| 77* | 2 | Concrete set 5 days under pressure of 100 lb. per sq. in..... | 387 | 457 | 546 | 634 | 671 | 673 | 570 | 477 | 674 |
| (b) 1-in. Cold-Rolled Rounds. | | | | | | | | | | | |
| 79 | 2 | Without pressure............ | 141 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | 141 |
| 81 | 1 | Concrete set 5 days under pressure of 6 lb. per sq. in....... | 222 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | 222 |
| 83 | 2 | Concrete set 5 days under pressure of 100 lb. per sq. in..... | 149 | ...... | ...... | ...... | ...... | ...... | 94 | 81 | 163 |
| (c) 1⅛-in. Corrugated Rounds. | | | | | | | | | | | |
| 85 | 2 | Without pressure. No spiral.. | 234 | 248 | 258 | 278 | 312 | 341 | ...... | ...... | 375 |
| 87 | 2 | Without pressure. Spiral—See Fig. 1 (b)............ | 244 | 279 | 314 | 359 | 412 | 517 | 809 | 1115 | 1115 |
| 89 | 2 | Without pressure, in steel pipe form.................... | 347 | 404 | 420 | 438 | 516 | 605 | 855 | 1070 | 1070 |
| 91* | 2 | Concrete set 5 days under pressure of 6 lb. per sq. in. ..... | 380 | 413 | 468 | 503 | 617 | 672 | ...... | ...... | 720 |
| 93* | 2 | Concrete set 5 days under pressure of 100 lb. per sq. in..... | 682 | 937 | 1180 | 1417 | 1600 | 1733 | .... | ..... | 1960 |
| 95 | 4 | Cast with long end of bar upward as in Fig. 1 (c); spiral.. | 408 | 508 | 590 | 687 | 793 | 881 | 1140 | 1250 | 1250 |

* Specimens molded in forms made of 8-in. sections of 8-in. steel pipe. The forms were cut off at age of 5 days.

° Tested with steel pipe form in place.

62. *Compression Tests of Concrete which Set under Pressure.*—In order to make a further study of the effect of allowing concrete to set under pressure, a series of compression tests was made on 8 by 16-in. cylinders. It is believed that these tests are of sufficient interest to warrant their inclusion in this report. Seventeen cylinders were made from a single batch of concrete. Two of these were allowed to set under normal conditions; three set under a pressure of 6 lb. per sq. in.; five set under 20 lb. per sq. in. and five under 100 lb. per sq. in. Some of the specimens remained under pressure for 1 day, some for 7 days and some during the entire period of storage. Two specimens set for seven days under 3 lb. per sq. in., after which they were placed in a testing machine and loaded at the rate of 100 lb. per sq. in. per

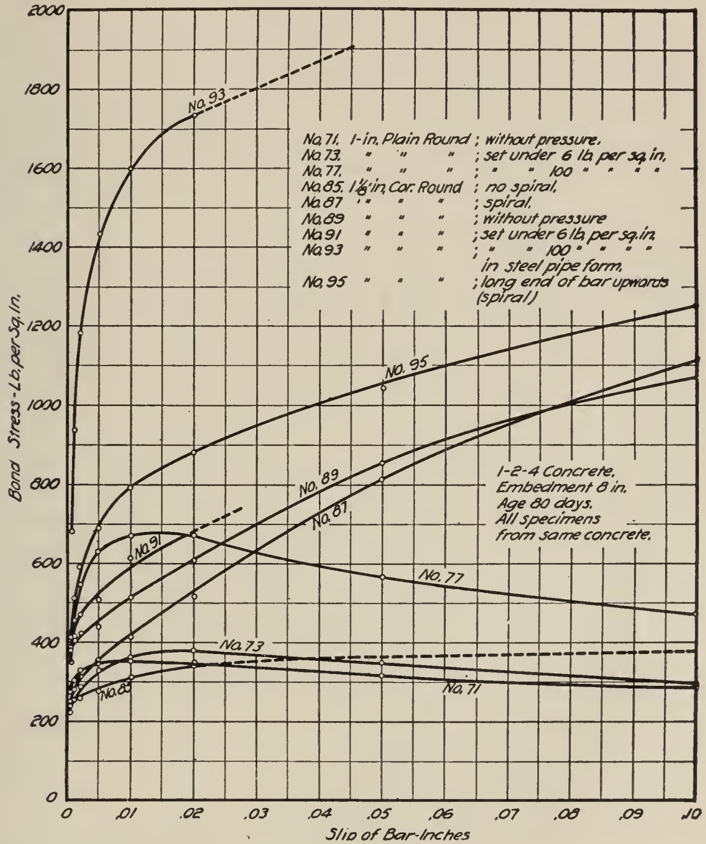


FIG. 45. LOAD-SLIP CURVES FOR MISCELLANEOUS PULL-OUT SPECIMENS.

week until a total load of 500 lb. per sq. in. had been applied; this load then remained on the cylinders until they were tested at the age of 80 days. The method of applying the pressure during the setting and hardening of the concrete was similar to that described in Art. 61 for the pull-out specimens.

Details of these tests are given in Table 23. Five of the cylinders were tested at 7 days; and the remainder at age of 80 days. The retest of the five cylinders which were originally tested to failure at 7 days will be discussed in Art. 63.

Deformations were measured over a 10-in. gage length by means of a wire-wound instrument which was a modified form of the Johnson extensometer. The weight of the concrete, the compressive strength, and the initial modulus of elasticity of the cylinders are given in the table. Typical stress-deformation curves are given in Fig. 46. The variation in compressive strength, initial modulus of elasticity, and density with the change in pressure are shown in Fig. 47.

The first notable feature of these tests is that it makes little or no difference in the strength and properties of the cylinders, whether the concrete remained under pressure for 1, 7 or 77 days. In other words, if the concrete takes its final set and hardening begins under pressure there is nothing to be gained by continuing the pressure for a longer period. For this reason the values for all the cylinders for each pressure which were tested for the first time at an age of 80 days were averaged in computing the percentages in Fig. 47. The consistency of the values indicates that the pressure probably had the effect of producing an unusually homogeneous concrete and justifies us in placing confidence in the results of the tests. The stress-deformation curves in the lower right section of Fig. 46 show the effect of different pressures on the compressive strength and the modulus of elasticity of these cylinders. The compressive strength was increased from 1840 lb. per sq. in. for no pressure to 3140 lb. per sq. in. for a pressure of 100 lb. per sq. in.—an increase of 73%. The compressive strength of the 8-in. cylinders setting without pressure is 91% of that for 6-in. cubes tested at the same age. The curve in Fig. 47 indicates that over one-half the increase occurred at pressures below 20 lb. per sq. in. It seems probable that the strength of the concrete which was subjected to a pressure of 100 lb. per sq. in. was considerably reduced by the loss of the water which was forced out when the pressure was first applied, owing to openings around the cover plate. On account of the danger of getting the

## TABLE 23.

### Compression Tests of Concrete which Set under Pressure.

The test pieces were cylinders 8 in. in diameter, 16 in. long. Concrete 1-2-4, machine-mixed. All specimens were made from the same batch. The cylinders were stored in damp sand after seven days except those setting under pressure for a longer period and those tested originally at age of seven days. The cylinders which set under pressure longer than seven days remained in the forms until they were prepared for testing; those tested at seven days were placed in water until the time of the second test.

The compressive strength of 6 6-in. cubes tested at 77 days was 2010 lb. per sq. in.

| Ref. No. | Age at Test days | Pressure During Setting lb. per sq. in. | Duration of Pressure days | Weight of Concrete lb. per cu. ft. | Compressive Strength lb. per sq. in. | Initial Modulus of Elasticity lb. per sq. in. |
|---|---|---|---|---|---|---|
| 501 | 7 | 0 | 0 | 142.5 | 725 | 1 500 000 |
| 501 | 80 | 0 | 0 | ..... | 1850* | 2 370 000* |
| 502 | 80 | 0 | 0 | ..... | 1820 | 3 100 000 |
| 503 | 80 | 6 | 1 | ..... | 2220 | 3 480 000 |
| 504 | 80 | 6 | 7 | ..... | 2170 | 3 600 000 |
| 505 | 80 | 6 | 77 | 142.5 | 2220 | 3 820 000 |
| 506 | 7 | 20 | 1 | 148.5 | 1240 | 2 000 000 |
| 506 | 80 | 20 | 1 | ..... | 2450* | 3 800 000* |
| 507 | 80 | 20 | 1 | ..... | 2350 | 3 880 000 |
| 508 | 7 | 20 | 7 | 148.0 | 1087 | 2 250 000 |
| 508 | 80 | 20 | 7 | ..... | 1935* | 3 240 000* |
| 509 | 80 | 20 | 7 | ..... | 2670 | 3 800 000 |
| 510 | 80 | 20 | 77 | 148.0 | 2850 | 3 520 000 |
| 511 | 7 | 100 | 1 | 148.2 | 1570 | 2 500 000 |
| 511 | 80 | 100 | 1 | ..... | 3060* | 4 400 000* |
| 512 | 80 | 100 | 1 | 147.0 | 3110 | 4 400 000 |
| 513 | 7 | 100 | 7 | 149.5 | 1450 | 2 500 000 |
| 513 | 80 | 100 | 7 | ..... | 2430* | 3 800 000* |
| 514 | 80 | 100 | 7 | 149.0 | 3140 | 4 000 000 |
| 515 | 80 | 100 | 77 | 148.0 | 3160 | 4 400 000 |
| 516° | 80 | ... | .. | 143.0 | 1865 | 2 640 000 |
| 517° | 80 | ... | .. | 145.0 | 2000 | 2 720 000 |

* Second test on the same specimen.

° Set under pressure of 3 lb. per sq. in. for seven days; forms removed and cylinders placed in testing machine, under pressure as follows:

100 lb. per sq. in. 7th to 14th day;
200 lb. per sq. in. 14th to 21st day;
300 lb. per sq. in. 21st to 28th day;
400 lb. per sq. in. 28th to 35th day;
500 lb. per sq. in. 35th to 80th day.

The storage of these cylinders differed from the others in that they were exposed to the air after age of seven days.

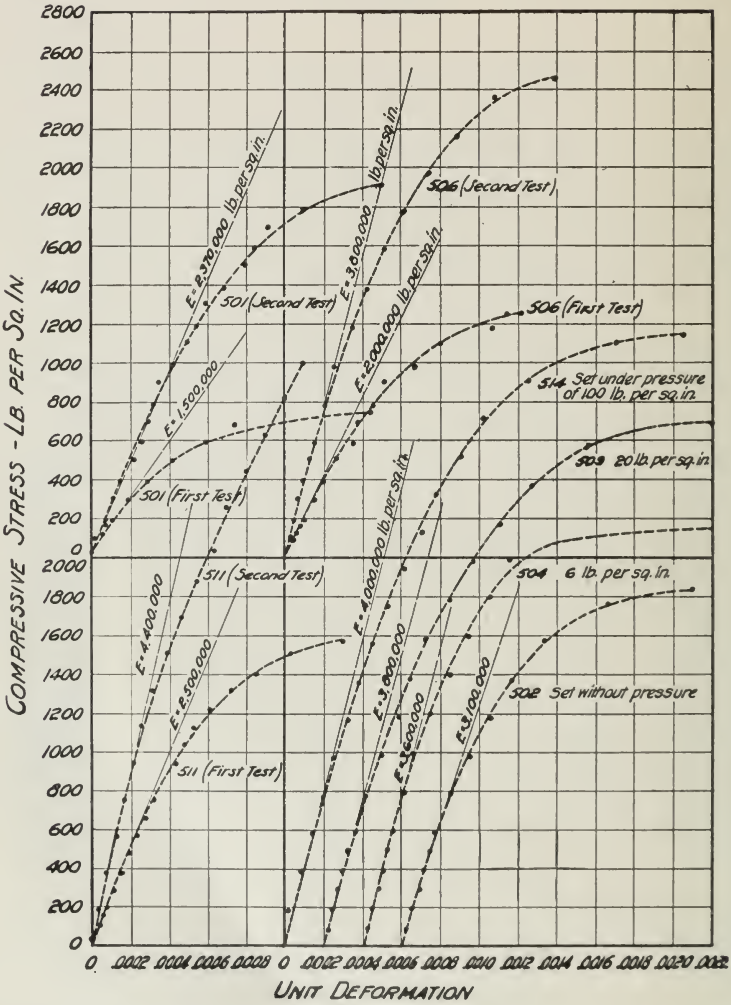


Fig. 46. Stress-deformation Curves for 8 by 16-in. Concrete Cylinders.

cover plate wedged in the form in applying the pressure to the fresh concrete, a close fit was impracticable. Under a pressure of 100 lb. per sq. in. the length of the 16-in. cylinder was shortened nearly 1 in. and as much as a quart of water was forced out.

The initial modulus of elasticity increased from 3 100 000 lb. per sq. in. for the cylinders without pressure to 4 300 000 lb. per sq. in. for those setting under 100 lb. per sq. in.; an increase of about 37%. As in the case of the compressive strength, over one-half of this increase occurred below a pressure of 20 lb. per sq. in. The cylinders which set for 7 days under a pressure of 3 lb. per sq. in. and were gradually loaded to 500 lb. per sq. in. (No. 516 and 517) gave about the same values as those setting without pressure. Apparently the difference in storage conditions nearly counteracted the effect due to pressure. The increase in compressive strength and modulus of elasticity with the pressure under which the concrete sets is more pronounced in the 7-day than in the 80-day tests.

The density of the concrete as determined by the weight of the cylinders was increased about 4% by the pressure of 100 lb. per sq. in.

63. *Effect of Loads Reapplied after Failure in Compression.*—An interesting instance of "autogenous healing" in concrete was found in the retest of five cylinders at the age of 80 days which had been loaded to their ultimate strength in compression at 7 days. One of the cylinders had set under no pressure; two set under 20 lb. per sq. in. and two under 100 lb. per sq. in. During the period from the 7th to the 80th day the cylinders were stored in water. The results of the first and second tests have been bracketed together in Table 23. Load-deformation curves for three of the cylinders are given in Fig. 46. In all the 7-day tests loading was continued until there was a distinct drop in the beam of the testing machine and the extensometer showed a rapid increase in deformation. The fact that the load-deformation curves are nearly horizontal and that a unit-deformation greater than 0.0012 in. was produced in all these cylinders show that they received their ultimate loads in the 7-day tests. Numerous vertical cracks and surface flaking could be seen on most of the cylinders. The compressive strength and modulus of elasticity for the 80-day tests averaged 93% and 64%, respectively, greater than for the 7-day tests of the same cylinders. The average compressive strength and modulus of elasticity for the retested cylinders are 91% and 92%, respectively, of the values for similar cylinders which were tested for the first time at 80 days. It is note-

worthy that the cylinders which set under pressure behaved in much the same way as those setting under normal conditions.

Apparently a compressive stress equal to the ultimate resistance of concrete applied as much as 7 days after mixing (as long as the specimen is not entirely shattered) does not necessarily permanently destroy its usefulness, if this failure is followed by a period of rest in the presence of sufficient water to permit hydraulicity to continue.

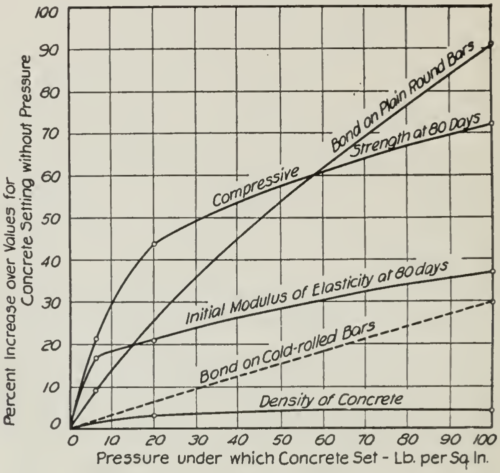


Fig. 47. Effect of Pressure During Setting on the Properties of Concrete.

64. *Effect of Reinforcing the Pull-out Specimen against Bursting.*—In many of the pull-out tests with deformed bars, the concrete blocks were reinforced against bursting by means of 6 or 7 turns of a ¼-in. wire spiral placed inside the forms. It was desired to determine what effect this restraint had on the load-slip relation and on the maximum bond stresses developed. Several specimens made with corrugated square and corrugated round bars were reinforced as indicated above and others from the same batch were without reinforcement. The results of these tests are given in Table 24. In nearly every case the specimens reinforced with the wire spiral gave higher bond stresses at all stages of the test than those without reinforcement. The difference is not very great for ages of 2 to 7 days. For the tests made at 28 days or over the bond resistance at a slip of 0.001 in. for the specimens with the spiral averaged about 15% higher than those not reinforced; at the maximum load the values are about 50% higher.

## TABLE 24.

## Effect of Reinforcing the Concrete Block against Splitting.

Pull-out tests, embedment 8 in. All specimens were stored in damp sand.
Stresses are given in pounds per square inch.

| Ref. No. | Age at Test days | Number of Tests | Remarks | Bond Stress at End Slip of—Inches | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | .0005 | .001 | .002 | .005 | .01 | .02 | .05 | .01 |
| (a) ¾-in. Corrugated Squares, 1-1-2 Hand-mixed Concrete (Batch 20). | | | | | | | | | | | |
| 1 | 2 | 2 | Spiral | 86 | 96 | 102 | 121 | 136 | 159 | 207 | 250 |
| 2 | 2 | 2 | Without spiral | 97 | 111 | 117 | 131 | 154 | 174 | 220 | 236 |
| 3 | 4 | 2 | Spiral | 145 | 157 | 174 | 208 | 234 | 268 | 315 | 351 |
| 4 | 4 | 2 | Without spiral | 108 | 129 | 148 | 177 | 196 | 230 | 258 | 259 |
| 5 | 7 | 2 | Spiral | 167 | 177 | 198 | 228 | 273 | 317 | 396 | 447 |
| 6 | 7 | 2 | Without spiral | 154 | 166 | 174 | 218 | 253 | 298 | 322 | 322 |
| 7 | 28 | 2 | Spiral | 309 | 329 | 343 | 377 | 422 | 507 | 683 | 827 |
| 8 | 28 | 2 | Without spiral | 258 | 286 | 316 | 393 | 480 | 527 | ...... | 527 |
| 9 | 60 | 5 | Spiral | 445 | 498 | 555 | 655 | 793 | 965 | 1101 | 1145 |
| 10 | 60 | 4 | Without spiral | 338 | 399 | 419 | 499 | ...... | ...... | ...... | 771 |
| 11 | 15 mo. | 2 | Spiral | 709 | 892 | 1015 | 1165 | 1340 | 1480 | ...... | 1525 |
| 12 | 15 mo. | 2 | Without spiral | 645 | 778 | 888 | ...... | ...... | ...... | ...... | 1025 |
| (b) ¾-in. Corrugated Squares, 1-1-2 Hand-mixed Concrete (Batch 23). | | | | | | | | | | | |
| 13 | 2 | 1 | Spiral | 86 | 92 | 102 | 138 | 166 | 200 | 270 | 312 |
| 14 | 2 | 1 | In 8-in. steel pipe form § | 91 | 114 | 119 | 140 | 166 | 195 | 271 | 326 |
| 15 | 2 | 2 | Without spiral | 93 | 105 | 112 | 129 | 142 | 180 | 211 | 214 |
| 17 | 4 | 1 | Spiral | 146 | 158 | 182 | 233 | 279 | 325 | 408 | 444 |
| 18 | 4 | 1 | In steel pipe § | 131 | 145 | 169 | 206 | 249 | 300 | 405 | 463 |
| 19 | 4 | 2 | Without spiral | 101 | 118 | 123 | 151 | 178 | 213 | 258 | 264 |
| 20 | 7 | 1 | Spiral | 192 | 216 | 246 | 310 | 375 | 450 | 533 | 555 |
| 21 | 7 | 1 | In steel pipe § | 148 | 171 | 190 | 243 | 300 | 376 | 495 | 562 |
| 22 | 7 | 2 | Without spiral | 150 | 159 | 177 | 194 | 210 | 214 | 242 | 274 |
| 23 | 39 | 1 | Spiral | 390 | 433 | 458 | 562 | 657 | 796 | 894 | 906 |
| 24 | 39 | 1 | In steel pipe § | 351 | 378 | 434 | 554 | 695 | 861 | 1075 | 1152 |
| 25 | 39 | 2 | Without spiral | 386 | 413 | 453 | 527 | 579 | ...... | ...... | 700 |
| 26 | 61 | 2 | Spiral | 577 | 660 | 749 | 887 | 1019 | 1104 | 1134 | 1152 |
| 27 | 61 | 2 | Without spiral | 378 | 439 | 489 | 607 | 717 | ...... | ...... | 819 |
| 28 | 5 mo. | 2 | Spiral | 516 | 586 | 657 | 829 | 1023 | 1195 | ...... | 1329 |
| 29 | 5 mo. | 2 | Without spiral | 474 | 564 | 639 | 770 | 894 | 1024 | ...... | 1070 |
| (c) 1⅛-in. Corrugated Round, 1-2-4 Hand-mixed Concrete. | | | | | | | | | | | |
| 30 | 76 | 5 | Spiral | 281 | 306 | 328 | 374 | 438 | 535 | 677 | 846 |
| 31 | 76 | 1 | Without spiral | 235 | 260 | 270 | 286 | 307 | 328 | 392 | 457 |
| (d) 1⅛-in. Corrugated Round, 1-2-4 Machine-mixed Concrete (1912). | | | | | | | | | | | |
| 32 | 80 | 2 | Spiral | 244 | 279 | 314 | 359 | 412 | 517 | 809 | 1115 |
| 33 | 80 | 2 | In 8-in. steel pipe § | 347 | 404 | 420 | 438 | 516 | 605 | 855 | 1070 |
| 34 | 80 | 2 | Without spiral | 234 | 248 | 258 | 278 | 312 | 341 | ...... | 375 |

§ Blocks were molded in an 8-in. length of 8-in. steel water pipe which remained in place during the test.

In order to study the effect of a greater amount of restraint against bursting, part of the specimens were molded and tested in forms consisting of 8-in. lengths of 8-in. steel water pipe. The specimens with solid pipe forms gave values about the same as those with the spiral, during the earlier stages of the tests, but as may have been expected, they gave much higher values of maximum bond resistance. These tests indicate that the restraint of the spiral reinforcement is effective in raising the bond resistance in pull-out tests with deformed bars.

Fig. 45 gives load-slip curves for a few tests which show the effect of reinforcing the concrete block against bursting, the effect of causing the concrete to set under pressure and the effect on the corrugated bar tests of molding the specimen with the long end of the bar upward.

## B. Reinforced Concrete Beam Tests.

65. *Preliminary.*—It is in the design of reinforced concrete beams that a correct knowledge of the bond resistance between the concrete and the steel is most important. The study of reinforced concrete beams with special reference to bond stresses was begun at the University of Illinois in 1909; additional series of beam tests designed for the study of bond stresses were made in 1911 and 1912. The results of 110 beam tests are given in this bulletin. The dimensions of the beams were: width 8 in., depth 12 in. (10 inches to the center of the steel) and span length 5 to 10 ft. Typical forms of reinforced concrete beams are shown in Fig. 2. The longitudinal reinforcement usually consisted of a single straight bar of large diameter placed in the middle of the width of the beam. These bars extended to the ends of the beams. Vertical stirrups were frequently used as web reinforcement. With a few exceptions which are noted in the tables, the beams were loaded at the one-third points of the span. The usual span length was 6 in. less than the total length of the beam; in some of the 1911 tests beams 7½ and 8½ ft. long were loaded on a span of 6 ft. with the ends overhanging the supports 9 in. or 15 in., instead of 3 in. as in the other beam tests. Bars of large diameter and beams of comparatively short span lengths were used in order to develop high bond stresses in comparison with the other stresses in the beam, and thus to produce bond failures.

66. *Measurements of Slip of Bar.*—In the beam tests the slip of the ends of the reinforcing bar was measured by means of Ames gages as in the pull-out tests. The instrument was carried by a wooden or

metal yoke which was attached to the ends of the beam in such a position that the movable head-piece of the instrument had a direct bearing against the end of the bar. As the test progressed, the amount of slip at each end of the bar was noted. In many of the beams of the series of 1911 and 1912 the slip of bar was measured at numerous points along the length. Openings about 1 in. in diameter were cut or formed in the concrete under the reinforcing bar at points where measurements of slip were desired. Plugs of $\frac{3}{8}$-in. square steel were screwed firmly into threaded holes in the bar. The gage was carried by a metal bracket attached to a steel plate which was fastened to the concrete by plaster of paris in two places on either side of the plug. The plunger of the gage rested against the steel plug so that a movement of the reinforcing bar in either direction with respect to the concrete would be indicated by the pointer. As many as 13 such instruments were used in some of the tests. The positions of these instruments are shown for typical tests in Fig. 56.

The amount of slip which has been termed "first slip of bar" in the tables corresponds to a movement of about 0.0002 in. We are justified in using a smaller quantity as a measure of first end slip in beams than was used in the pull-out tests, for the following reasons: (1) Measurements from the yoke to points on the concrete of the end of the beam near the bar do not show appreciable deformation in the concrete at any stage of the test. This is probably due to the unstressed condition of the concrete at the end of a beam. (2) The load-slip curves for the beam tests show that as soon as the slip at the end of the bar reaches about 0.0002 in., it rapidly increases with a further application of load. (3) Tests made on beams by allowing the load which produced a very small amount of slip at the end of the bar to remain constant for several hours or days indicate that the ultimate bond resistance probably would be developed under the indefinitely continued application of loads which produce at first an end slip as small at 0.0002 in. The tests in which slip of bar was measured at intermediate points are discussed in detail in Art. 94.

67. *Computation of Stresses in a Reinforced Concrete Beam.*—An analysis of the stresses in a reinforced concrete beam was given in Bulletin No. 4 of the University of Illinois Engineering Experiment Station, "Tests of Reinforced Concrete Beams," by A. N. Talbot, and reference may be made to that bulletin for methods of computing the stresses given in the following tables. In that analysis it was shown

that for the assumptions of beam theory the bond between concrete and steel in a reinforced concrete beam is a function of the vertical shear. The bond unit-stress was expressed by

$$u = \frac{v}{m\, o\, d'}$$

Where $v$ = the total vertical shear,
$m$ = the number of bars,
$o$ = the perimeter of one bar,
$d'$ = the effective depth of the beam, or the distance between the center of the steel and the centroid of the compressive stresses.

The formula assumes that the reinforcing bar ends at the point of support. As in the test beams the bar extended beyond the point of support 3 in. or more, the amount of available bond surface will be greater than that assumed in the analysis, but generally in the calculations this additional bond surface will be disregarded and the formula used as given above without modification.

It is recognized that this analysis does not consider all the phenomena of bond action. It will be seen below that after slip of bar becomes appreciable, the bond stress in a beam is not distributed as indicated by the formula. However, instead of attempting to take into account the effect of slip of bar and the cracking of the concrete, the values computed by the above formula will be used. These nominal values of bond resistance form a useful basis of comparison in a series of tests in which the dimensions and make-up of the beams are similar.

68. *Phenomena of Beam Tests.*—If a bar which is embedded in a prism of concrete is subjected to a pull at its ends a part of the tensile force is transmitted to the concrete and a bond stress is developed between the steel and the concrete which may be said to be due to the stiffness of the concrete or its resistance to stretch. When the concrete has been distended to its limit of stretch or to its limit of tensile strength, a break occurs and the concrete adjoining the break springs back toward the unbroken concrete, there is a slipping along the bar, and a minute crack forms at the break. The size of the crack and the amount of the slip will depend upon the relative dimensions of concrete and steel. It is evident that some tensile stress will remain in the unbroken concrete and some bond stress between concrete and steel even after slip has occurred, and that with increased tension in the bar there will be further

stretch in the concrete or further slip of the bar or more cracks in the concrete. For want of a better one, the term "anti-stretch slip" may be used for this slip of bar which is due to the stiffness or springiness of the concrete after cracking. At what strain this anti-stretch slip occurs and how far apart the cracks will be may be expected to be a function of the size and periphery of bar and the condition of its surface and the quality of the concrete and the section of concrete which is tributary to the reinforcing bar.

It is well here to call attention to the fact that a tension break in the concrete may occur without its presence being visible to the eye. The stretched concrete to the side of the break will act as a spring and tend to pull the concrete back from the break. If this elastic force is greater than the bond resistance slip will occur and the crack will widen and become perceptible. Such cracks are very fine. The whitewashing of the surface of the beams materially assisted in detecting these cracks at an early stage of their development, and doubtless many of them would not have been seen on the natural surface of the concrete. It may be added that the term "anti-stretch slip" has been used sometimes in the bulletin to cover the elastic force which goes with the slip.

In the case of a beam which is loaded symmetrically at the one-third points, the region between the two loads carries no vertical shear and there is no bond stress due to normal beam action as it is usually analyzed. However, this is the region of greatest longitudinal stress and hence the region in which a phenomenon similar to that just described will exist to a considerable degree. At the beginning of loading, the concrete itself will carry the greater part of the tensile stress and a part will be transmitted to the reinforcing bar. In the later stages, after cracks have formed, it may be simpler to consider the tension to be carried primarily by the bar and the concrete to form the web and the enveloping medium. At the formation of tension cracks, the phenomenon of anti-stretch slip described in the preceding paragraph will exist. The reinforced concrete beam tests to be described indicate that a slip of bar took place in this region at loads below those at which minute cracks on the whitewashed face of the beam appeared, and considerably below the loads at which they have been noted in beams having only the usual concrete surface for inspection. In general, this early interior slip of bar was noted at a tensile stress in the bar of about 6000 lb. per sq. in. The distance apart of these cracks and their relation to the form and size of bar will be discussed with the details of the tests.

For the region outside the load points, that is, in the outer thirds of the span length, for the method of loading generally used, shearing and other web stresses exist, and due to the beam action bond stresses between the concrete and steel are developed which may be termed beam bond stresses. By the usual analysis, for the loading used, these bond stresses are nominally uniform from load point to support. The anti-stretch slip may be expected to exist here also, especially in the part near the load points, and the existence of this slip and bond stress concurrently with the beam bond stress makes a considerable complication and may be expected greatly to modify the distribution of the bond stresses over the length of the bar, and to affect resistance to beam bond stresses.

In the tests of beams loaded at the one-third points, the beam exhibits considerable stiffness up to the load which causes the first tension cracks in the concrete. After this point, the beam deflection increases more rapidly and continues at a nearly constant rate until failure is imminent. A marked change in the rate of deflection follows the diminution in effective tensile resistance of the concrete and is coincident with the stage when anti-stretch slip first develops prominently in the middle third of the beam. Due to the inequalities in the tensile resistance of the concrete and to the development of anti-stretch slip the tension cracks form at certain points instead of being closely spaced, and a large part of the increase in length of the lower fibers is concentrated at these cracks. An examination of the photographs and sketches of the beams after failure will show that tension cracks form through the region of the middle third and generally a short distance outside with usually other tension cracks farther out.

For the stage of the test at which cracks first appeared at the load points or a short distance outside, the bar showed a measurable slip just beyond the crack (nearer the end). It was necessary to increase the load 50% to 300% before slip was produced at the end of the bar; the amount of increase varied principally with the distance between the load point and the support. We may expect then that a bond stress nearly as great as the ultimate bond resistance was being developed for a short distance beyond the crack, much of which was due to the condition which produces anti-stretch slip of bar. At this stage of the test the bond stress at the supports was only a small part of the maximum bond resistance. The tests indicate that when the maximum bond stress was first developed outside the load points the bond stress at the support was not more than, say, 15% to 40% of the maximum bond resistance. As the bar was further stressed by an additional load applied to the beam, the bond stress near the crack decreased on account of excessive slip of bar and the region of full bond resistance was thrown more and more toward the support. Later in the test, due to the combination of beam bond stress and anti-stretch slip, another crack was formed a few inches nearer the support. The opening of a second crack had the effect of reducing the bond stress between the cracks and the tensile stress in the bar was increased toward the support. As the loading progressed this process continued with the piecemeal development of the maximum bond resistance and the subsequent reduction of bond due to excessive slip over the portion of the bar affected, until the effective embedded length of the bar was no longer able to withstand the bond stresses developed and failure from slip of bar soon followed. It is clear that the unbroken embedded length of bar at the ends of the beam which takes the principal portion of the total bond stress during the last stages of the test, will depend upon the relative dimensions of the bars and the beam and upon the bond, tensile and shearing resistance of the concrete. The beam tests discussed below show how this distance varied for a variety of conditions.

Fig. 57 to 63 show the appearance of representative beams after testing. The surfaces of the beams had been whitewashed before the tests in order to facilitate the tracing of cracks. The positions of the load-points and supports are shown by vertical arrows. The surface cracks, which were generally very fine, were traced with black paint on the surface of the beams in order that the location of the cracks may be shown on the photographs. The numbers near the cracks indicate the extension of the cracks as the test progressed expressed in thousands of pounds load. In the beams in which slip of bar was measured at intermediate points, the positions of the instruments are indicated by the numbers inside circles; the points at which tensile deformations in the longitudinal bar were measured in the tests of certain beams are indicated in the same way.

The load-slip curves for representative beams in each group are given in Fig. 69 to 76, inclusive; load-deflection and end-slip-of-bar curves are given in Fig. 77 to 86, inclusive. These diagrams give important indications as to the effect of the different variables in the make-up or loading of the beams.

### *a. 1909 Beam Tests.*

69. *Outline of Series.*—In 1909 eleven reinforced concrete beams were tested with special reference to a study of bond stresses. Data of the beams and tests are given in Tables 25 and 26. These beams were considered as a preliminary series; no companion pull-out specimens were made.

## TABLE 25.

### DATA OF REINFORCED CONCRETE BEAMS—1909 SERIES.

1-2-4 hand-mixed concrete; Chicago AA portland cement.
Each beam was reinforced with a single longitudinal bar.
All beams were 8 in. wide; total depth 12 in.; depth to center of steel 10 in.; length 6½ ft.

| Beam No. | Longitudinal Reinforcement | | Stirrups (Round bars 4 in. apart) | Mixture by Weight | Beam from Same Batch | Compression Tests of 6-in. Cubes | |
|---|---|---|---|---|---|---|---|
| | Kind of Bar | Per cent | | | | Age at Test days | Average of 3 Tests lb. per sq. in. |
| 120 | 1-in. plain round............. | 0.98 | ⅝ in. | 1-2.35-4.15 | 121 | 82 | 1420 |
| 121 | 1-in. plain round............. | 0.98 | ⅝ in. | 1-2.35-4.15 | 120 | 82 | 1420 |
| 201 | 1-in. plain round............. | 0.98 | ⅝ in. | 1-2.53-4.40 | 202 | 62 | 1757 |
| 84 | 1¼-in. plain round........... | 1.53 | ⅝ in. | 1-2.34-4.27 | 85 | 65 | 1722 |
| 203 | 1¼-in. plain round........... | 1.53 | ½ in. | 1-2.52-4.47 | 204 | 63 | 1573 |
| 204 | 1¼-in. plain round........... | 1.53 | ½ in. | 1-2.52-4.47 | 203 | 63 | 1573 |
| 117 | 1-in. cup.................... | 1.25 | ⅝ in. | 1-2.23-3.93 | 118 | 87 | 1960 |
| 62 | 1⅛-in. corrugated round...... | 1.25 | ⅝ in. | 1-2.44-4.23 | ... | 65 | 1762 |
| 202 | 1⅛-in. corrugated round...... | 1.25 | ⅝ in. | 1-2.53-4.40 | 201 | 62 | 1757 |
| 85 | ¾-in. corrugated square...... | 0.70 | ⅝ in. | 1-2.34-4.27 | 84 | 65 | 1722 |
| 118 | 1-in. twisted square.......... | 1.25 | ⅝ in. | 1-2.23-3.93 | 117 | 87 | 1960 |

The average compressive strength of 6 sets of 6-in. cubes was 1700 lb. per sq. in.

All the beams were identical as to materials and external dimensions. 1-2-4 hand-mixed concrete made with Chicago AA cement was used. Each beam was reinforced with a single bar which extended flush with the ends of the beam. The size and spacing of vertical stirrups is given in Table 25. The age at test averaged about 100 days. All beams were loaded at the one-third points of a 6-ft. span. In all but one test (Beam No. 84) the load was applied progressively to failure. In the test of Beam No. 84, load was applied until one end of the bar showed an appreciable slip; the load was then released and reapplied. This test is discussed in detail in Art. 90.

70. *Bond with Plain Round Bars.*—End slip began in the beams reinforced with a 1-in. round bar at an average bond stress of 192 lb. per sq. in.; and for the 1¼-in. round at 211 lb. per sq. in. The average maximum bond resistance was 279 lb. per sq. in. for the 1-in. bars and 303 lb. per sq. in. for the 1¼-in. bars. The load-deflection and load-slip curves for Beams 121 and 204 are given in Fig. 77.

## TABLE 26.

### TESTS OF REINFORCED CONCRETE BEAMS—1909 SERIES.

**All beams were loaded at the one-third points of a 6-ft. span; overhang 3 in. at each end.**

**Loads are given in pounds; stresses in pounds per square inch.**

**In computing unit stresses, the weight of the beam was considered.**

| Beam No. | Age at Test days | Load at First Outer Crack | At First Slip of End of Bar° | | At Maximum Load | | | | Failure of Beam |
|---|---|---|---|---|---|---|---|---|---|
| | | | Applied Load | Computed Bond Stress | Applied Load | Tensile Stress in Steel | Vertical Shearing Stress | Computed Bond Stress | |
| 120 | 112 | 11 000 | 10 000 | 198 | 11 700 | 22 100 | 89 | 230 | Slow bond failure at S. end |
| 121 | 112 | 10 000 | 10 000 | 198 | 14 500 | 27 100 | 111 | 281 | Bond at S. end. |
| 201 | 88 | 10 000 | 10 000 | 180 | 16 900 | 31 400 | 128 | 327 | Bond at S. end |
| Av. | 104 | 10 300 | 10 000 | 192 | 14 370 | 26 870 | 109 | 279 | |
| 84* | 104 | 17 000 | 15 000 | 243 | 20 500 | 24 900 | 158 | 327 | Bond at S. end. |
| 203 | 91 | 13 000 | 11 200 | 185 | 19 300 | 23 500 | 149 | 309 | Bond at S. end. |
| 204 | 90 | 13 000 | 12 500 | 204 | 17 000 | 20 800 | 132 | 273 | Bond at S. end. |
| Av. | 95 | 14 300 | 12 900 | 211 | 18 900 | 23 070 | 146 | 303 | |
| 117 | 118 | 19 000 | 19 000 | 292 | 33 000 | 48 100 | 250 | 500 | Bond at S. end. |
| 62 | 56 | 12 000 | .. ... | ... | 22 500 | 33 200 | 172 | 390 | Diagonal tension and bond at S. end. |
| 202 | 88 | 14 000 | 12 000 | 213 | 19 000 | 28 100 | 146 | 332 | Bond at S. end. |
| Av. | 72 | 13 000 | .......... | ....... | 20 750 | 30 650 | 159 | 361 | |
| 85 | 105 | .. ... | 6 000 | 128 | 28 700 | 72 500 | 209 | 568 | Tension in steel. |
| 118 | 118 | 20 000 | 15 000 | 233 | 32 100 | 46 700 | 241 | 488 | Bond at S. end. |

° Corresponding to a slip of 0.0002 in.

* The load was released and reapplied. See Art. 90 and Fig. 51.

71. *Bond with Deformed Bars.*—One beam in the 1909 series was reinforced with a 1-in. cup bar, one with a ¾-in. corrugated square bar (type B) and two with 1⅛-in. corrugated rounds. Beam No. 118 reinforced with a 1-in. twisted square bar will be referred to in the discussion of other beams of the same kind in the 1912 series (see Art. 84).

Beam No. 117, reinforced with one 1-in. cup bar, carried the highest load in this series—33 000 lb. End slip began at an applied load of 19 000 lb. and a computed bond stress of 292 lb. per sq. in. This is about the same load that caused the first diagonal crack. The maximum bond resistance was 500 lb. per sq. in., corresponding to a steel stress of 48 100 lb. per sq. in. The load-end-slip and load-deflection curves for this test are given in Fig. 77. The slip measured at the end where failure occurred was 0.006 in. at the maximum load; the slip at the opposite end was 0.001 in.

Beam No. 85, reinforced with one ¾-in. corrugated square bar (type B) showed first end slip at a computed bond stress of 128 lb. per sq. in. The load-slip curve for this beam (Fig. 77) shows a very rapid slip at one end following an applied load of about 7000 lb. The end slip had reached 0.001 in. at a load of 8000 lb.; at the maximum load the slip was about 0.02 in. at one end and 0.004 in. at the other. While this beam failed finally by tension in the steel, it is plain that a bond resistance much higher than the value given—568 lb. per sq. in.—could not have been developed.

Beams No. 62 and 202 were each reinforced with one 1⅛-in. corrugated round bar. Slip of bar was not measured in Beam No. 62. In Beam No. 202 end slip began at a load of 12 000 lb. and bond stress of 213 lb. per sq. in. After a load of 12 000 lb., slipping at both ends was very pronounced. At failure, at a load of 19 000 lb., bond stress of 332 lb. per sq. in., one end of the bar had slipped 0.014 in.; the other 0.007 in. The bond stress at the beginning of end slip of Beam No. 202 was about 5% higher than the average of the six beams in this series which were reinforced with plain rounds. The average bond stress at the maximum load for the two beams reinforced with 1⅛-in. corrugated rounds (361 lb. per sq. in.) was 24% higher than the average of the six beams with plain rounds. However, the beams with corrugated bars were tested at a somewhat earlier age; on the other hand the cube tests show them to be made of concrete of somewhat higher compressive strength than those with plain bars.

### *b. 1911 Beam Tests.*

72. *Outline of Series.*—Thirty-six reinforced concrete beams were included in the 1911 series. Plain round and corrugated bars were used for longitudinal reinforcement. All the beams, except two, were tested with a 6-ft. span. Beam No. 1045.2 and 1045.3 were tested with an 8-ft. span. Several of the beams in this series were tested with ends overhanging the supports 9 in. or 15 in. In one group, the depth of concrete below the steel was varied. Fifteen of the beams had no web reinforcement; the remainder were provided with V-shaped stirrups of ¼-in. or ½-in. plain rounds. In the 1911 beams the concrete was hand-mixed; the age at test averaged about 8 months.

Pull-out specimens and 6-in. cubes were made from the same materials as were used in many of the beams. The pull-out specimens were stored in the open air with the beams, until tested; the cubes were stored in damp sand.

Details of the make-up of the beams, their dimensions, the materials used and the strength of the concrete will be found in Table 27. Table 28 gives the data of the tests and some of the calculated stresses in the beams as well as notes on failure. The computed bond stresses developed in the beams and in the pull-out tests in this series at various amounts of end slip will be found in Table 29. A summary of the bond stresses in the beam and pull-out tests is given in Table 30.

In several of the beams in this series measurements were made on slip of bar at points other than the ends. Discussion of the slip found at internal points will be given with the discussion of similar tests in the 1912 series (Art. 94).

73. *Basis of Comparison of Bond Resistance in Beam and Pull-out Tests.*—In the 1911 and 1912 beam series, the corresponding beams and pull-out specimens were made from the same batch and stored under the same conditions. In comparing the bond resistance in beam and pull-out tests, it is evident that the bond stresses corresponding to definite amounts of slip in each case should be considered. The amount of slip in both kinds of specimens was measured at the free end of the bar, and at other points in some of the beam tests. Differences in bond resistance developed in the two forms of test specimens may be expected to be due principally to differences in the secondary stresses in the

## TABLE 27.

## DATA OF REINFORCED CONCRETE BEAMS—1911 SERIES.

1-2-4 hand-mixed concrete. The longitudinal reinforcement in each beam consisted of a single bar of high-carbon steel. All beams were 8 in. wide; total depth 12 in., unless otherwise noted; depth to center of steel, 10-in.

| Group | Beam No. | Length feet | Longitudinal Reinforcement | | Stirrups (Round Bars 6 in. apart) | Beam from same Batch | Cement | Mixture by Weight | Compression Tests of 6-in. Cubes[°] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Kind of Round Bar | Per cent | | | | | Age at Test, mo. | Average of 3 Tests lb. per sq. in. |
| 1 | 1037.1 | 6½ | 1-in. plain | 0.98 | None | 1032.1 | Universal | 1-2.10-3.67 | 9 | 2733 |
| | 1037.2 | 6½ | 1-in. plain | 0.98 | None | 1039.2 | Universal | 1-2.08-3.46 | 8½ | 3977 |
| | 1037.3 | 6½ | 1-in. plain | 0.98 | None | 1043.2 | Universal | 1-2.12-3.56 | 7 | 3647 |
| | 1040.3 | 6½ | 1-in. plain | 0.98 | None | 1032.3 | Lehigh | 1-2.13-3.72 | 8½ | 2533 |
| 2 | 1039.1 | 6½ | 1-in. plain | 0.98 | ¼-in. | 1043.1 | Universal | 1-2.02-3.63 | 9 | 2610 |
| | 1039.2 | 6½ | 1-in. plain | 0.98 | ¼-in. | 1037.2 | Universal | 1-2.08-3.46 | 8½ | 3977 |
| | 1039.3 | 6½ | 1-in. plain | 0.98 | ¼-in. | 1035.3 | Lehigh | 1-2.06-3.62 | 7 | 4117 |
| 3 | 1031.1 | 6½ | 1-in. plain | 0.98 | ½-in. | 1034.1 | Universal | 1-2.11-3.67 | 9 | 3173 |
| | 1031.2 | 6½ | 1-in. plain | 0.98 | ½-in. | 1035.2 | Universal | 1-2.20-3.72 | 8 | 2307 |
| | 1032.1 | 6½ | 1-in. plain | 0.98 | ½-in. | 1037.1 | Universal | 1-2.10-3.67 | 9 | 2733 |
| | 1032.2 | 6½ | 1-in. plain | 0.98 | ½-in. | 1045.1 | Universal | 1-2.13-3.51 | 7½ | 3220 |
| 4 | 1031.3 | 6½ | 1-in. plain | 0.98 | ½-in. | 1037.3 | Universal | 1-2.12-3.56 | 7 | 3647 |
| | 1032.3 | 6½ | 1-in. plain | 0.98 | ½-in. | 1040.3 | Lehigh | 1-2.13-3.72 | 8½ | 2533 |
| 5 | 1044.1[+] | 6½ | 1-in. plain | 0.98 | None | 1044.5 | Universal | 1-2.05-3.40 | 7 | 3157 |
| | 1044.2[+] | 6½ | 1-in. plain | 0.98 | None | 1045.2 | Lehigh | 1-2.08-3.68 | 8½ | 2920 |
| | 1044.3[+] | 6½ | 1-in. plain | 0.98 | None | 1042.3 | Lehigh | 1-2.07-3.56 | 8 | 2800 |
| 6 | 1044.5[•] | 6½ | 1-in. plain | 0.98 | None | 1044.1 | Universal | 1-2.05-3.40 | 7 | 3657 |
| | 1044.6[•] | 6½ | 1-in. plain | 0.98 | None | 1045.3 | Lehigh | 1-2.08-3.51 | 8 | 2517 |
| | 1044.7[•] | 6½ | 1-in. plain | 0.98 | None | ...... | Lehigh | 1-2.03-3.77 | ... | .... |

[°] The average strength of 6-in. cubes tested at about 8 months was as follows: 11 sets from Universal cement, 3100 lb. per sq. in.; 6 sets from Lehigh cement, 2937 lb. per sq. in

[+] Total depth 11 in.; depth to center of steel 10 in.

[•] Total depth 14 in.; depth to center of steel 10-in.

*(Table 27 continued on page 136)*

# TABLE 28.

## TESTS OF REINFORCED CONCRETE BEAMS—1911 SERIES.

The loads were applied at the one-third points of the span, unless otherwise noted.

Loads are given in pounds; stresses are given in pounds per square inch. In computing unit stresses the weight of the beam was considered.

| Group | Beam No. | Length feet | Test Span feet | Over-hang at each End in. | Age at Test mo. | Load at First Outer Crack | At First Slip of End of Bar | | At Maximum Load | | | | Maximum Bond Resistance in Pull-out Tests | Failure of Beam |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Applied Load | Computed Bond Stress | Applied Load | Tensile Stress in Steel | Vertical Shearing Stress | Computed Bond Stress | | |
| 1 | 1037.1 | 6½ | 6 | 3 | 8 | 12 000 | 14 000 | 272 | 15 000 | 27 800 | 115 | 291 | ... | Diag. tension followed by bond. |
| | 1037.2 | 6½ | 6 | 3 | 8 | 10 000 | 8 000 | 159 | 18 500 | 34 200 | 140 | 356 | 568 | Sudden failure by bond, S. end. |
| | 1037.3 | 6½ | 6 | 3 | 8½ | 10 000 | 15 000 | 291 | 21 400 | 39 300 | 162 | 410 | ... | Bond failure at S. end. |
| | 1040.3 | 6½ | 6 | 3 | 7 | 10 000 | 12 000 | 235 | 18 900 | 34 800 | 143 | 363 | 373 | Bond and diag. tens. at N. end. |
| | Average.. | ....... | ..... | ..... | 8 | 10 500 | 12 200 | 239 | 18 450 | 34 000 | 140 | 355 | 470 | |
| 2 | 1039.1 | 6½ | 6 | 3 | 8 | 10 000 | 12 000 | 235 | 17 300 | 32 000 | 132 | 334 | 457 | Bond at S. end. |
| | 1039.2 | 6½ | 6 | 3 | 8 | 8 000 | 14 000 | 272 | 27 800 | 50 800 | 209 | 529 | 568 | Bond at S. end. |
| | 1039.3† | 6½ | 6 | 3 | 7½ | 12 000 | 20 000 | 384 | 26 000† | 47 600 | 196 | 496† | 687 | Bond at S. end |
| | Average.. | ....... | ..... | ..... | 8 | 10 000 | 15 300 | 297 | 23 700 | 43 500 | 179 | 453 | 571 | |
| 3 | 1031.1 | 6½ | 6 | 3 | 8 | 8 000 | 11 000 | 216 | 15 900 | 29 500 | 123 | 307 | 477 | Bond; bar pulled out at N. end. |
| | 1031.2 | 6½ | 6 | 3 | 8 | 7 000 | 9 000 | 179 | 16 400 | 30 400 | 125 | 317 | 566 | Bond followed by diag. tens. |
| | 1032.1 | 6½ | 6 | 3 | 8 | 6 000 | 10 000 | 197 | 15 000 | 27 800 | 115 | 290 | ... | Bond at S. end. |
| | 1032.2 | 6½ | 6 | 3 | 7 | 14 500. | 15 000 | 291 | 21 500 | 39 500 | 163 | 411 | 542 | Bond at S. end. |
| | Average.. | ....... | ..... | ..... | 8 | 8 900 | 11 200 | 221 | 17 200 | 32 400 | 142 | 331 | 528 | |
| 4 | 1031.3* | 6½ | 6 | 3 | 8 | 11 000 | 10 000 | 197 | 18 900 | 43 500 | 143 | 363* | ... | Probably bond at S. end. |
| | 1032.3* | 6½ | 6 | 3 | 7 | 8 000 | 10 000 | 197 | 20 500 | 47 100 | 155 | 393* | 373 | Bond at S. end. |
| | Average.. | ....... | ..... | ..... | 7½ | 9 500 | 10 000 | 197 | 19 700 | 45 300 | 149 | 378 | ... | |
| 5 | 1044.1 | 6½ | 6 | 3 | 7 | 12 000 | 17 000 | 327 | 20 000 | 36 800 | 152 | 383 | 371 | Bond at N. end. |
| | 1044.2 | 6½ | 6 | 3 | 7 | 12 000 | 14 000 | 272 | 17 800 | 32 800 | 135 | 345 | ... | Bond at N. end. |
| | 1044.3 | 6½ | 6 | 3 | 7 | 8 000 | 12 000 | 234 | 17 850 | 32 900 | 136 | 344 | ... | Bond at S. end. |
| | Average.. | ....... | ..... | ..... | 7 | 10 700 | 14 300 | 274 | 18 550 | 34 200 | 141 | 357 | ... | |
| 6 | 1044.5 | 6½ | 6 | 3 | 10 | ...... | 23 000 | 439 | 27 400 | 50 100 | 205 | 522 | 371 | Bond at N. end. |
| | 1044.6 | 6½ | 6 | 3 | 7 | 10 000 | 24 000 | 458 | 29 700 | 54 200 | 222 | 564 | ... | Bond at N. end. |
| | 1044.7 | 6½ | 6 | 3 | 7 | ...... | 21 000 | 403 | 30 700 | 56 000 | 230 | 583 | ... | Bond at N. end. |
| | Average.. | ....... | ..... | ..... | 8 | 10 000 | 22 700 | 433 | 29 300 | 53 400 | 219 | 556 | ... | |

† The load on Beam No. 1039.3 was maintained at 26 000 lb. until the bar pulled out; see Art. 90 and Fig. 52.

* Loaded at points 6 in. each side of middle.

(*Table 28 continued on page 137*)

## TABLE 27—CONTINUED FROM PAGE 134.

## DATA OF REINFORCED CONCRETE BEAMS—1911 SERIES.

1-2-4 hand-mixed concrete. The longitudinal reinforcement in each beam consisted of a single bar of high-carbon steel. All beams were 8 in. wide; total depth 12 in., unless otherwise noted; depth to center of steel, 10-in.

| Group | Beam No. | Length feet | Longitudinal Reinforcement | | Stirrups (Round Bars 6 in. apart) | Beam from same Batch | Cement | Mixture by Weight | Compression Tests of 6-in. Cubes | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Kind of Round Bar | Per cent | | | | | Age at Test, mo. | Average of 3 Tests lb. per sq. in. |
| 7 | 1042.1 | 7½ | 1-in. plain | 0.98 | ½-in. | 1035.1 | Universal | 1-2.12-3.63 | 9 | 2813 |
| | 1042.2 | 7½ | 1-in. plain | 0.98 | ½-in. | 1043.3 | Lehigh | 1-2.13-3.58 | 8½ | 2737 |
| 8 | 1043.1 | 8½ | 1-in. plain | 0.98 | None | 1039.1 | Universal | 1-2.02-3.63 | 9 | 2610 |
| | 1043.2 | 8½ | 1-in. plain | 0.98 | None | 1031.3 | Universal | 1-2.12-3.56 | 7 | 3647 |
| 9 | 1035.1 | 6½ | 1¼-in. plain | 1.53 | ½-in. | 1042.1 | Universal | 1-2.12-3.63 | 9 | 2813 |
| | 1035.2 | 6½ | 1¼-in. plain | 1.53 | ½-in. | 1031.2 | Universal | 1-2.20-3.72 | 8 | 2307 |
| | 1035.3 | 6½ | 1¼-in. plain | 1.53 | ½-in. | 1039.3 | Lehigh | 1-2.06-3.62 | 7 | 4117 |
| 10 | 1043.3 | 8½ | 1¼-in. plain | 1.53 | None | 1042.2 | Lehigh | 1-2.13-3.58 | 8½ | 2737 |
| | 1045.1 | 8½ | 1¼-in. plain | 1.53 | ½-in. | 1032.2 | Universal | 1-2.13-3.51 | 7½ | 3220 |
| 11 | 1045.2 | 8½ | 1¼-in. plain | 1.53 | ½-in. | 1044.2 | Lehigh | 1-2.08-3.68 | 8½ | 2920 |
| | 1045.3 | 8½ | 1¼-in. plain | 1.53 | ½-in. | 1044.6 | Lehigh | 1-2.08-3.51 | 8 | 2517 |
| 12 | 1034.1 | 6½ | 1⅛-in. corrugated | 1.24 | ½-in. | 1031.1 | Universal | 1-2.11-3.67 | 9 | 3173 |
| | 1034.2 | 6½ | 1⅛-in. corrugated | 1.24 | ½-in. | 1040.1 | Universal | 1-2.13-3.85 | 8 | 2630 |
| | 1034.3 | 6½ | 1⅛-in. corrugated | 1.24 | ½-in. | 1040.2 | Universal | 1-2.13-3.52 | 8½ | 3807 |
| 13 | 1040.1 | 7½ | 1⅛-in. corrugated | 1.24 | None | 1034.2 | Universal | 1-2.13-3.85 | 8 | 2630 |
| | 1040.2 | 7½ | 1⅛-in. corrugated | 1.24 | None | 1034.3 | Universal | 1-2.13-3.52 | 8½ | 3807 |
| | 1042.3 | 7½ | 1⅛-in. corrugated | 1.24 | ½-in. | 1044.3 | Lehigh | 1-2.07-3.56 | 8 | 2800 |

[a] The average strength of 6-in. cubes tested at about 8 months was as follows: 11 sets from Universal cement, 3100 lb. per sq. in.; 6 sets from Lehigh cement, 2937 lb. per sq. in.

## TABLE 28—CONTINUED FROM PAGE 135.

### TESTS OF REINFORCED CONCRETE BEAMS—1911 SERIES.

The loads were applied at the one-third points of the span.

Loads are given in pounds; stresses are given in pounds per square inch. In computing unit stresses the weight of the beam was considered.

| Group | Beam No. | Length feet | Test Span feet | Overhang at each End in. | Age at Test mo. | Load at First Outer Crack | At First Slip of End of Bar | | At Maximum Load | | | | Maximum Bond Resistance in Pull-out Tests | Failure of Beam |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Applied Load | Computed Bond Stress | Applied Load | Tensile Stress in Steel | Vertical Shearing Stress | Computed Bond Stress | | |
| 7 | 1042.1 | 7½ | 6 | 9 | 8 | 8 000 | 20 500 | 394 | 25 600 | 46 800 | 193 | 488 | 375 | Bond at S. end. |
| | 1042.2 | 7½ | 6 | 9 | 11 | 11 000 | 19 000 | 365 | 26 900 | 49 200 | 202 | 511 | ... | Bond at S. end. |
| | Average.. | ....... | ..... | ..... | 9½ | 9 500 | 19 750 | 379 | 26 250 | 48 000 | 197 | 500 | ... | |
| 8 | 1043.1 | 8½ | 6 | 15 | 8 | 14 600 | 20 000 | 384 | 23 600 | 43 200 | 178 | 451 | 457 | Diag. tens. followed by bond. |
| | 1043.2 | 8½ | 6 | 15 | 8½ | 10 000 | 26 000 | 496 | 26 200 | 48 000 | 197 | 498 | ... | Diag. tens. at S. end. |
| | Average.. | ....... | ..... | ..... | 8 | 12 300 | 23 000 | 440 | 24 900 | 45 600 | 187 | 475 | ... | |
| 9 | 1035.1 | 6½ | 6 | 3 | 8 | 12 000 | 14 000 | 226 | 19 200 | 23 300 | 149 | 307 | 395 | Bond at N. end. |
| | 1035.2 | 6½ | 6 | 3 | 8 | 8 000 | 10 000 | 165 | 21 100 | 25 500 | 163 | 336 | 480 | Bond at S. end. |
| | 1035.3 | 6½ | 6 | 3 | 7 | 12 000 | 14 000 | 226 | 38 000 | 45 500 | 291 | 598 | 536 | Diag. tens. followed by bond. |
| | Average.. | ....... | ..... | ..... | 7½ | 10 700 | 12 700 | 206 | 26 100 | 31 400 | 201 | 413 | 470 | |
| 10 | 1043.3 | 8½ | 6 | 15 | 10½ | 12 000 | 22 000 | 351 | 22 500 | 27 200 | 174 | 358 | ... | Diagonal tension. |
| | 1045.1 | 8½ | 6 | 15 | 7 | 12 000 | 28 000 | 444 | 36 400 | 43 600 | 271 | 572 | 542 | Diag. tens. followed by bond. |
| | Average.. | ....... | ..... | ..... | 9 | 12 000 | 25 000 | 397 | 29 450 | 35 400 | 226 | 465 | ... | |
| 11 | 1045.2 | 8½ | 8 | 3 | 11 | 8 000 | 18 000 | 288 | 23 900 | 38 600 | 186 | 380 | ... | Bond at N. end. |
| | 1045.3 | 8½ | 8 | 3 | 10 | 8 000 | 14 000 | 227 | 23 500 | 38 200 | 182 | 374 | ... | Bond at N. end. |
| | Average.. | ....... | ..... | ..... | 10½ | 8 000 | 16 000 | 257 | 23 700 | 38 400 | 184 | 377 | ... | |
| 12 | 1034.1 | 6½ | 6 | 3 | 8 | 14 000 | 14 000 | 245 | 28 200 | 41 200 | 214 | 486 | 1374 | Bond at N. end. |
| | 1034.2 | 6½ | 6 | 3 | 7 | 10 000 | 14 500 | 255 | 25 000 | 36 600 | 190 | 432 | 1444 | Bond at N. end. |
| | 1034.3 | 6½ | 6 | 3 | 9 | 10 000 | 10 000 | 179 | 33 600 | 49 000 | 251 | 577 | 471 | Bond and diag. tens. at both ends. |
| | Average.. | ....... | ..... | ..... | 8 | 11 300 | 12 800 | 226 | 28 900 | 42 300 | 219 | 498 | ... | |
| 13 | 1040.1 | 7½ | 6 | 9 | 7 | 13 400 | 18 000 | 314 | 25 500 | 37 300 | 194 | 441 | 1444 | Diag. tens. followed by bond. |
| | 1040.2 | 7½ | 6 | 9 | 8 | 10 000 | 21 000 | 364 | 27 700 | 40 500 | 210 | 478 | 471 | Diagonal tension. |
| | 1042.3 | 7½ | 6 | 9 | 7 | 10 000 | 28 000 | 483 | 32 500 | 47 300 | 246 | 559 | ... | Tension in steel. |
| | Average.. | ....... | ..... | ..... | 7½ | 11 100 | 22 400 | 387 | 28 600 | 41 700 | 217 | 493 | ... | |

## TABLE 29.

### COMPARISON OF BOND RESISTANCE IN BEAMS AND PULL-OUT TESTS—1911 SERIES.

The measurements of slip of bar in the beam tests refer to the end showing the greater slip.
The bond stresses given for the pull-out specimens are the average of three tests on bars embedded 8 in.
Stresses are given in pounds per square inch.

| Group | Beam No. | Size and Kind of Round Bar | BEAM TESTS | | | | | | | PULL-OUT TESTS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Age at Test mo. | Bond Stress at End Slip of (inches) | | | | | Computed Bond Stress at Maximum Load | Age at Test mo. | Bond Stress at End Slip of (inches) | | | | | | | Maximum Bond Resistance |
| | | | | .0002 | .0005 | .001 | .002 | .005 | | | .0002 | .0005 | .001 | .002 | .005 | .010 | .020 | |
| 1 | 1037.1 | 1-in. plain | 8 | 272 | 280 | ... | ... | ... | 291 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1037.2 | 1-in. plain | 8 | 159 | 291 | 340 | ... | ... | 356 | 9 | 280 | 348 | 398 | 449 | 518 | 555 | 556 | 568 |
| | 1037.3 | 1-in. plain | 8½ | 291 | 365 | 394 | ... | ... | 410 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1040.3 | 1-in. plain | 7 | 235 | 330 | ... | ... | ... | 363 | 7 | 194 | 233 | 254 | 285 | 338 | 363 | 368 | 373 |
| | Average | | 8 | 239 | 316 | ... | ... | ... | 352 | 8 | 237 | 290 | 326 | 367 | 428 | 459 | 462 | 470 |
| 2 | 1039.1 | 1-in. plain | 8 | 235 | 272 | 310 | 319 | 280 | 334 | 6 | 224 | 281 | 334 | 374 | 437 | 456 | 437 | 457 |
| | 1039.2 | 1-in. plain | 8 | 275 | 367 | 479 | 510 | 525 | 531 | 9 | 280 | 348 | 398 | 449 | 518 | 555 | 556 | 558 |
| | 1039.3 | 1-in. plain | 7½ | 421 | 478 | 496 | ... | ... | 496 | 8 | 355 | 491 | 557 | 598 | 632 | 668 | 667 | 687 |
| | Average | | 8 | 310 | 372 | 428 | ... | ... | 454 | 8 | 286 | 373 | 430 | 474 | 529 | 560 | 553 | 567 |
| 3 | 1031.1 | 1-in. plain | 8 | 190 | 200 | 233 | 255 | 280 | 307 | 5 | 235 | 289 | 345 | 394 | 461 | 470 | 465 | 477 |
| | 1031.2 | 1-in. plain | 8 | 189 | 235 | 253 | 281 | ... | 317 | 8 | 210 | 316 | 393 | 468 | 543 | 566 | 551 | 566 |
| | 1032.1 | 1-in. plain | 8 | 197 | 220 | 231 | 253 | 280 | 290 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1032.2 | 1-in. plain | 7 | 291 | 328 | 345 | 383 | 402 | 411 | 9 | 306 | 379 | 436 | 484 | 529 | 540 | 530 | 542 |
| | Average | | 8 | 217 | 246 | 266 | 293 | ... | 331 | 7½ | 250 | 328 | 391 | 448 | 511 | 525 | 515 | 525 |
| 4 | 1031.3 | 1-in. plain | 8 | 197 | 309 | 356 | ... | ... | 363 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1032.3 | 1-in. plain | 7 | 197 | 291 | 347 | 374 | 393 | 393 | 7 | 194 | 233 | 254 | 285 | 338 | 363 | 368 | 373 |
| | Average | | 7½ | 197 | 300 | 351 | ... | ... | 378 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| 5 | 1044.1 | 1-in. plain | 7 | 300 | 347 | 364 | 383 | ... | 383 | 6 | 191 | 230 | 274 | 303 | 352 | 365 | 346 | 371 |
| | 1044.2 | 1-in. plain | 7 | 272 | 328 | 340 | ... | ... | 345 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1044.3 | 1-in. plain | 7 | 233 | 315 | 330 | 340 | 344 | 344 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | Average | | 7 | 268 | 330 | 345 | ... | ... | 357 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| 6 | 1044.5 | 1-in. plain | 10 | 439 | 445 | 460 | 485 | 512 | 522 | 6 | 191 | 230 | 274 | 303 | 352 | 365 | 346 | 371 |
| | 1044.6 | 1-in. plain | 7 | 458 | 520 | 533 | 542 | 561 | 566 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1044.7 | 1-in. plain | 7 | 403 | 478 | 530 | 541 | 580 | 583 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | Average | | 8 | 433 | 481 | 508 | 523 | 551 | 557 | . | . | ... | ... | ... | ... | ... | ... | . |

## TABLE 29—Continued.

## Comparison of Bond Resistance in Beams and Pull-out Tests—1911 Series.

| Group | Beam No. | Size and Kind of Round Bar | BEAM TESTS | | | | | | | PULL-OUT TESTS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Age at Test mo. | Bond Stress at End Slip of (inches) | | | | | Computed Bond Stress at Maximum Load | Age at Test mo. | Bond Stress at End Slip of (inches) | | | | | | | Maximum Bond Resistance |
| | | | | .0002 | .0005 | .001 | .002 | .005 | | | .0002 | .0005 | .001 | .002 | .005 | .010 | .020 | |
| 7 | 1042.1 | 1-in. plain.......... | 8 | 394 | 425 | 459 | 480 | ... | 488 | 8 | 160 | 180 | 208 | 248 | 329 | 364 | 372 | 375 |
| | 1042.2 | 1-in. plain.......... | 11 | 365 | 402 | 430 | 444 | 486 | 511 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | Average | .................... | 9½ | 380 | 414 | 445 | 462 | ... | 500 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| 8 | 1043.1 | 1-in. plain.......... | 8 | 384 | 421 | 410 | 420 | 403 | 451 | 6 | 224 | 281 | 334 | 374 | 437 | 456 | 437 | 457 |
| | 1043.2 | 1-in. plain.......... | 8½ | 496 | ... | 498 | 477 | 498 | 498 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | Average | .................... | 8 | 440 | ... | 454 | ... | 440 | 474 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| 9 | 1035.1 | 1¼-in. plain........ | 8 | 226 | 230 | ... | 240 | 307 | 307 | 8 | 160 | 180 | 208 | 248 | 328 | 364 | 372 | 375 |
| | 1035.2 | 1¼-in. plain........ | 8 | 165 | 227 | 258 | 273 | 319 | 336 | 7 | 204 | 326 | 398 | 449 | 478 | ... | 424 | 480 |
| | 1035.3 | 1¼-in. plain........ | 7 | 227 | 319 | 400 | 549 | ... | 596 | 7½ | 284 | 368 | 419 | 467 | 502 | 526 | 515 | 536 |
| | Average | .................... | 7½ | 206 | 259 | ... | 354 | ... | 413 | 7½ | 216 | 291 | 342 | 404 | 436 | ... | 455 | 495 |
| 10 | 1043.3 | 1¼-in. plain........ | 10½ | 350 | 358 | ... | ... | ... | 358 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | 1045.1 | 1¼-in. plain........ | 7 | 444 | 474 | 515 | 534 | 555 | 572 | 7 | 306 | 379 | 436 | 484 | 529 | 540 | 530 | 542 |
| | Average | .................... | 9 | 397 | 416 | ... | ... | ... | 465 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| 11 | 1045.2 | 1¼-in. plain........ | 11 | 288 | 351 | 366 | ... | ... | 378 | 9 | 306 | 379 | 436 | 484 | 529 | 540 | 530 | 542 |
| | 1045.3 | 1¼-in. plain........ | 10 | 227 | 289 | 340 | 358 | 365 | 374 | 9 | 306 | 379 | 436 | 484 | 529 | 540 | 530 | 542 |
| | Average | .................... | 10½ | 267 | 320 | 353 | ... | ... | 376 | 9 | 306 | 379 | 436 | 484 | 529 | 540 | 530 | 542 |
| 12 | 1034.1 | 1⅛-in. corrugated... | 8 | 245 | 290 | 320 | 350 | 412 | 482 | 9 | 121 | 162 | 216 | 300 | 456 | 571 | 774 | 1374* |
| | 1034.2 | 1⅛-in. corrugated... | 7 | 240 | 260 | 296 | 313 | 363 | 430 | 7 | 222 | 289 | 363 | 404 | 450 | 491 | 585 | 1444* |
| | 1034.3 | 1⅛-in. corrugated... | 9 | 179 | 264 | 330 | 367 | 430 | 576 | 7 | 201 | 275 | 320 | 351 | 389 | 412 | 418 | 471 |
| | Average | .................... | 8 | 221 | 271 | 317 | 343 | 402 | 496 | 7½ | 181 | 242 | 300 | 352 | 432 | 491 | 592 | 1096 |
| 13 | 1040.1 | 1⅛-in. corrugated... | 7 | 314 | 360 | 410 | 433 | 400 | 441 | 7 | 222 | 289 | 363 | 404 | 450 | 491 | 585 | 1444* |
| | 1040.2 | 1⅛-in. corrugated... | 8 | 364 | 435 | 450 | 475 | ... | 478 | 7 | 201 | 275 | 320 | 351 | 389 | 412 | 418 | 471 |
| | 1042.3 | 1⅛-in. corrugated... | 7 | 483 | 520 | 559 | ... | ... | 559 | . | ... | ... | ... | ... | ... | ... | ... | ... |
| | Average | .................... | 7½ | 387 | 438 | 473 | ... | ... | 493 | 7 | 211 | 282 | 341 | 377 | 419 | 451 | 501 | 907 |

* Blocks reinforced with 6 turns of ¼-in. wire in the form of a spiral.

specimen and in a much less degree to the form of the specimen. The anti-stretch slip mentioned in a preceding paragraph is a manifestation of one of the secondary stresses. Since the beams were loaded symmetrically and failure was practically always due to bond or some cause involving bond, the bar pulled out at the end where the bond resistance was the weaker. In other words the beam tests give in each case the smaller of two possible values of bond resistance resulting from the same combination of materials. In a few tests one end of the bar showed a greater slip during the earlier stages of loading, only to be overtaken by the other end as the loading progressed. In two or three tests both ends of the beam behaved almost identically in this respect. These observations apply to beams reinforced both with plain and deformed bars. The pull-out tests for comparison with the reinforced concrete beams were made in sets of three.

Since we have automatically rejected the higher of the two bond resistances in the beams, it will make the comparison more nearly correct to reject the highest one of the pull-out specimens. The average values for the lowest two from each set of pull-out specimens are given in Tables 29 and 30. These are the values which have been used for comparison in the discussion of beam and pull-out tests. It will be seen that this method gives values for the pull-out tests which are a little too high; but the error due to this cause is not of enough consequence to justify further refinement in the computations.

74. *Bond with Plain Round Bars.*—In the 1911 series 23 beams were reinforced with 1-in. plain rounds and 7 beams with 1¼-in. plain rounds. The effect of size of bar, vertical stirrups, depth of concrete below the steel, overhang of ends, etc., in beams reinforced with plain rounds will be discussed in detail in the following articles. It will be seen later that the presence of vertical stirrups has no considerable effect on the bond resistance of beams of this kind, hence for purposes of the present discussion all beams in the 1911 series reinforced with 1-in. plain round bars, except those having a total depth of 14 in. and those with unusual overhang of ends, will be grouped together.

The groups with 1-in. plain rounds include 16 tests as shown in groups 1 to 5, Table 30. End slip became appreciable at a bond stress of 247 lb. per sq. in., 67% of the maximum bond resistance. The largest amount of end slip which was observed in every test in these groups was 0.001 in., corresponding to a bond stress of 344 lb. per sq. in., 92% of

## TABLE 30.

## SUMMARY OF BOND STRESSES IN BEAM AND PULL-OUT TESTS—1911 SERIES.

Stresses are given in pounds per square inch.

| Group | Characteristics | No. of Tests | Age at Test mo. | Bond Stress at End Slip of (inches) | | | | | | Computed Bond Stress at Maximum Load |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | .0002 | .0005 | .001 | .002 | .005 | .01 | |
| | **Beam Tests; 1-in. Plain Rounds.** | | | | | | | | | |
| 1–5 | All beams except as noted* | 16 | 7½ | 247 | 310 | 344 | ..... | ..... | ..... | 371† |
| 7 | Beams with 9-in. overhang | 2 | 9½ | 380 | 414 | 445 | 462 | ..... | ..... | 500 |
| 8 | Beams with 15-in. overhang | 2 | 8 | 440 | ..... | 454 | ..... | 440 | ..... | 474 |
| 6 | Beams 14 in. total depth | 3 | 8 | 433 | 481 | 508 | 523 | ..... | ..... | 557 |
| | **Pull-Out Tests; 1-in. Plain Rounds.** | | | | | | | | | |
| 1–8 | Lowest two from each set of three tests° | 14 | ..... | 225 | 298 | 353 | 399 | 457 | 481 | 493 |
| | All pull-out tests | 21 | 7 | 242 | 312 | 365 | 410 | 469 | 491 | 500 |
| | Highest from each set | 7 | ..... | 275 | 340 | 390 | 433 | 491 | 509 | 511 |
| | Lowest from each set | 7 | ..... | 201 | 284 | 338 | 385 | 430 | 475 | 491 |
| | **Beam Tests; 1¼-in. Plain Rounds.** | | | | | | | | | |
| 9 | 6-ft. beams with 3-in. overhang | 3 | 7½ | 206 | 259 | ..... | 354 | ..... | ..... | 413 |
| 10 | 8-ft. beams with 3-in. overhang | 2 | 8½ | 397 | 416 | ..... | ..... | ..... | ..... | 465 |
| 11 | 6-ft. beams with 15-in. overhang | 2 | 10½ | 267 | 320 | 353 | ..... | ..... | ..... | 376 |
| | **Pull-out Tests; 1¼-in. Plain Rounds.** | | | | | | | | | |
| 9–11 | Lowest two from each set of 3 tests° | 8 | ..... | 231 | 301 | 342 | 389 | 446 | ..... | 474 |
| | All pull-out tests | 12 | 8 | 239 | 313 | 365 | 412 | 459 | 483 | 483 |
| | Highest from each set | 4 | ..... | 253 | 338 | 413 | 459 | 487 | ..... | 502 |
| | Lowest from each set | 4 | ..... | 226 | 266 | 313 | 360 | 410 | ..... | 441 |
| | **Beam Tests; 1⅛-in. Corrugated Rounds.** | | | | | | | | | |
| 12 | Beams with 3-in. overhang | 3 | 8 | 221 | 271 | 317 | 343 | 402 | .... | 496 |
| 13 | Beams with 9-in. overhang | 3 | 7½ | 387 | 438 | 473 | ..... | ..... | ..... | 493 |
| | **Pull-out Tests; 1⅛-in. Corrugated Rounds.** | | | | | | | | | |
| 12, 13 | Lowest two from each set of 3 tests° | 6 | ..... | 184 | 233 | 270 | 313 | 400 | 440 | ✦ |
| | All pull-out tests | 9 | 7½ | 181 | 242 | 300 | 354 | 432 | 491 | ✦ |
| | Highest from each set | 3 | ..... | 141 | 262 | 360 | 464 | 526 | 598 | ✦ |
| | Lowest from each set | 3 | ..... | 190 | 242 | 280 | 312 | 367 | 409 | ✦ |

* Includes all beams reinforced with 1-in. plain rounds, except those in groups 6, 7 and 8.

† The maximum bond resistance for 5 beams made from Lehigh cement averaged 390 lb. per sq. in. See cube tests, Table 27.

° Use these values for comparison with bond stresses in beam tests. See Art. 73.

✦ Omitted on account of part of the specimens being reinforced against bursting and part unreinforced. See Table 29 for details of these tests.

the maximum bond resistance. It is evident that when the end slip of the bar has reached 0.001 in. the stress distribution is such that the beam generally has passed its usefulness for carrying load.

In the case of five beams reinforced with 1¼-in. plain rounds (Table 30) end slip began at a computed bond stress of 282 lb. per sq. in., 65% of the maximum resistance. In this group an end slip of 0.0005 in. was measured in all the tests, corresponding to a bond stress of 86% of the maximum bond resistance.

A comparison of the bond stresses developed in the beams reinforced with 1-in. and 1¼-in. plain rounds in Table 30 and Fig. 49 with the pull-out tests on bars of the same sizes (the average of the lowest two tests from each set) shows values for the two kinds of tests which are similar at each amount of slip given in the table up to an end slip of about 0.001 in. The maximum bond resistance for the beams in these two groups is about the same as the bond stress developed in the corresponding pull-out specimens at a slip of 0.002 in. Fig. 78 shows the load-deflection and load-slip curves for typical beams reinforced with plain round bars. The tests do not show any material difference in bond unit resistance due to size of bar.

A summary of the bond stresses developed in all the 6-ft. beams reinforced with plain round bars and loaded at one-third points is given in Table 35.

75. *Effect of Vertical Stirrups on Bond Resistance.*—Beam groups 1, 2 and 3, in Table 28, may be used to see whether the presence of vertical stirrups influences bond resistance in beams of this kind. The beams in group 1 had no stirrups; those in groups 2 and 3 were reinforced throughout the outer thirds with V-shaped stirrups of ¼-in. and ½-in. rounds, respectively, spaced 6 in. apart as shown in Fig. 2 (b). Each of these beams was reinforced with one 1-in. plain round, and was tested on a 6-ft. span. The relation of bond resistance due to the presence of web reinforcement is not well defined. The beams with ¼-in. stirrups gave higher values for bond resistance at all stages of the tests than either of the other groups. In all these tests end slip of bar became appreciable at about 65% (range 64% to 67%) of the ultimate load. At an end slip of 0.0005 in. the average bond resistance of the beams is 82% of the ultimate. It will be seen in Tables 27 and 28 that both the cube tests and pull-out tests for the beams in group 2 indicate a better quality of concrete than that in the beams of groups

1 and 3. The results indicate that bond stress is the primary cause of failure in all these tests. Although the diagonal tensile stresses were fairly high, it would appear that if the reinforcement had not slipped, failure by diagonal tension would not have occurred until a higher load had been applied. Of course stirrups of this kind may not be expected to be effective when the bond stresses are high. Attention is called to the discussion in Art. 78.

Load deflection and load-slip curves for beams with and without stirrups are given in Fig. 78 and 79. The appearance of some of the beams after failure is shown in Fig. 57.

76. *Effect of Depth of Concrete below the Reinforcement.*—In general, the depth of concrete below the center of the longitudinal reinforcement was 2 in. In two groups of tests this depth was made 1 in. and 4 in., respectively; that is, the total depth of the beams was 11 and 14 in., with the depth from top of beam to center of bar 10 in., as usual. These beams were not provided with vertical stirrups; they were each reinforced with one 1-in. plain round bar and were tested on a 6-ft. span. See groups 5 (1-in. depth), 1 (2-in. depth) and 6 (4-in. depth), Table 28. It is seen that there is little difference in the values for the beams with depths of 1 and 2 in., but the beams with 4 in. of concrete below the steel show a very great increase in strength. The maximum bond resistance for the 1-in. and 2-in. depth averaged 356 lb. per sq. in., while for the 4-in. depth the maximum was 557 lb. per sq. in. The load-deflection and load-slip curves for these beams are given in Fig. 78. The deflection of beam is greatest for the beams with 1-in. thickness and least for those with 4-in. thickness. Beams in group 6 show great stiffness up to a load of about 6000 lb. The photograph, Fig. 58a, shows the appearance of these beams after test. The numbers opposite the cracks indicate the growth of the cracks at loads expressed in thousands of pounds. An interesting feature of the tests in group 6 is the absence of cracks in the outer thirds of the beam length. It seems evident that the anti-stretch slip has occurred from the cracks near the load points and that slip has progressed outward from these cracks toward the ends of the beam during the remainder of the test instead of from the several cracks which usually appear in the outer thirds subsequently in the tests. The first end slip came at a materially higher load in this group. It seems probable that the distribution of

bond stress along the bar beyond the outer cracks in this case is not materially different from that found in a pull-out test. The full bond resistance for the embedded length of bar beyond the crack is developed, or at least that for a small amount of slip, instead of having part of the bond resistance consumed by stresses in opposite directions due to anti-stretch slip. The presence of the large mass of concrete below the bar acts to maintain the integrity of the concrete; its total tensile strength is greater than the available residual resistance to slip along the bar, and hence the concrete will slip along the bar, probably still maintaining some tensile stress. In the case of a small depth of concrete the total tensile strength of the concrete is insufficient to overcome the bond resistance, the concrete breaks in tension at another point and springs back both ways from the point of rupture, forming a crack. With further addition of load further slip will occur at the crack, and with still more load other cracks may appear. In this case a part of the bond resistance is consumed by stresses in opposite directions due to anti-stretch slip and due to the slip the bond resistance utilized is less.

For the beams of ordinary depth in groups 1 to 5 in Table 28 the outer cracks at the ends where failure occurred cross the plane of the steel at distances ranging from about 12 to 22 in. (average 17 in.) from the end of the beam. For the beams in group 6 with 4 in. of concrete below the center of the reinforcement, the principal failure cracks came at 21 to 30 in. (average 26 in.) from the end of the beam. It would appear that after the cracks have lengthened, the main effective bond resistance lies on the part of the bar between the outermost crack and the end of the beam.

77. *Effect of Span Length.*—The beams in group 11, Table 28, were loaded with an 8-ft. span. The beams were reinforced with 1¼-in. plain round bars and ½-in. vertical stirrups. These tests may be compared with the beams in group 9, loaded on a 6-ft. span. In the 6-ft. beams the first outer cracks appeared at an average load of 10 700 lb., and the first end slip of bar at 12 700 lb.; the corresponding loads for the 8-ft. beams were 8000 lb. and 16 000 lb., respectively. These loads are significant. The first outer crack appeared in both groups of beams at loads corresponding to a stress of about 13 500 lb. per sq. in. in the longitudinal steel at mid-span. The steel stresses at mid-span at beginning of slip at the end of the bar are about 15 600 and 26 000 lb. per sq. in. for the 6-ft. and 8-ft. beams, respectively, corresponding to

a computed bond stress of 206 and 257 lb. per sq. in. The average distances of the cracks from the ends of the beams were about 17 in. and 25 in., respectively. This suggests that when end slip of bar began the main effective bond resistance for beams of both lengths lies principally in the portion of the bar having an ubroken embedment. The fact that the maximum bond resistance for the 8-ft. beams is lower than for the 6-ft. beams is probably due to the opening of other outer cracks than the first one, nearer the supports, causing a further concentration of the bond resistance toward the ends of the bar.

Tests of other beams in which the span length varied are discussed in Art. 86.

78. *Effect of Length of Overhang of Ends of Beam.*—It has been the practice at the University of Illinois in designing test beams of the kind described herein to make the beam 6 in. longer than the test span. In making the usual calculations of bond stress no allowance has been made for this additional 3 in. of embedment of the reinforcing bar. In order to study the effect on bond resistance, some of the beams in the 1911 series were tested with the ends overhanging the supports for 9 in. or 15 in., as shown in Fig. 2 (c) and with the bars extending to the ends of the beams. 1-in. and 1¼-in. plain rounds and 1⅛-in. corrugated rounds were used in these tests; see groups 7, 8, 10 and 13 in Table 28. Figs. 58 and 59 show representative beams after failure. A summary of the tests is given in Table 30. These beams differed somewhat in web reinforcement, even in the same groups, and these differences must be kept in mind in making comparisons.

The effect of varying the length of overhang may be seen by comparing the values for the different functions given in Table 28 and also the bond stresses corresponding to different amounts of end slip in the beam and pull-out tests in Table 30. The applied loads at first outer crack show whether the appearance of anti-stretch slip was affected by overhang. The applied loads and the bond stresses developed at first end slip and at the maximum load indicate the influence of the overhang on the bond resistance of the beams.

One-inch plain rounds were used in beams with the ends overhanging the supports 3, 9 and 15 in. The tests with 3-in. overhang include 16 beams; the others two or three each. 1¼-in. plain rounds were used in beams with 3 in. and 15 in. overhang, and 1⅛-in. corru-

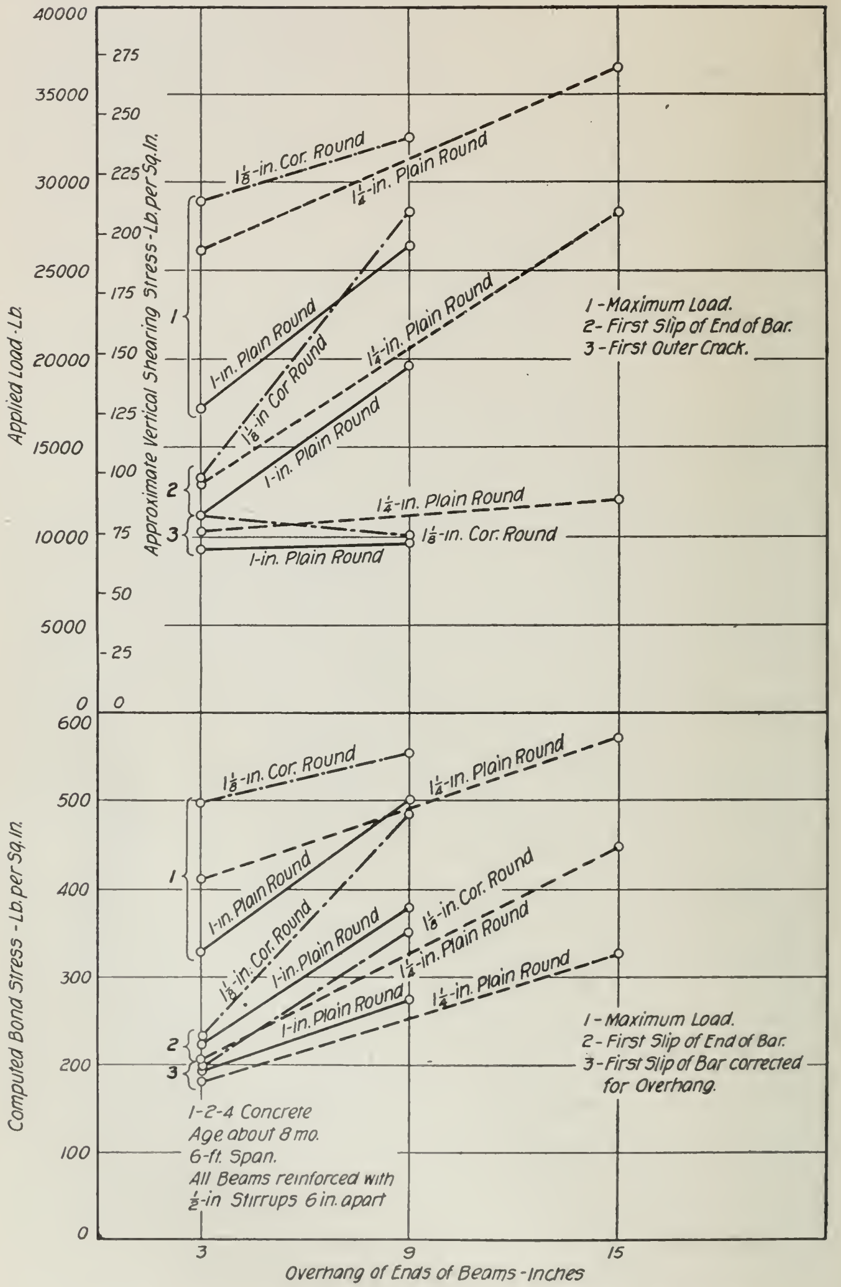


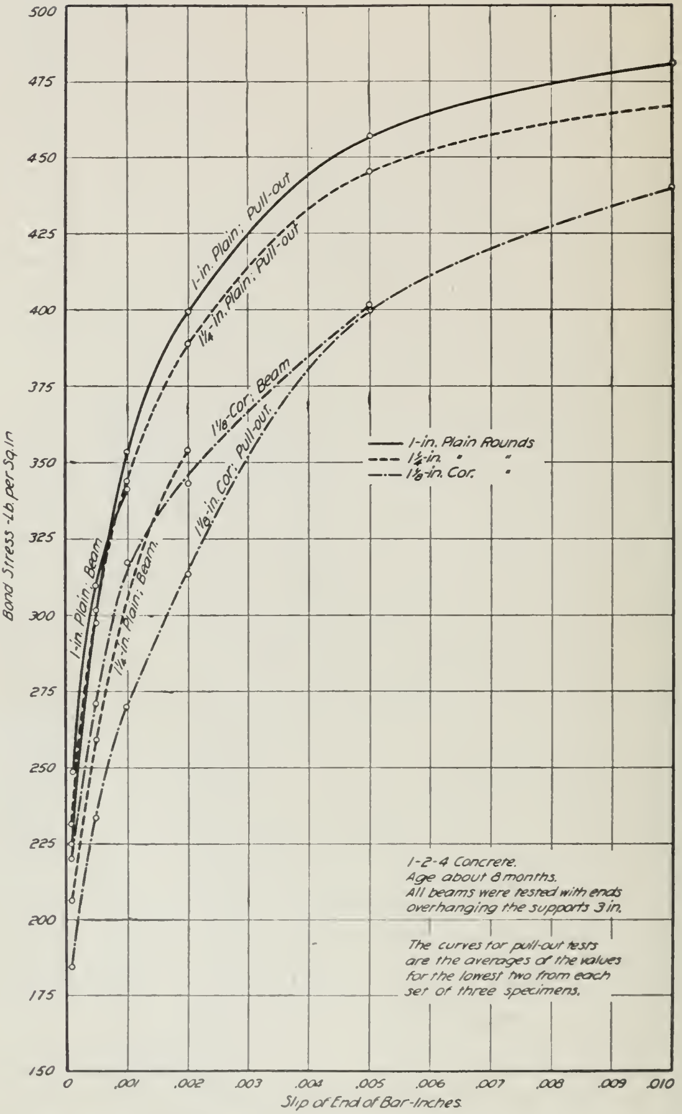
Fig. 48. Effect of Length of Overhang of Ends of Beams.

gated rounds in beams with 3 and 9 in. overhang. The values for bond stress given for these beams in the tables have all been computed in the same way without reference to the length of overhang of the ends.

A study of the data shows that the amount of the overhang of the ends had practically no influence on the load causing the opening of the first outer cracks and that the load at which first outer cracks appeared was independent of the kind and size of bar. The load at which first end slip of bar was found was greatly influenced by the length of overhang. In beams without stirrups the additional overhang seems to have little, if any, effect upon resistance to diagonal tension. For the beams recorded as failing in diagonal tension, the vertical shearing stress averaged 178 lb. per sq. in. for beams without stirrups and 288 lb. per sq. in. for beams with stirrups.

The relation between the bond stresses is somewhat different from that of the shearing stresses on account of the different diameters of the bars used. In Fig. 48 the values of the computed bond stresses have been plotted for the beams which were reinforced with vertical stirrups. It will be seen that the beams with 9-in. and 15-in. overhang developed a considerably higher computed bond stress than the beams with 3-in. overhang. If we reduce the computed bond stresses in these beams by considering that the bond stress at any load is uniformly distributed over the overhanging portion of the bar in the same way as is assumed for the outer thirds of the span (equivalent to multiplying the computed bond stress in beams with 3-in. overhang by 24/27, that in beams with 9-in. overhang by 24/33, and that in beams with 15-in. overhang by 24/39), it will be found that such corrected bond stresses will generally be greater for the longer overhangs. It should be noted that this method of correcting the bond stresses is subject to error in one respect, since slip was measured at the ends of the bars in all cases. The beginning of end slip with 15-in. overhang may be expected to represent quite a different state of stress in the beam proper than exists in a beam with 3-in. overhang when end slip begins.

79. *Bond on Corrugated Bars.*—1⅛-in. corrugated round bars were used for longitudinal reinforcement in 6 beams of the 1911 series—groups 12 and 13. These tests were referred to in the preceding articles; they will be included also in the discussion of similar tests from the 1912 beam series, Art. 85.



1-2-4 Concrete.
Age about 8 months.
All beams were tested with ends overhanging the supports 3 in.

The curves for pull-out tests are the averages of the values for the lowest two from each set of three specimens.

FIG. 49. COMPARISON OF BOND STRESSES IN BEAMS AND PULL-OUT SPECIMENS—1911 SERIES.

80. *Bond Resistance in 1911 Beam and Pull-out Tests.*—Table 30 gives a summary of the bond stresses in the 1911 beam and pull-out tests. The load-slip curves for many of these groups have been plotted in Fig. 49. As noted in Art. 73 the average stresses from the lowest two from each set of three pull-out specimens is used in all comparisons between the beam and pull-out tests. For the 1-in. plain rounds in beams with 3 in. overhang the bond stress up to an end slip of 0.001 in. is about the same as for the pull-out specimens for the same end slip. The bond stress at an end slip of 0.001 in. is 93% of the maximum bond resistance for the beam tests and 72% for the pull-out tests. The 1 and 1¼-in. plain rounds give nearly identical values in the pull-out tests. It should be borne in mind that with the uneven distribution of bond stress developed in beam tests the bond stress developed with a given end slip is dependent upon the relative dimensions of bar and beam and hence that the relation between the values of bond in beam tests and in pull-out tests here given may be to a certain extent accidental.

In the tests with 1¼-in. plain rounds the 6-ft. beams with 3-in. overhang gave values which are somewhat lower than the values from the pull-out specimens at the earlier stages of the tests; at the maximum load the average computed beam bond stress is 87% of the maximum bond resistance found in the pull-out tests.

For the 1⅛-in. corrugated round bars the bond stresses in the beam tests at end slips up to 0.005 in. are higher than in the pull-out tests for the same end slip. The maximum bond stresses in the two forms of specimen can not be compared directly, since part of the pull-out specimens were reinforced against bursting; in the one set not so reinforced the maximum bond resistance was 471 lb. per sq. in. as compared with 496 lb. per sq. in. in the beam tests. The corrugated bars in the beam tests give stresses which are about intermediate between those for the 1-in. and the 1¼-in. plain rounds for end slip less than about 0.002 in. The maximum bond resistance for the corrugated bars is about 25% higher than for the plain rounds.

Considering the pull-out tests only, the plain rounds give higher bond resistances than the corrugated rounds for all stages of the test up to the end slip which corresponds to the maximum bond resistance in the plain bars. A discussion of the relation of bond resistance in beams and pull-out specimens from another series of tests is given in Art. 92.

### *c. 1912 Beam Tests.*

81. *Outline of Series.*—Sixty-three reinforced concrete beams were included in the 1912 series. Generally the span length was 6 ft.; one group of beams was tested on each of the following spans: 5, 7 and 8 ft.; four groups were tested on a 10-ft. span. The loads were generally applied at the one-third points; groups of 6-ft. beams were tested with two symmetrical loads spaced the following distances apart: 2, 2½, 3, 3½ and 4 ft. Plain round, plain square, twisted square, and corrugated round bars were used for longitudinal reinforcement. In all but six of the beams the longitudinal reinforcement consisted of a single bar. Three beams were reinforced with four ⅝-in. and three with three ¾-in. plain rounds.

The beams in this series were made in groups of three; the first beam in each group was made of hand-mixed concrete, the remainder of machine-mixed concrete. All the beams were provided with vertical stirrups of ½-in. rounds. In the beams reinforced with a single longitudinal bar, the stirrups were V-shaped; in the other beams U-shaped stirrups were used. See Fig. 2. The average age at test was 63 days.

Pull-out specimens and 6-in. cubes were made with nearly all of the beams. The pull-out specimens were stored in the open air with the beams; the cubes were stored in damp sand.

Details of the make-up of the beams, their dimensions, the materials used and data on the strength of the concrete will be found in Table 31. Table 32 gives data of the tests and some of the calculated stresses in the beams, as well as notes on the failures. The bond stresses corresponding to given amounts of end slip for both beam and pull-out tests will be found in Table 33. A summary of the bond stresses developed in this series of tests is given in Table 34.

In comparing the results of pull-out tests for the square, twisted and the deformed bars in this series with those made earlier it should be borne in mind that the concrete cylinders in this series were not reinforced against bursting.

82. *Bond Resistance with Plain Round Bars.*—Thirty-seven beams in the 1912 series were reinforced with 1-in. plain round bars. The tests to determine the effect of span length, effect of variation in the position of the loads, and on the sizes of bar are discussed elsewhere.

Table 35 gives a summary of the results of tests of 6-ft. beams reinforced with 1 and 1¼-in. plain round bars from the series of 1909, 1911 and 1912. The 1909 tests gave lower values than the other series, but the concrete was of lower strength, as may be seen from the cube tests. All but one of the beams included in this table failed in bond. Disregarding the differences in the age of the beams, the mean values for bond resistances are found as given in the table. First slip of bar was measured at the end of the beam at a mean bond stress of 236 lb. per sq. in.—66% of the maximum bond resistance. The mean bond stress at an end slip of 0.001 in. for the beams in these three series was 324 lb. per sq. in.—91% of the maximum bond resistance. The mean maximum bond resistance for the 34 beams considered was 356 lb. per sq. in. These values seem to be representative for the conditions present in the tests. However, it should be noted that the bond resistance developed in beams, as given by the ordinary methods of computation, appears to be dependent upon the relation of size of bar to the dimensions of the beam and to the span length and that the computed values found in these tests may not apply to bars of smaller diameters or to beams of other size and span.

83. *Bond Resistance with Plain Square Bars.*—Six beams were reinforced with 1-in. plain square bars. In group 29 the bars were placed with their sides horizontal; in group 30 the bars were placed with a diagonal line horizontal. Comparing the bond stresses in the beam and pull-out tests for groups 29 and 30 in Table 32 and in the summary, Table 34, it will be seen that while the bars placed with a diagonal horizontal give a higher bond resistance, the pull-out and cube tests show that on the average the concrete in the second group of tests was inferior in strength to that in the first group. Beam No. 1046.5 gave abnormally low bond resistance. It is probable that no difference in bond resistance can be ascribed to the difference in the position of the bars.

The bond stresses at various end slips in the beams reinforced with 1-in. square bars are about 75% of those for similar beams reinforced with 1-in. rounds. See groups 14-15 and 29 and 30, Tables 32 and 34. A comparison of the pull-out tests gives about the same relation. End slip of bar begins in the pull-out tests at a lower stress than in the beam tests; at a slip of 0.001 in. the stresses are about the same. At the maximum loads the calculated bond stresses in the beams average 86% of those in the pull-out tests. This is about the same ratio as for the tests with 1-in. round bars in this series.

## TABLE 31.

## Data of Reinforced Concrete Beams—1912 Series.

1-2-4 concrete; Universal portland cement, sand, and crushed limestone. The concrete for the first beam of each group was mixed by hand; all others were machine-mixed.

All beams were 8 in. wide; total depth 12 in.; depth to center of steel, 10 in.

Each beam, except those in Groups 27 and 28, was reinforced with a single longitudinal bar. All beams were provided with vertical stirrups of ½-in. plain rounds, spaced 6-in. apart, outside the load points. See Fig. 2 (f).

| Group | Beam No. | Length feet | Longitudinal Reinforcement | | Beam from same Batch | Mixture by Weight | Compression Tests of 6-in. Cubes* | |
|---|---|---|---|---|---|---|---|---|
| | | | Size and Kind of Bar | Per cent | | | Age days | Average of 3 Tests lb. per sq. in. |
| 14 | 1052.1 | 6½ | 1-in. plain round.......... | 0.98 | 1051.1 | 1-1.95-3.34 | 70 | 2120 |
| | 1052.2 | 6½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.95-3.34 | 61 | 2730 |
| | 1052.3 | 6½ | 1-in. plain round.......... | 0.98 | 1057.3 | 1-1.93-3.23 | .. | .... |
| 15 | 1056.1 | 6½ | 1-in. plain round.......... | 0.98 | 1057.1<br>1058.1 | 1-1.94-3.29 | 62 | 2140 |
| | 1056.2 | 6½ | 1-in. plain round.......... | 0.98 | ...... | 1-2.00-3.20 | 68 | 3500 |
| | 1056.3 | 6½ | 1-in. plain round.......... | 0.98 | 1047.2 | 1-2.00-3.36 | .. | .... |
| 16 | 1052.4 | 6½ | 1-in. plain round<br>4 ⅜-in. auxiliary bars<br>at each end of beam.... | 0.98 | 1055.1 | 1-2.07-3.48 | 61 | 2065 |
| | 1052.5 | 6½ | | 0.98 | ...... | 1-1.92-3.20 | 66 | 2640 |
| | 1052.6 | 6½ | | 0.98 | 1058.3 | 1-1.93-3.32 | 60 | 2960 |
| 17 | 1057.1 | 6½ | 1-in. plain round.......... | 0.98 | 1056.1<br>1058.1 | 1-1.94-3.29 | 62 | 2140 |
| | 1057.2 | 6½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.98-3.34 | .. | .... |
| | 1057.3 | 6½ | 1-in. plain round.......... | 0.98 | 1052.3 | 1-1.93-3.23 | .. | .... |
| 18 | 1058.1 | 6½ | 1-in. plain round.......... | 0.98 | 1056.1<br>1057.1 | 1-1.94-3.29 | 62 | 2140 |
| | 1058.2 | 6½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.96-3.27 | 66 | 2730 |
| | 1058.3 | 6½ | 1-in. plain round.......... | 0.98 | 1052.6 | 1-1.93-3.32 | 60 | 2960 |
| 19 | 1059.1 | 6½ | 1-in. plain round.......... | 0.98 | 1061.1<br>1048.1 | 1-2.07-3.46 | 60 | 2680 |
| | 1059.2 | 6½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.99-3.32 | 66 | 2600 |
| | 1059.3 | 6½ | 1-in. plain round.......... | 0.98 | 1062.3 | 1-1.92-3.31 | 67 | 2600 |
| 20 | 1060.1 | 6½ | 1-in. plain round.......... | 0.98 | 1059.1<br>1048.1 | 1-2.07-3.46 | 60 | 2680 |
| | 1060.2 | 6½ | 1-in. plain round.......... | 0.98 | 1055.5 | 1-2.01-3.23 | 64 | 3140 |
| | 1060.3 | 6½ | 1-in. plain round.......... | 0.98 | 1063.3<br>1064.3 | 1-1.98-3.30 | 63 | 3250 |
| 21 | 1051.1 | 5½ | 1-in. plain round.......... | 0.98 | 1052.1 | 1-1.95-3.34 | 70 | 2120 |
| | 1051.2 | 5½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.94-3.26 | 66 | 2800 |
| | 1051.3 | 5½ | 1-in. plain round.......... | 0.98 | 1053.3 | 1-1.98-3.28 | 68 | 2760 |
| 22 | 1053.1 | 7½ | 1-in. plain round.......... | 0.98 | 1050.4 | 1-1.95-3.09 | 60 | 2170 |
| | 1053.2 | 7½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.98-3.15 | 62 | 2200 |
| | 1053.3 | 7½ | 1-in. plain round.......... | 0.98 | 1051.3 | 1-1.98-3.29 | .. | .... |
| 23 | 1054.1 | 8½ | 1-in. plain round.......... | 0.98 | 1046.1 | 1-1.93-3.26 | 62 | 2580 |
| | 1054.2 | 8½ | 1-in. plain round.......... | 0.98 | ...... | 1-1.96-3.28 | 65 | 2800 |
| | 1054.3 | 8½ | 1-in. plain round.......... | 0.98 | 1061.6 | 1-1.98-3.30 | 63 | 2870 |

* The average compressive strength of 36 6-in. cubes of hand-mixed concrete was 2200 lb. per sq. in.; of 93 cubes of machine-mixed concrete, 2800 lb. per sq. in.

*(Table 31 continued on page 154)*

## TABLE 32.

### TESTS OF REINFORCED CONCRETE BEAMS—1912 SERIES.

All beams had 3 in. overhang at each end.
In computing unit stresses the weight of the beam was considered.
Loads are given in pounds; stresses are given in pounds per square inch.

| Group | Beam No. | Test Span feet | Distance between Loads feet | Age at Test days | Load at First Outer Crack | At First Slip of End of Bar | | At Maximum Load | | | | Maximum Bond Resistance in Pull-out Tests | Failure of Beam |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Applied Load | Computed Bond Stress | Applied Load | Tensile Stress in Steel | Vertical Shearing Stress | Computed Bond Stress | | |
| 14 | 1052.1 | 6 | 2 | 67 | 8 000 | 8 000 | 154 | 19 940 | 36 800 | 149 | 383 | 356 | Bond at S. end |
| | 1052.2 | 6 | 2 | 59 | 11 000 | 11 000 | 211 | 18 000 | 33 200 | 136 | 347 | 405 | Bond at N. end |
| | 1052.3 | 6 | 2 | 64 | 6 000 | 16 000 | 307 | 20 000 | 36 800 | 150 | 385 | ...... | Tension in steel |
| | Av. | ...... | ...... | 63 | 8 300 | 11 700 | 224 | 19 300 | 35 600 | 145 | 372 | 395 | |
| 15 | 1056.1 | 6 | 2 | 65 | 14 000 | 13 000 | 250 | 15 750 | 29 200 | 119 | 305 | 454 | Bond at N. end |
| | 1056.2 | 6 | 2 | 65 | 6 000 | 12 500 | 240 | 19 700 | 36 300 | 149 | 379 | 392 | Bond and tension in steel |
| | 1056.3 | 6 | 2 | 65 | 10 000 | 13 500 | 260 | 21 000 | 38 600 | 158 | 403 | 443 | Bond at N. end |
| | Av. | ...... | ...... | 65 | 10 000 | 13 000 | 250 | 18 800 | 34 700 | 142 | 362 | 430 | |
| 16 | 1052.4 | 6 | 2 | 61 | 14 000 | 18 000 | 347 | 20 600 | 37 900 | 155 | 395 | 443 | Bond at N. end |
| | 1052.5 | 6 | 2 | 63 | 10 000 | 11 500 | 220 | 18 500 | 34 100 | 139 | 356 | 415 | Bond at N. end |
| | 1052.6 | 6 | 2 | 65 | 10 000 | 14 000 | 269 | 18 800 | 34 700 | 142 | 383 | 381 | Bond at S. end |
| | Av. | ...... | ...... | 63 | 11 300 | 14 500 | 279 | 19 300 | 35 600 | 145 | 372 | 413 | |
| 17 | 1057.1 | 6 | 2½ | 68 | 8 000 | 12 000 | 231 | 19 500 | 31 400 | 147 | 375 | 323 | Bond at N. end |
| | 1057.2 | 6 | 2½ | 64 | 8 000 | 13 000 | 250 | 19 100 | 30 800 | 144 | 368 | ...... | Bond at S. end |
| | 1057.3 | 6 | 2½ | 63 | 8 000 | 13 000 | 250 | 25 000 | 40 000 | 187 | 478 | 322 | Tension in steel |
| | Av. | ...... | ...... | 65 | 8 000 | 12 700 | 244 | 21 200 | 34 100 | 166 | 407 | 322 | |
| 18 | 1058.1 | 6 | 3 | 68 | 8 000 | 14 000 | 272 | 18 900 | 26 100 | 142 | 364 | 454 | Bond at N. end |
| | 1058.2 | 6 | 3 | 63 | 6 000 | 10 000 | 198 | 14 000 | 19 500 | 106 | 272 | 252 | Bond at both ends |
| | 1058.3 | 6 | 3 | 67 | ..... | 13 000 | 254 | 18 600 | 25 700 | 140 | 359 | 381 | Bond at N. end |
| | Av. | ...... | ...... | 66 | 7 000 | 12 300 | 241 | 17 200 | 23 800 | 129 | 332 | 363 | |
| 19 | 1059.1 | 6 | 3½ | 64 | 26 000 | 12 000 | 235 | 33 750 | 38 400 | 251 | 642 | 591 | Bond at N. end |
| | 1059.2 | 6 | 3½ | 59 | ...... | 10 000 | 198 | 17 700 | 20 400 | 134 | 341 | ...... | Bond at S. end |
| | 1059.3 | 6 | 3½ | 62 | 14 000 | 12 000 | 235 | 22 000 | 25 200 | 165 | 421 | 416 | Bond at N. end |
| | Av. | ...... | ...... | 62 | 20 000 | 11 300 | 223 | 24 400 | 28 000 | 183 | 468 | 504 | |
| 20 | 1060.1 | 6 | 4 | 70 | 21 000 | 16 000 | 309 | 32 000 | 29 100 | 238 | 608 | 591 | Bond at S. end |
| | 1060.2 | 6 | 4 | 62 | 21 000 | 18 000 | 347 | 24 700 | 22 600 | 185 | 472 | 599 | Bond at S. end |
| | 1060.3 | 6 | 4 | 61 | ...... | 17 000 | 329 | 30 000 | 27 400 | 224 | 573 | 579 | Bond at N. end |
| | Av. | ...... | ...... | 64 | 21 000 | 17 000 | 328 | 28 900 | 26 400 | 215 | 551 | 590 | |
| 21 | 1051.1 | 5 | 1⅔ | 68 | 8 000 | 16 000 | 290 | 16 800 | 25 800 | 128 | 324 | 386 | Bond at S. end |
| | 1051.2 | 5 | 1⅔ | 60 | 6 000 | 13 000 | 252 | 20 000 | 30 600 | 151 | 383 | 342 | Bond at S. end |
| | 1051.3 | 5 | 1⅔ | 63 | 8 000 | 15 000 | 290 | 20 100 | 30 800 | 152 | 385 | 480 | Bond at N. end |
| | Av. | ...... | ...... | 64 | 7 300 | 13 700 | 277 | 19 000 | 29 100 | 144 | 364 | 403 | |
| 22 | 1053.1 | 7 | 2⅓ | 62 | 8 000 | (No re | adings) | 17 100 | 37 000 | 130 | 331 | 366 | Tension in steel |
| | 1053.2 | 7 | 2⅓ | 63 | 8 000 | 12 000 | 236 | 16 600 | 36 000 | 126 | 322 | 400 | Bond and tension in steel |
| | 1053.3 | 7 | 2⅓ | 63 | 8 000 | 16 000 | 310 | 18 400 | 39 800 | 139 | 356 | ...... | Tension in steel |
| | Av. | ...... | ...... | 63 | 8 000 | ...... | ...... | 17 400 | 37 600 | 132 | 336 | 383 | |
| 23 | 1054.1 | 8 | 2⅔ | 60 | 6 000 | 10 000 | 210 | 16 000 | 40 000 | 121 | 312 | 459 | Tension in steel |
| | 1054.2 | 8 | 2⅔ | 63 | 5 000 | 12 000 | 227 | 15 100 | 37 800 | 114 | 296 | 525 | Tension in steel |
| | 1054.3 | 8 | 2⅔ | 60 | 8 000 | 13 000 | 256 | 15 400 | 38 500 | 117 | 301 | 596 | Tension in steel |
| | Av. | ...... | ...... | 61 | 6 300 | 12 000 | 231 | 15 500 | 38 600 | 117 | 303 | 527 | |

(*Table 32 continued on page 155.*)

## TABLE 31—CONTINUED FROM PAGE 152.

## DATA OF REINFORCED CONCRETE BEAMS—1912 SERIES.

| Group | Beam No. | Length feet | Longitudinal Reinforcement | | Beam from same Batch | Mixture by Weight | Compression Tests of 6-in. Cubes* | |
|---|---|---|---|---|---|---|---|---|
| | | | Size and Kind of Bar | Per cent | | | Age days | Average of 3 Tests lb. per sq. in. |
| 24 | 1055.1 | 10½ | 1-in. plain round........... | 0.98 | 1052.4 | 1-2.07-3.48 | 61 | 2065 |
| | 1055.2 | 10½ | 1-in. plain round........... | 0.98 | ...... | 1-1.99-3.28 | .. | .... |
| | 1055.3 | 10½ | 1-in. plain round........... | 0.98 | ...... | 1-1.98-3.37 | .. | .... |
| 25 | 1055.4 | 10½ | 1-in. plain round........... | 0.98 | 1048.4 | 1-2.03-3.28 | 61 | 1940 |
| | 1055.5 | 10½ | 1-in. plain round........... | 0.98 | 1060.2 | 1-2.05-3.13 | .. | .... |
| | 1055.6 | 10½ | 1-in. plain round........... | 0.98 | ...... | 1-1.97-3.27 | .. | .... |
| 26 | 1055.7 | 10½ | 1-in. plain round........... | 0.98 | 1050.1 | 1-2.01-3.32 | 60 | 2220 |
| | 1055.8 | 10½ | 1-in. plain round........... | 0.98 | ...... | 1-2.05-3.44 | 65 | 2980 |
| | 1055.9 | 10½ | 1-in. plain round........... | 0.98 | ...... | 1-1.99-3.31 | 67 | 2870 |
| 27 | 1050.4 | 6½ | 4 ⅝-in. plain rounds...... | 1.53 | 1053.1 | 1-1.95-3.09 | 60 | 2170 |
| | 1050.5 | 6½ | 4 ⅝-in. plain rounds...... | 1.53 | ...... | 1-2.02-3.14 | 62 | 2700 |
| | 1050.6 | 6½ | 4 ⅝-in. plain rounds...... | 1.53 | 1061.3 | 1-1.95-3.32 | 60 | 2390 |
| 28 | 1050.1 | 6½ | 3 ¾-in. plain rounds...... | 1.66 | 1055.7 | 1-2.01-3.32 | 60 | 2220 |
| | 1050.2 | 6½ | 3 ¾-in. plain rounds...... | 1.66 | ...... | 1-2.02-3.29 | 66 | 3290 |
| | 1050.3 | 6½ | 3 ¾-in. plain rounds...... | 1.66 | 1047.3 | 1-1.95-3.36 | .. | .... |
| 29 | 1046.1 | 6½ | 1-in. square bar............<br>(Side horizontal) | 1.25 | 1054.1 | 1-1.93-3.26 | 62 | 2580 |
| | 1046.2 | 6½ | 1-in. square bar............<br>(Side horizontal) | 1.25 | ...... | 1-1.91-3.20 | 68 | 3790 |
| | 1046.3 | 6½ | 1-in. square bar............<br>(Side horizontal) | 1.25 | 1048.3 | 1-1.93-3.25 | 59 | 2800 |
| 30 | 1046.4 | 6½ | 1-in. square bar............<br>(Placed on edge) | 1.25 | 1047.1 | 1-1.96-3.29 | 60 | 2190 |
| | 1046.5 | 6½ | 1-in. square bar............<br>(Placed on edge) | 1.25 | ...... | 1-1.96-3.35 | 68 | 1890 |
| | 1046.6 | 6½ | 1-in. square bar............<br>(Placed on edge) | 1.25 | 1048.6 | 1-1.98-3.39 | 64 | 2080 |
| 31 | 1047.1 | 6½ | 1-in. twisted square bar.....<br>(1 twist per lineal ft.) | 1.25 | 1046.4 | 1-1.96-3.29 | 60 | 2190 |
| | 1047.2 | 6½ | 1-in. twisted square bar.....<br>(1 twist per lineal ft.) | 1.25 | 1056.3 | 1-1.98-3.34 | 71 | 2980 |
| | 1047.3 | 6½ | 1-in. twisted square bar.....<br>(1 twist per lineal ft.) | 1.25 | 1050.3 | 1-1.95-3.36 | 59 | 2670 |
| 32 | 1048.1 | 6½ | 1⅛-in. corrugated round ... | 1.24 | 1059.1<br>1060.1 | 1-2.07-3.46 | 60 | 2680 |
| | 1048.2 | 6½ | 1⅛-in. corrugated round ... | 1.24 | ...... | 1-1.96-3.24 | 61 | 3150 |
| | 1048.3 | 6½ | 1⅛-in. corrugated round ... | 1.24 | 1046.3 | 1-1.91-3.30 | .. | .... |
| 33 | 1048.4 | 6½ | 1⅛-in. corrugated round ... | 1.24 | 1055.4 | 1-2.03-3.28 | 61 | 1940 |
| | 1048.5 | 6½ | 1⅛-in. corrugated round .. | 1.24 | ..... | 1-2.05-3.19 | 65 | 3260 |
| | 1048.6 | 6½ | 1⅛-in. corrugated round .. | 1.24 | 1046.6 | 1-1.98-3.34 | .. | .... |
| 34 | 1049.1 | 10½ | 1⅛-in. corrugated round .. | 1.24 | ...... | 1-1.97-3.28 | 63 | 2420 |
| | 1049.2 | 10½ | 1⅛-in. corrugated round .. | 1.24 | ...... | 1-2.01-3.19 | 62 | 2440 |
| | 1049.3 | 10½ | 1⅛-in. corrugated round ... | 1.24 | .... | 1-1.95-3.32 | .. | .... |

* The average compressive strength of 36 6-in. cubes of hand-mixed concrete was 2200 lb. per sq. in.; of 93 cubes of machine-mixed concrete, 2800 lb. per sq. in.

## TABLE 32—CONTINUED FROM PAGE 153.
## TESTS OF REINFORCED CONCRETE BEAMS—1912 SERIES.

| Group | Beam No. | Test Span feet | Distance between Loads feet | Age at Test days | Load at First Outer Crack | At First Slip of End of Bar | | At Maximum Load | | | | Maximum Bond Resistance in Pull-out Tests | Failure of Beam |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Applied Load | Computed Bond Stress | Applied Load | Tensile Stress in Steel | Vertical Shearing Stress | Computed Bond Stress | | |
| 24 | 1055.1 | 10 | 3⅓ | 64 | 4 500 | (No en | d slip) | 12 300 | 39 300 | 94 | 242 | 443 | Tension in steel |
| | 1055.2 | 10 | 3⅓ | 60 | 4 000 | 12 000 | 240 | 12 300 | 39 300 | 94 | 246 | ... | Tension in steel |
| | 1055.3 | 10 | 3⅓ | 2 yr. | 4 000 | 10 400 | 205 | 11 100 | 34 300 | 86 | 220 | 560† | Tension in steel |
| | Av. | ...... | ...... | ...... | 4 000 | 11 200 | 222 | 11 900 | 37 600 | 91 | 236 | 501 | |
| 25 | 1055.4 | 10 | 3⅓ | 64 | 4 000 | 10 800 | 218 | 10 800 | 34 800 | 83 | 218 | 459 | Tension in steel |
| | 1055.5 | 10 | 3⅓ | 65 | 4 000 | 9 500 | 190 | 9 900 | 32 200 | 76 | 201 | 599 | Tension in steel |
| | 1055.6‡ | 10 | 3⅓ | 2 yr. | 4 000 | 10 400 | 215 | 12 100 | 38 700 | 92 | 236 | 497† | Tension in steel |
| | Av. | ...... | ...... | ...... | 4 000 | 10 800 | 208 | 10 900 | 35 200 | 84 | 218 | 518 | |
| 26 | 1055.7 | 10 | 3⅓ | 62 | 4 000 | 12 100 | 242 | 12 100 | 38 700 | 92 | 242 | 562 | Tension in steel |
| | 1055.8* | 10 | 6 | 93 | 4 000 | 10 000 | 203 | 12 800* | 24 300 | 97 | 255 | 415 | Bond at S. end |
| | 1055.9° | 10 | 6 | 85 | 4 000 | 12 000 | 240 | 16 000° | 30 200 | 121 | 315 | 349 | Bond at S. end |
| 27 | 1050.4 | 6 | 2 | 60 | 10 000 | 24 000 | 195 | 33 700 | 40 300 | 259 | 269 | 405 | Tension in steel |
| | 1050.5 | 6 | 2 | 62 | 12 000 | 19 000 | 156 | 34 000 | 40 700 | 261 | 271 | 591 | Tension and bond |
| | 1050.6 | 6 | 2 | 60 | 8 000 | 20 000 | 164 | 27 900 | 33 500 | 215 | 225 | 503 | Bond at N. end |
| | Av. | ...... | ...... | 61 | 10 000 | 21 000 | 172 | 31 900 | 38 200 | 245 | 255 | 500 | |
| 28 | 1050.1 | 6 | 2 | 61 | 12 000 | 24 000 | 217 | 34 500 | 38 600 | 265 | 308 | 524 | Bond at S. end |
| | 1050.2 | 6 | 2 | 62 | 12 000 | 20 000 | 183 | 26 000 | 29 200 | 201 | 235 | 519 | Bond at N. end |
| | 1050.3 | 6 | 2 | 57 | 10 000 | 19 000 | 174 | 31 200 | 34 900 | 241 | 278 | 642 | Bond at S. end |
| | Av. | ...... | ...... | 60 | 11 300 | 21 000 | 191 | 30 600 | 34 200 | 235 | 274 | 562 | |
| 29 | 1046.1 | 6 | 2 | 62 | 8 000 | 11 000 | 174 | 19 000 | 27 800 | 146 | 293 | 336 | Bond at S. end |
| | 1046.2 | 6 | 2 | 62 | 10 000 | 11 000 | 174 | 19 300 | 28 200 | 148 | 298 | 287 | Bond at S. end |
| | 1046.3 | 6 | 2 | 57 | 10 000 | 14 000 | 219 | 20 500 | 30 000 | 157 | 316 | 465 | Bond at S. end |
| | Av. | ...... | ...... | 60 | 9 300 | 12 000 | 189 | 19 600 | 28 700 | 150 | 302 | 363 | |
| 30 | 1046.4 | 6 | 2 | 61 | 6 000 | 12 000 | 189 | 20 000 | 29 200 | 154 | 308 | 340 | Bond at S. end |
| | 1046.5 | 6 | 2 | 62 | 8 000 | 11 000 | 177 | 12 000 | 17 800 | 94 | 188 | 275 | Bond at N. end |
| | 1046.6 | 6 | 2 | 64 | 10 000 | 13 000 | 204 | 16 700 | 24 500 | 129 | 263 | 325 | Bond at N. end |
| | Av. | ...... | ...... | 62 | 8 000 | 12 000 | 190 | 16 200 | 23 800 | 129 | 253 | 313 | |
| 31 | 1047.1 | 6 | 2 | 62 | 11 000 | 15 000 | 237 | 21 800 | 31 800 | 167 | 335 | 447 | Bond at N. end |
| | 1047.2 | 6 | 2 | 65 | 6 000 | 15 000 | 233 | 23 400 | 34 100 | 179 | 359 | 510 | Bond at N. end |
| | 1047.3 | 6 | 2 | 60 | 6 000 | 12 500 | 196 | 20 600 | 30 100 | 158 | 317 | 443 | Bond at N. end |
| | Av. | ...... | ...... | 62 | 7 700 | 14 200 | 222 | 21 900 | 32 000 | 168 | 337 | 467 | |
| 32 | 1048.1 | 6 | 2 | 70 | 13 000 | 15 000 | 258 | 31 550 | 48 000 | 239 | 541 | 781 | Bond at N. end |
| | 1048.2 | 6 | 2 | 59 | 8 000 | 15 000 | 258 | 27 600 | 40 300 | 209 | 475 | 652 | Bond at N. end |
| | 1048.3 | 6 | 2 | 60 | 8 000 | 18 000 | 310 | 28 300 | 41 300 | 215 | 486 | 696 | Bond at S. end |
| | Av. | ...... | ...... | 63 | 9 700 | 16 000 | 275 | 29 100 | 41 900 | 231 | 501 | 710 | |
| 33 | 1048.4 | 6 | 2 | 62 | 8 000 | 12 000 | 206 | 28 000 | 40 800 | 210 | 482 | 721 | Bond at S. end |
| | 1048.5 | 6 | 2 | 63 | 8 000 | 14 000 | 240 | 27 700 | 40 500 | 209 | 477 | 706 | Bond at N. end |
| | 1048.6 | 6 | 2 | 64 | 8 000 | 17 000 | 292 | 25 000 | 36 600 | 191 | 431 | 614 | Bond at N. end |
| | Av. | ...... | ...... | 63 | 8 000 | 14 300 | 246 | 26 900 | 39 300 | 203 | 463 | 680 | |
| 34 | 1049.1 | 10 | 3⅓ | 61 | 6 000 | 18 000 | 310 | 21 000 | 52 500 | 160 | 369 | 587 | Tension in steel |
| | 1049.2 | 10 | 3⅓ | 63 | 8 000 | 15 000 | 258 | 21 100 | 52 700 | 161 | 371 | 521 | Tension in steel |
| | 1049.3‡ | 10 | 3⅓ | 400 | ...... | 16 000 | 275 | 23 200 | 58 300 | 175 | 400 | 619 | Tension in steel |

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* A load of 10 000 lb. (which caused a slip of 0.0002 in. at S. end), was maintained for 32 days; the load was then gradually increased until the bar pulled out on the 60th day at 12 800 lb. See Art 90.

° A load of 16 000 lb. (which caused a slip of 0.0002 in. at S. end), was maintained till the bar pulled out. See Art 90.

† Tested at about 80 days.

‡ Each load was released before a higher load was applied. See Fig. 64 and 67.

# TABLE 33.

## Comparison of Bond Resistance in Beams and Pull-out Tests—1912 Series.

For data concerning the span and the position of the loads in the beam tests, see Tables 31 and 32.
The measurements of slip of bar in the beam tests refer to the end showing the greater slip.
The bond stresses given for the pull-out specimens are the average of three tests on bars embedded 8 in.
Stresses are given in pounds per square inch.

| Group | Beam No. | Size and Kind of Bar | BEAM TESTS | | | | | | | PULL-OUT TESTS | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Age at Test days | Bond Stress at End Slip of (inches) .0002 | .0005 | .001 | .002 | .005 | Computed Bond Stress at Maximum Load | Age at Test days | Bond Stress at End Slip of (inches) .0002 | .0005 | .001 | .002 | .005 | Maximum Bond Resistance |
| 14 | 1052.1 | 1-in. plain round..... | 67 | 154 | 216 | 291 | 349 | ... | 383 | 67 | 211 | 300 | 331 | 361 | 375 | 386 |
| | 1052.2 | 1-in. plain round.... | 59 | 211 | 300 | 328 | ... | ... | 347 | 66 | 161 | 316 | 363 | 382 | 400 | 405 |
| | 1052.3 | 1-in. plain round..... | 64 | 307 | 338 | 385 | ... | ... | 385 | .. | ... | ... | ... | ... | ... | ... |
| | Average | .................... | 63 | 224 | 285 | 335 | ... | ... | 372 | 66 | 186 | 308 | 349 | 371 | 387 | 395 |
| 15 | 1056.1 | 1-in. plain round..... | 65 | 250 | 291 | 300 | ... | ... | 305 | 66 | 159 | 332 | 405 | 427 | 447 | 454 |
| | 1056.2 | 1-in. plain round..... | 65 | 240 | 298 | 314 | 345 | 367 | 379 | 69 | 182 | 312 | 354 | 369 | 388 | 392 |
| | 1056.3 | 1-in. plain round..... | 65 | 260 | 338 | 403 | ... | ... | 403 | 64 | 128 | 266 | 367 | 388 | 422 | 443 |
| | Average | .................... | 65 | 250 | 309 | 339 | ... | ... | 362 | 66 | 156 | 303 | 375 | 395 | 419 | 430 |
| 16 | 1052.4 | 1-in. plain round..... | 61 | 347 | 371 | 384 | ... | ... | 395 | 64 | 211 | 344 | 397 | 417 | 441 | 443 |
| | 1052.5 | 1-in. plain round.... | 63 | 220 | 328 | 356 | ... | ... | 356 | 71 | 149 | 307 | 347 | 376 | 405 | 415 |
| | 1052.6 | 1-in. plain round..... | 65 | 269 | 329 | 340 | 352 | 362 | 363 | 71 | 141 | 308 | 348 | 360 | 373 | 381 |
| | Average | .................... | 63 | 279 | 343 | 360 | ... | ... | 371 | 69 | 167 | 320 | 363 | 384 | 406 | 413 |
| 17 | 1057.1 | 1-in. plain round..... | 68 | 231 | 309 | 353 | 367 | 372 | 375 | 64 | 88 | 239 | 271 | 287 | 307 | 323 |
| | 1057.2 | 1-in. plain round..... | 64 | 250 | 317 | 347 | 362 | ... | 368 | .. | ... | ... | ... | ... | ... | ... |
| | 1057.3 | 1-in. plain round..... | 63 | 250 | 376 | 478 | ... | ... | 478 | 64 | 109 | 240 | 275 | 288 | 308 | 322 |
| | Average | .................... | 65 | 244 | 334 | 393 | ... | ... | 407 | 64 | 99 | 239 | 273 | 287 | 307 | 322 |
| 18 | 1058.1 | 1-in. plain round..... | 68 | 272 | 314 | 331 | 347 | ... | 364 | 66 | 159 | 332 | 405 | 427 | 447 | 454 |
| | 1058.2 | 1-in. plain round..... | 63 | 198 | 208 | 222 | 242 | 261 | 272 | 68 | 119 | 224 | 241 | 249 | 253 | 254 |
| | 1058.3 | 1-in. plain round..... | 67 | 254 | 291 | 310 | 331 | 349 | 359 | 71 | 141 | 308 | 348 | 360 | 373 | 381 |
| | Average | .................... | 66 | 241 | 271 | 288 | 307 | ... | 332 | 68 | 140 | 288 | 331 | 345 | 358 | 363 |
| 19 | 1059.1 | 1-in. plain round..... | 64 | 235 | 281 | 384 | 477 | 627 | 642 | 64 | 170 | 421 | 525 | 564 | 586 | 591 |
| | 1059.2 | 1-in. plain round..... | 59 | 198 | 235 | 272 | 304 | ... | 341 | .. | ... | ... | ... | ... | ... | ... |
| | 1059.3 | 1-in. plain round..... | 62 | 235 | 309 | 384 | 421 | ... | 421 | 64 | 151 | 306 | 362 | 396 | 410 | 416 |
| | Average | .................... | 62 | 223 | 275 | 347 | 401 | ... | 468 | 64 | 161 | 364 | 444 | 480 | 498 | 504 |

## TABLE 33—CONTINUED.

| | | | BEAM TESTS | | | | | | | PULL-OUT TESTS | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Group | Beam No. | Size and Kind of Bar | Age at Test days | Bond Stress at End Slip of (inches) .0002 | .0005 | .001 | .002 | .005 | Computed Bond Stress at Maximum Load | Age at Test days | Bond Stress at End Slip of (inches) .0002 | .0005 | .001 | .002 | .005 | Maximum Bond Resistance |
| 20 | 1060.1 | 1-in. plain round..... | 70 | 309 | 496 | ... | ... | ... | 608 | 64 | 170 | 421 | 525 | 564 | 586 | 591 |
| | 1060.2 | 1-in. plain round..... | 62 | 347 | 384 | 415 | 441 | 460 | 472 | 64 | 186 | 408 | 507 | 566 | 594 | 599 |
| | 1060.3 | 1-in. plain round..... | 61 | 329 | 413 | 488 | 530 | ... | 573 | 64 | 165 | 385 | 467 | 511 | 556 | 579 |
| | Average | .................. | 64 | 328 | 431 | ... | ... | ... | 551 | 64 | 174 | 405 | 500 | 547 | 579 | 590 |
| 21 | 1051.1 | 1-in. plain round..... | 68 | 290 | 300 | 309 | 311 | 320 | 324 | 67 | 211 | 300 | 331 | 361 | 375 | 386 |
| | 1051.2 | 1-in. plain round..... | 60 | 252 | 308 | 346 | ... | ... | 383 | 60 | 176 | 256 | 275 | 294 | 316 | 342 |
| | 1051.3 | 1-in. plain round..... | 63 | 290 | 327 | 346 | 370 | 385 | 385 | 65 | 124 | 340 | 409 | 433 | 464 | 480 |
| | Average | .................. | 64 | 277 | 312 | 334 | ... | ... | 364 | 64 | 170 | 295 | 338 | 363 | 385 | 403 |
| 22 | 1053.1 | 1-in. plain round..... | 62 | No | measur | ements | | | 331 | 59 | 131 | 250 | 279 | 305 | 344 | 366 |
| | 1053.2 | 1-in. plain round..... | 63 | 236 | 273 | 292 | 300 | 316 | 322 | 63 | 202 | 324 | 359 | 369 | 389 | 400 |
| | 1053.3 | 1-in. plain round..... | 63 | 310 | 348 | ... | ... | ... | 356 | .. | ... | ... | ... | ... | ... | ... |
| | Average | .................. | 63 | 273 | 310 | ... | ... | ... | 339 | 62 | 166 | 287 | 319 | 337 | 366 | 383 |
| 23 | 1054.1 | 1-in. plain round..... | 60 | 210 | 275 | 297 | 312 | ... | 312 | 60 | 161 | 329 | 373 | 398 | 433 | 459 |
| | 1054.2 | 1-in. plain round..... | 63 | 227 | 296 | ... | ... | ... | 296 | 65 | 149 | 335 | 406 | 450 | 490 | 525 |
| | 1054.3 | 1-in. plain round..... | 60 | 256 | ... | ... | ... | ... | 301 | 64 | 154 | 415 | 509 | 561 | 590 | 596 |
| | Average | .................. | 61 | 231 | ... | ... | ... | ... | 303 | 63 | 155 | 360 | 429 | 470 | 504 | 527 |
| 24 | 1055.1 | 1-in. plain round..... | 64 | ... | ... | ... | ... | ... | 242 | 64 | 211 | 344 | 397 | 417 | 441 | 443 |
| | 1055.2 | 1-in. plain round..... | 60 | 240 | ... | ... | ... | ... | 246 | .. | ... | ... | ... | ... | ... | ... |
| | 1055.3 | 1-in. plain round..... | 2 yr. | 205 | | ... | ... | | 220 | 82 | 206 | 364 | 452 | 505 | 546 | 560 |
| | Average | .................. | | ... | ... | ... | ... | ... | 236 | 73 | 208 | 354 | 424 | 461 | 493 | 501 |
| 25 | 1055.4 | 1-in. plain round..... | 64 | 218 | ... | ... | ... | ... | 218 | 63 | 194 | 329 | 373 | 398 | 433 | 459 |
| | 1055.5 | 1-in. plain round..... | 65 | 190 | ... | ... | ... | ... | 201 | 64 | 186 | 408 | 507 | 566 | 594 | 599 |
| | 1055.6 | 1-in. plain round..... | 2 yr. | 208 | 205 | ... | ... | | 236 | 81 | 144 | 318 | 406 | 447 | 485 | 497 |
| | Average | .................. | | | ... | ... | ... | ... | 218 | 69 | 175 | 352 | 425 | 470 | 504 | 518 |
| 26 | 1055.7 | 1-in. plain round..... | 62 | 242 | ... | ... | ... | ... | 242 | 62 | 184 | 339 | 442 | 527 | 561 | 562 |
| | 1055.8* | 1-in. plain round..... | 93 | 203 | 203 | 203 | 203 | 223 | 255 | 64 | 172 | 297 | 348 | 371 | 394 | 415 |
| | 1055.9* | 1-in. plain round..... | 85 | 240 | 315 | ... | ... | ... | 315 | 65 | 127 | 252 | 292 | 309 | 336 | 349 |
| | Average | .................. | 80 | ... | ... | ... | ... | ... | ... | 64 | 161 | 296 | 361 | 402 | 430 | 442 |
| 27 | 1050.4 | ⅝-in. plain round+... | 60 | 195 | 241 | ... | ... | ... | 269 | 70 | 214 | 306 | 340 | 363 | 393 | 405 |
| | 1050.5 | ⅝-in. plain round+... | 62 | 156 | 203 | 233 | 256 | ... | 271 | 63 | 300 | 444 | 529 | 544 | 578 | 591 |
| | 1050.6 | ⅝-in. plain round+... | 60 | 164 | 172 | 182 | 195 | 213 | 225 | 61 | 150 | 376 | 434 | 471 | 492 | 503 |
| | Average | .................. | 61 | 172 | 205 | ... | ... | ... | 255 | 65 | 221 | 375 | 435 | 460 | 488 | 500 |

* Load continued for several days. See Art. 90.

+ Four bars were used in each of the beams in Group 27.

*(Table 33 continued on page 158.)*

### TABLE 33—CONTINUED.

| Group | Beam No. | Size and Kind of Bar | BEAM TESTS | | | | | | | PULL-OUT TESTS | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Age at Test | Bond Stress at End Slip of (inches) | | | | | Computed Bond Stress at Maximum Load | Age at Test | Bond Stress at End Slip of (inches) | | | | | Maximum Bond Resistance |
| | | | days | .0002 | .0005 | .001 | .002 | .005 | | days | .0002 | .0005 | .001 | .002 | .005 | |
| 28 | 1050.1 | ¾-in. plain round[+]... | 61 | 217 | 235 | 269 | 287 | ... | 308 | 62 | 191 | 377 | 430 | 467 | 502 | 524 |
| | 1050.2 | ¾-in. plain round[+]... | 62 | 183 | 200 | 227 | ... | ... | 235 | 67 | 220 | 403 | 445 | 472 | 499 | 519 |
| | 1050.3 | ¾-in. plain round[+]... | 57 | 174 | 200 | 229 | 252 | 269 | 278 | 59 | 248 | 429 | 546 | 600 | 633 | 642 |
| | Average | .................... | 60 | 191 | 212 | 242 | ... | ... | 274 | 63 | 220 | 403 | 474 | 513 | 545 | 562 |
| 29 | 1046.1 | 1-in. plain square.... | 62 | 174 | 204 | 238 | 254 | 279 | 293 | 60 | 157 | 233 | 258 | 275 | 306 | 336 |
| | 1046.2 | 1-in. plain square.... | 62 | 174 | 219 | 263 | 280 | 293 | 298 | 69 | 122 | 202 | 224 | 243 | 257 | 287 |
| | 1046.3 | 1-in. plain square.... | 57 | 219 | 263 | 285 | 298 | 310 | 316 | 59 | 173 | 279 | 371 | 422 | 457 | 465 |
| | Average | .................... | 60 | 189 | 229 | 262 | 277 | 294 | 302 | 63 | 151 | 238 | 285 | 313 | 340 | 363 |
| 30 | 1046.4 | 1-in. plain square.... | 61 | 189 | 263 | 298 | ... | ... | 308 | 61 | 106 | 192 | 217 | 242 | 290 | 340 |
| | 1046.5 | 1-in. plain square.... | 62 | 177 | 174 | 174 | ... | ... | 188 | 69 | 131 | 194 | 212 | 229 | 256 | 275 |
| | 1046.6 | 1-in. plain square.... | 64 | 204 | 219 | 233 | 241 | 251 | 263 | 65 | 113 | 225 | 255 | 275 | 304 | 334 |
| | Average | .................... | 62 | 190 | 219 | 235 | ... | ... | 253 | 65 | 117 | 204 | 228 | 249 | 273 | 316 |
| 31 | 1047.1 | 1-in. twisted square.. | 62 | 237 | 249 | 293 | 300 | 330 | 335 | 69 | 163 | 248 | 272 | 303 | 349 | 447 |
| | 1047.2 | 1-in. twisted square.. | 65 | 233 | 263 | 293 | 325 | 345 | 359 | 71 | 101 | 266 | 309 | 336 | 386 | 443 |
| | 1047.3 | 1-in. twisted square.. | 60 | 196 | 249 | 282 | 304 | 310 | 317 | 59 | 182 | 325 | 410 | 446 | 479 | 443 |
| | Average | ........ ........ | 62 | 222 | 254 | 289 | 310 | 330 | 337 | 66 | 149 | 280 | 330 | 362 | 405 | 467 |
| 32 | 1048.1 | 1⅛-in. cor. round.... | 70 | 270 | 347 | 405 | 439 | 491 | 541 | 64 | 170 | 400 | 531 | 646 | 717 | 781 |
| | 1048.2 | 1⅛-in. cor. round.... | 59 | 263 | 330 | 381 | 425 | 448 | 475 | 66 | 185 | 326 | 391 | 439 | 494 | 652 |
| | 1048.3 | 1⅛-in. cor. round.... | 60 | 313 | 364 | 406 | 448 | 480 | 486 | 66 | 140 | 380 | 511 | 604 | 672 | 696 |
| | Average | .................... | 63 | 282 | 347 | 397 | 437 | 473 | 501 | 65 | 168 | 369 | 478 | 563 | 628 | 710 |
| 33 | 1048.4 | 1⅛-in. cor. round.... | 62 | 213 | 288 | 338 | 374 | 398 | 482 | 62 | 200 | 397 | 514 | 607 | 694 | 721 |
| | 1048.5 | 1⅛-in. cor. round.... | 63 | 246 | 321 | 372 | 405 | 451 | 477 | 65 | 169 | 350 | 491 | 558 | 619 | 706 |
| | 1048.6 | 1⅛-in. cor. round.... | 64 | 297 | 354 | 392 | 416 | 421 | 431 | 65 | 94 | 289 | 362 | 397 | 447 | 614 |
| | Average | .................... | 63 | 252 | 321 | 367 | 398 | 423 | 463 | 64 | 155 | 339 | 456 | 521 | 587 | 680 |
| 34 | 1049.1 | 1⅛-in. cor. round.... | 61 | 318 | 364 | 369 | ... | ... | 369 | 68 | 140 | 355 | 445 | 490 | 537 | 590 |
| | 1049.2 | 1⅛-in. cor. round.... | 63 | 268 | 307 | 329 | 353 | 363 | 371 | 69 | 146 | 306 | 365 | 390 | 414 | 521 |
| | 1049.3 | 1⅛-in. cor. round.... | 400 | 275 | 328 | 370 | ... | ... | 400 | 79 | 141 | 365 | 452 | 518 | 567 | 619 |
| | Average | .................... | .. | ... | ... | ... | ... | ... | ... | 72 | 142 | 342 | 421 | 466 | 506 | 577 |

[+] Three bars were used in each of the beams in Group 28.

84. *Bond Resistance with Twisted Square Bars.*—The beams in group 31 were reinforced with one 1-in. twisted square bar. These bars were twisted cold (one twist per lineal foot) from the same stock as was used in the beams reinforced with plain square bars. Three pull-out specimens were made for each of the beams; the blocks were not reinforced against bursting. At an end slip of 0.0005 in. the bond stresses were 254 and 266 lb. per sq. in. for the beam tests and pull-out tests, respectively. Table 36 gives a summary of the bond stresses developed in the tests with twisted square, plain square, and plain round bars in the 1912 series.

For the two stages of the tests included in the table the twisted bars give values about 20% higher than the plain squares in the beam tests and about 34% higher in the pull-out tests. The values for the twisted bars are about 12% lower than the plain rounds in the beam tests; in the pull-out tests the twisted bars are 8% lower than the plain rounds at a slip of 0.001 in. and show about the same difference at the maximum. The maximum load for the pull-out tests with the square twisted bars came at a slip of about 0.1 in. and was accompanied by the bursting of the concrete blocks while the maximum load for the plain rounds came at a slip of about 0.01 in. The bond stresses for the square twisted and the plain round bars at a slip of 0.01 in. were 407 and 408 lb. per sq. in., respectively. These values are about the same as the maximum bond stresses for the plain rounds in the 1909 pull-out tests (see Table 14).

Load-deflection and load-slip curves for these beams are given in Fig. 85. It is seen that after end slip of about 0.001 in. is developed the action of the beams with twisted bars is quite similar to that in beams with plain square bars.

Beam No. 118, series of 1909 (see Fig. 77), reinforced with one 1-in. twisted square bar acted in much the same way as the beams just discussed, except that after an end slip of 0.004 in. the bond resistance began to increase and a second maximum load about 3000 lb. higher than the first was carried. This phenomenon is similar to that observed in pull-out tests of twisted square bars as described in Art. 41.

85. *Bond Resistance with Corrugated Bars.*—Six 6-ft. beams and three 10-ft. beams were reinforced with 1⅛-in. corrugated rounds (groups 32, 33 and 34). All the 6-ft. beams gave bond failures; the 10-ft. beams failed in tension.

## TABLE 34.

## SUMMARY OF BOND STRESSES IN BEAM AND PULL-OUT TESTS—1912 SERIES.

Stresses are given in pounds per square inch.

| Group | Characteristics | No. of Tests | Age at Test days | Bond Stress at End Slip of (inches) | | | | | | Computed Bond Stress at Maximum Load |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | .0002 | .0005 | .001 | .002 | .005 | .01 | |
| | **Beam Tests; 1-in. Plain Rounds.** | | | | | | | | | |
| 14-16 | 6-ft. span; Loads 2 ft. apart......... | 9 | 64 | 251 | 312 | 345 | ..... | ..... | ..... | 369 |
| 17 | 6-ft. span; Loads 2½ ft. apart......... | 3 | 65 | 247 | 334 | 398 | ..... | ..... | ..... | 407 |
| 18 | 6-ft. span; Loads 3 ft. apart......... | 3 | 66 | 241 | 277 | 288 | 307 | ..... | ..... | 332 |
| 19 | 6-ft. span; Loads 3½ ft. apart......... | 3 | 62 | 223 | 275 | 347 | 401 | ..... | ..... | 468 |
| 20 | 6-ft. span; Loads 4 ft. apart ........ | 3 | 64 | 328 | 431 | ..... | ..... | ..... | ..... | 551 |
| 21 | 5-ft. span; Loads 1⅔ ft. apart......... | 3 | 64 | 277 | 312 | 334 | ..... | ..... | ..... | 364 |
| 22 | 7-ft. span; Loads 2⅓ ft. apart......... | 3 | 63 | 273 | 310 | .... | ..... | ..... | ..... | 336 |
| 23 | 8-ft. span; Loads 2⅔ ft. apart......... | 3 | 62 | 231 | ..... | ..... | ..... | ..... | ..... | 303 |
| 24-26 | 10-ft. span; Loads 3⅓ ft. apart......... | 7 | ..... | 218 | ..... | ..... | ..... | ..... | ..... | 230 |
| 26 | 10-ft. span; Loads 6 ft. apart......... | 2 | 89 | 222 | 259 | ..... | ..... | ..... | ..... | 285 |
| | **Pull-out Tests; 1-in. Plain Rounds.** | | | | | | | | | |
| 14-26 | Lowest two from each set of three tests.°.... | 54 | ..... | 144 | 306 | 362° | 389 | 410 | 417 | 426° |
| | All pull-out tests......................... | 81 | 66 | 158 | 319 | 377 | 408 | 434 | 447 | 448 |
| | Highest from each set...................... | 27 | ..... | 192 | 347 | 413 | 450 | 480 | 491 | 494 |
| | Lowest from each set....................... | 27 | ..... | 123 | 289 | 340 | 368 | 384 | 392 | 399 |
| | **Beam Tests; ⅝-in. Plain Rounds.** | | | | | | | | | |
| 27 | All beam tests........................... | 3 | 61 | 172 | 205 | 235 | ..... | ..... | ..... | 255 |
| | **Pull-out Tests; ⅝-in. Plain Rounds.** | | | | | | | | | |
| 27 | Lowest two from each set of three tests°..... | 6 | ..... | 194 | 337 | 394° | 424 | 455 | 461 | 467° |
| | All pull-out tests......................... | 9 | 64 | 221 | 375 | 435 | 460 | 490 | 490 | 500 |
| | Highest from each set ..................... | 3 | ..... | 280 | 452 | 505 | 531 | 560 | 561 | 565 |
| | Lowest from each set....................... | 3 | ..... | 175 | 293 | 345 | 363 | 400 | 402 | 415 |
| | **Beam Tests; ¾-in. Plain Rounds.** | | | | | | | | | |
| 28 | All beam tests........................... | 3 | 60 | 186 | 212 | 242 | ..... | ..... | ..... | 274 |
| | **Pull-out Tests; ¾-in. Plain Rounds.** | | | | | | | | | |
| 28 | Lowest two from each set of three tests°..... | 6 | ..... | 179 | 390 | 455° | 490 | 519 | 536 | 537° |
| | All pull-out tests......................... | 9 | 63 | 220 | 403 | 474 | 513 | 545 | 558 | 562 |
| | Highest from each set...................... | 3 | ..... | 301 | 430 | 510 | 559 | 595 | 604 | 611 |
| | Lowest from each set....................... | 3 | ..... | 125 | 365 | 441 | 476 | 510 | 524 | 524 |

° Used for comparison with bond stresses in reinforced concrete beams.

## TABLE 34—CONTINUED.

| Group | Characteristics | No. of Tests | Age at Test days | Bond Stress at End Slip of (inches) | | | | | | Computed Bond Stress at Maximum Load |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | .0002 | .0005 | .001 | .002 | .005 | .01 | |
| | **Beam Tests; 1-in. Plain Squares.** | | | | | | | | | |
| 29-30 | All beam tests | 6 | 61 | 190 | 224 | 248 | ..... | ..... | ..... | 278 |
| | **Pull-out Tests; 1-in. Plain Squares.** | | | | | | | | | |
| 29-30 | Lowest two from each set of three tests° | 12 | ..... | 115 | 209 | 241° | 266 | 294 | 309 | 323° |
| | All pull-out tests | 18 | 64 | 134 | 221 | 256 | 281 | 312 | 328 | 340 |
| | Highest from each set | 6 | ..... | 171 | 244 | 285 | 312 | 367 | 366 | 372 |
| | Lowest from each set | 6 | ..... | 104 | 194 | 220 | 241 | 266 | 283 | 298 |
| | **Beam Tests; 1-in. Twisted Squares.** | | | | | | | | | |
| 31 | All beam tests | 3 | 61 | 222 | 254 | 289 | 310 | 330 | ..... | 337 |
| | **Pull-out Tests; 1-in. Twisted Squares.** | | | | | | | | | |
| 31 | Lowest two from each set of three tests° | 8 | ..... | 131 | 266 | 325° | 351 | 386 | 407 | 428° |
| | All pull-out tests | 12 | 65 | 145 | 278 | 330 | 362 | 403 | 433 | 467 |
| | Highest from each set | 4 | ..... | 167 | 311 | 348 | 407 | 459 | 493 | 543 |
| | Lowest from each set | 4 | ..... | 113 | 252 | 303 | 325 | 363 | 382 | 405 |
| | **Beam Tests; 1⅛-in. Corrugated Rounds.** | | | | | | | | | |
| 32-33 | 6-ft. span; loads 2 ft. apart | 6 | 63 | 261 | 334 | 382 | 418 | 458 | 465 | 482 |
| 34 | 10-ft. span; loads 3⅓ ft. apart | 2 | 62 | 293 | 335 | 349 | ..... | ..... | ..... | 370 |
| | **Pull-out Tests; 1⅛-in. Corrugated Rounds.** | | | | | | | | | |
| 32-34 | Lowest two from each set of three tests° | 18 | ..... | 142 | 332 | 429° | 490 | 548 | 571 | 630° |
| | All pull-out tests | 27 | 67 | 154 | 353 | 452 | 516 | 574 | 595 | 654 |
| | Highest from each set | 9 | ..... | 177 | 394 | 498 | 570 | 625 | 639 | 698 |
| | Lowest from each set | 9 | ..... | 132 | 304 | 407 | 470 | 522 | 546 | 617 |
| | **Pull-out Tests; 1-in. Rounds with Standard V-shaped Threads.** | | | | | | | | | |
| | Lowest two from each set of three tests | 16 | ..... | 181 | 380 | 470 | 535 | 592 | 613 | 677 |
| | All pull-out tests | 24 | 66 | 206 | 410 | 505 | 580 | 642 | 685 | 725 |
| | Highest from each set | 8 | ..... | 254 | 473 | 576 | 670 | 738 | 757 | 813 |
| | Lowest from each set | 8 | ..... | 177 | 351 | 431 | 488 | 530 | 551 | 614 |

° Used for comparison with bond stresses in reinforced concrete beams.

Load-deflection and load-slip curves for the 6-ft. beams are given in Fig. 86. The ends of the bar in the 6-ft. beams give a slip of 0.001 in. at a computed bond stress of 382 lb. per sq. in. Beams with plain round bars give a bond stress of 345 lb. per sq. in. at the same amount of slip. The maximum bond resistance of the 6-ft. beams was 482 lb. per sq. in. as compared with 369 lb. per sq. in. for similar beams with 1-in. plain rounds. The load at an end slip of 0.001 in. was 79% of the maximum for the beams with corrugated bars and 94% for the beams with plain round bars. The loads at first outer cracks are about the same for the 6-ft. beams reinforced with corrugated bars as for other beams tested on the same span, whether plain round, plain square, or corrugated round—about 9000 lb. The load at first end slip of bar averaged 15 200 lb. for the beams with corrugated bars and 13 100 lb. for the beams with plain rounds. The average bond stress at first slip of end of bar was 261 lb. per sq. in. for the corrugated bars and 251 lb. per sq. in. for the plain rounds. It is seen that the vertical shearing stresses developed at first outer crack noted are about the same for the two forms of bar, regardless of the difference in area of bars. At the maximum load the bond resistance of the beams reinforced with corrugated bars was about 30% higher than those with plain rounds.

## TABLE 35.

### BOND RESISTANCE IN 6-FT. BEAMS REINFORCED WITH PLAIN ROUND BARS.

Each beam was reinforced with a single round bar 1 in. or 1¼ in. in diameter.
All beams were loaded at the one-third points of the span.
Stresses are given in pounds per square inch.

| Series | Groups | Number of Tests | Age at Test | Bond Stress at Slip of End of Bar of | | | Maximum Bond Resistance | Compressive Strength of 6-in. Cubes |
|---|---|---|---|---|---|---|---|---|
| | | | | .0002 in. | .0005 in. | .001 in. | | |
| 1909 | ........ | 6 | 100 days | 202 | 244 | 265 | 291 | 1575 |
| 1911 | 1-5, 9 | 19 | 8 mo. | 240 | 306 | 335 | 377 | 3030 |
| 1912 | 14-16 | 9 | 63 days | 251 | 312 | 345 | 369 | 2630 |
| Mean | ........ | ........ | .......... | 236 | 295 | 324 | 356 | |

The beams tested on a 10-ft. span showed end slip to begin at about the same bond stress as in the 6-ft. beams. The 10-ft. beams reinforced with corrugated bars showed cracks much nearer the supports than were found in the beams with plain bars; see Figs. 59c and 63d.

86. *Effect of Span Length.*—Besides the beams tested on a 6-ft. span, a group was tested on each of the following spans—5, 7, 8, and 10 ft. All the beams were reinforced with 1-in. plain rounds and were loaded at the one-third points of the span. Load-slip curves are given in Fig. 71 and 72. See the photographs in Fig. 63 for the appearance of these beams after failure. Nearly all the beams with spans of 7, 8 and 10 ft. failed in tension and hence did not develop their maximum resistance to bond stresses. Notwithstanding this, the longer beams give data on the slip of bar at ends and at intermediate points and on the deflections and general behavior of such beams.

## TABLE 36.

### Bond Resistance with Twisted Square, Square, and Round Bars

6-ft. beam tests from series of 1912; see Tables 31, 32, 33 and 34.
Stresses are given in pounds per square inch.

| Size and Kind of Bar | Reinforced Concrete Beam Tests | | | Pull-out Tests† | | |
|---|---|---|---|---|---|---|
| | Number of Tests | At End Slip of 0.001 in. | At Maximum Load | Number of Tests | At End Slip of 0.001 in. | At Maximum Load |
| 1 in. twisted square | 3 | 289 | 337 | 8 | 325 | 428 |
| 1 in. plain square | 6 | 248* | 278* | 12 | 241* | 323* |
| 1 in. plain round | 9 | 345 | 369 | 54 | 362 | 426 |

† As stated in Art. 73, the bond stresses considered in comparing the pull-out tests with the beam tests are the averages obtained after rejecting the strongest specimen from each set of three tests. See Table 34.

* Beam No. 1046.5 gave abnormally low results. Omitting this test, the values for the beam tests with plain square bars are 283 and 295 lb. per sq. in., and for the pull-out tests, 265 and 355 lb. per sq. in., respectively.

The computed bond stresses at first slip at the end of the bar were as follows:—5-ft. beams, 277; 6-ft., 257; 7-ft., 273; 8-ft., 231; 10-ft., 222 lb. per sq. in. If correction is made for the added embedded length due to the 3 in. overhang of the ends of the beam the following values of bond stress at first end slip are obtained:—5-ft., 235; 6-ft., 222; 7-ft., 244; 8-ft., 209; 10-ft., 206 lb. per sq. in. The stresses developed at the maximum loads were:—5-ft. beams, 364; 6-ft., 369; 7-ft., 336; 8-ft., 303; 10-ft., 230 lb. per sq. in. The last three values, however, do not measure the maximum bond resistance, since the beams failed by tension in the longitudinal steel.

It is seen that there is not much difference in the computed bond stresses for beams of the span lengths tested, during the early stages of the tests. This indicates that the distribution of bond stress is similar in beams of these lengths during the early stages of the test. However, it should be borne in mind that these computed values for bond represent only the average bond stress and that they give no information as to the distribution of the bond stress or the actual bond stresses developed at various points. The discussion in Art. 68 and 95 indicates the distribution of bond stress in beams of this kind.

87. *Effect of Position of Loads on Beam.*—The beams in groups 17, 18, 19 and 20 (span 6 ft.) were tested in order to study the effect of the position of the loads with respect to the beam supports. The beams in groups 21, 22 and 23, which were tested by loads applied at the one-third points of spans varying from 5 ft. to 8 ft., will be included in this discussion. All these beams had the same cross-section—8 in. wide and 10 in. effective depth—and were each reinforced with a single 1-in. plain round bar. The position of the loads with respect to the supports may be expressed in terms of the ratio of the effective depth of the beam, $a$, to the horizontal distance from the support to the load point, $b$. This ratio for the beams included in the discussion varied from 1.2 to 3.2.

To make the comparison more nearly accurate, it will be desirable in the calculations to take into account the length of embedment of the bar beyond the point of support, which in all the beams was 3 in. To do this it is assumed that the full calculated tensile stress of the bar at the load point is taken off by a bond stress which is uniform in intensity from the load point to the end of the bar. The values so calculated will be less than the computed bond stresses given in Table 32.

Fig. 50 has been plotted with the average computed bond stress for each group of beams as ordinates and with the ratio $b/a$ as abscissas. The closed circles represent the bond stresses developed at first end slip of bar; and the open circles represent the maximum bond stresses. It will be seen that the computed bond stress at beginning of end slip is nearly the same throughout the range of the tests. This confirms the conclusion stated in the preceding paragraph in the discussion of another series of tests, that during the early stages of the test the bond stress was probably distributed in a similar manner in these beams in which the loads were applied at different points on the span. The computed bond resistances at the maximum load are greater for the smaller values of the ratio $b/a$; except for group 19 and 20 this difference is not large.

A study of the position of the cracks in these beams shown in Fig. 60 and 61 gives no indication that the location of the cracks is productive of these differences in the maximum bond resistance developed by the loads in different positions. Added friction due to the increased bearing pressures obtained with the beams having smaller values of $b/a$ may increase the bond resistance somewhat, but this would be expected to affect the static friction element of bond at the beginning of slip and there is no evidence of such effect. It would seem, however, that the variation must be due to the presence of other than normal beam action.

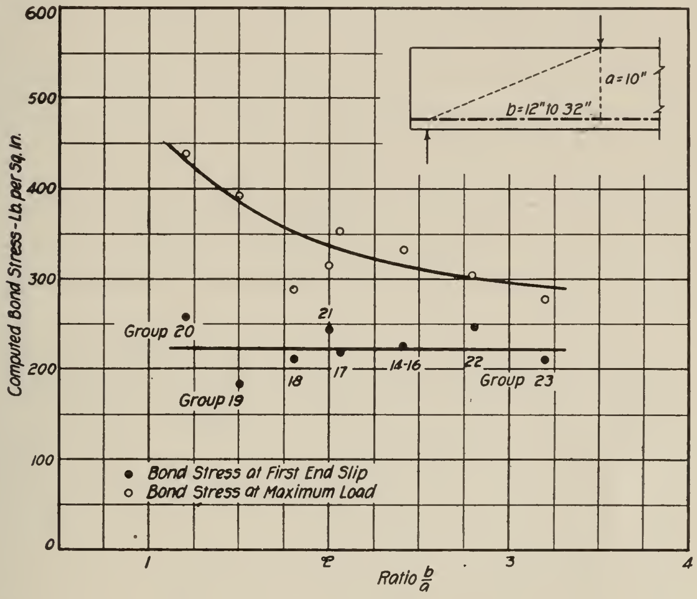


FIG. 50. EFFECT OF POSITION OF LOADS ON REINFORCED CONCRETE BEAMS.

88. *Effect of Auxiliary Reinforcement in Ends of Beams.*—In addition to the main longitudinal reinforcement, consisting of 1-in. plain round bars, the beams in Group 16 were provided with two loops of ⅜-in. round bars at each end extending from a point 3 in. inside the load points to the ends of the beam. The arrangement is shown in Fig. 2 (g). This increased the tension reinforcement outside the load points by 56%. The purpose of these tests was to discover the effect produced on the bond resistance as a result of retarding the opening of the usual outer cracks. The load-slip curves for these beams are shown

in Fig. 69, and the load-deflection curves in Fig. 81. Group 16 may be compared with Groups 14 and 15, which were the same except they did not have the auxiliary bars. The load at first crack outside the load points in Group 16 averaged 11 300 lb. as compared with 9100 lb. for Groups 14 and 15; an increase of 24%. The bond stress at first end slip of bar, computed on the area of the main bar only, is raised from 240 to 282 lb. per sq. in., an increase of 17%. The maximum bond resistance, based on the main bar only, is practically the same for each of these groups of beams.

The photograph of these beams after failure (Fig. 60b) shows a different distribution of the cracks in the beams having auxiliary bars. Generally speaking, the main cracks came inside the load points near the end of the auxiliary bars. The characteristic sloping cracks of such beams are generally absent, although it seems probable that interior cracks which did not appear on the surface may have opened to an extent that permitted the same distribution of bond stress as was found in the other beams. The most striking effect arising from use of the auxiliary bars is found in the uniformity and regularity of the slip of bar at intermediate points of the bar as shown in Fig. 69 and discussed in Art. 94.

89. *Effect of Variation in Number and Size of Reinforcing Bars.* —Four ⅝-in. plain rounds were used for longitudinal reinforcement in Group 27 and three ¾-in. rounds in Group 28. This corresponds to 1.53% and 1.66% of longitudinal steel, respectively, as compared with about 1% for the 1-in. plain rounds used in most of the beams. These beams were provided with U-shaped stirrups as shown in Fig. 2(h). The measurements of end slip were made on the two outside bars. There was a marked uniformity in the behavior of the two outside bars in nearly all the tests; generally the two instruments at the same end of the beams gave exactly the same reading; in other cases the larger value was used in plotting the load-slip curves in Fig. 84.

The relation of the bond stresses developed in the beam and pull-out tests may be seen in Table 34. In the beam tests the ⅝ and ¾-in. bars show end slip to begin at a much lower computed bond stress than for the beams with 1-in. bars, but it is worth noting that this first end slip came at loads which gave the same tensile stress in the steel at mid-span and hence presumably about the same amount of stretch of bar between load point and support. For an end slip of 0.001 in. (using the values in Groups 14 to 16 for the 1-in. bars) the average computed bond stresses are: ⅝-in. bars, 235 lb. per sq. in.; ¾-in. bars, 242; 1-in. bars, 345 lb. per sq. in. At the maximum loads the values are: 255, 274 and 369 lb. per sq. in., respectively. From this it would seem that slip is a function of amount of stretch of bar as well as of bond surface. The loads at first outer crack are about the same for each of the groups of beams. The concrete may be expected to fail in tension at about the same load, and it seems that the anti-stretch cracks form at once. The appearance of one group of beams after test is shown in Fig. 62a. By comparing the photographs of the beams in Group 27 with Group 14, it will be seen that the progress of these outer cracks may assist in explaining the difference in the bond stresses developed. For the beams reinforced with 1-in. plain rounds (Groups 14 and 15), the average distance of the outermost cracks which had appeared previous to the first slip of bar at the end where bond failure occurred, was 18 in. from the end of the beams; in Group 27, reinforced with four ⅝-in. rounds, the corresponding distance is 11 in.; in group 28, reinforced with three ¾-in. rounds, 13 in.

A consideration of the position of these outer cracks indicates that after such cracks are formed in beams of this length a large proportion of the bond stress is carried by the embedded length lying between the crack and the end of the beam, while a smaller proportion is carried by the part between the crack and the load point. This assumption makes the bond stresses of the outer embedded length more nearly the same for the three groups. It seems probable that the actual bond resistance developed in the outer embedded lengths approaches the values developed in the pull-out tests. After cracks have formed in the concrete near the load points, it may be expected that an increasae of load will produce other cracks still nearer the support due partly to anti-stretch slip. This cracking progresses step by step toward the end of the beam until the remaining unbroken embedment of the bar is no longer able to furnish necessary resistance, and the beam finally fails by the bars pulling out. The progress of the formation of cracks is very well illustrated in the beams of Groups 27 and 28. In Beam No. 1050.2 a crack opened about 3 in. outside the N. load at 8000 lb.; another appeared 5 in. farther out at 12 000 lb.; and a third at 16 000 lb. In Beam No. 1050.3 cracks appeared at the S. end at 14 000, 18 000, 29 000 and 31 000 lb.

These tests indicate that the distribution of bond stresses in beams is influenced by the relation of the dimensions of the cross-section of the beam to the diameters of the reinforcing bars.

90. *Effect of Repeated and Continued Loads on Beams.*—In testing Beam No. 84 of the 1909 series, reinforced with one 1¼-in. plain round, the load was applied progressively as in other tests, until one end of the bar had slipped about 0.0002 in. under an applied load of 15 000 lb. The load was then released to 500 lb. The residual center deflection was 0.015 in. (see Fig. 51). At first loading, 6400 lb. load was required to produce this deflection. The load was then increased to 17 000 lb., causing a slip at the north end of the bar of 0.0006 in. Upon releasing the load to 900 lb. the residual deflection was over 0.02 in. There was no recovery in the amount of the end slip of the bar upon release of the loads, although the bar had slipped an appreciable amount. The load was then reapplied up to a maximum of 20 500 lb., when the bar pulled out at the end which at first had shown the smaller slip. This phenomenon was observed in a few tests in the later series, showing that the bond resistance at the two ends was so nearly equal that failure was likely to come at either end, or at both ends at the same time. In most of the beam tests slipping was much more pronounced at one end than the other.

The lack of recovery in the slip at the end of the bar in this beam test and in the repeated load tests on pull-out specimens is significant, and seems to be characteristic of plain bars. It will be seen in the test of Beam No. 1049.3 that corrugated bars do show some recovery of slip upon release of load.

Beam No. 1055.9, reinforced with one 1-in. plain round bar, was loaded on a 10-ft. span at two points 6 ft. apart. The test progressed as usual until a load of 16 000 lb. (bond stress, 315 lb. per sq. in.) had been applied, which caused a slip of 0.0004 in. at the south end of the bar. This load was maintained until failure occurred after about 34 hours. The changes in slip of bar with the time are plotted in Fig. 52. As was seen in the pull-out tests, after slipping becomes general, there is nearly a constant difference between the total slip at the end of the bar and at point (3). The slip at (1) was probably affected by the large diagonal crack which opened at that point.

Beam No. 1039.3, reinforced with one 1-in. plain round bar, was loaded at the one-third points of a 6-ft. span. This beam was 7½ months old at the time of test. A load of 26 000 lb. caused a slip of 0.001 in. at the north end. This load was maintained constant until failure occurred after 66 hours. The end slip during the period the beam was under load is shown in Fig. 52.

Beam No. 1055.8, 10-ft. span, reinforced with one 1-in. plain round bar, was loaded at two points 6 ft. apart. The load was increased as usual until the north end of the bar showed a slip of 0.0007 in. under a load of 10 000 lb. The load was maintained at 10 000 lb. for 32 days, during which frequent observations of slip of bar and deflection were taken. After 32 days the load was increased 100 lb. per day until failure occurred on the 60th day of loading under a load of 12 800 lb. The slip of bar and the center deflections are plotted in Fig. 53. Slip-

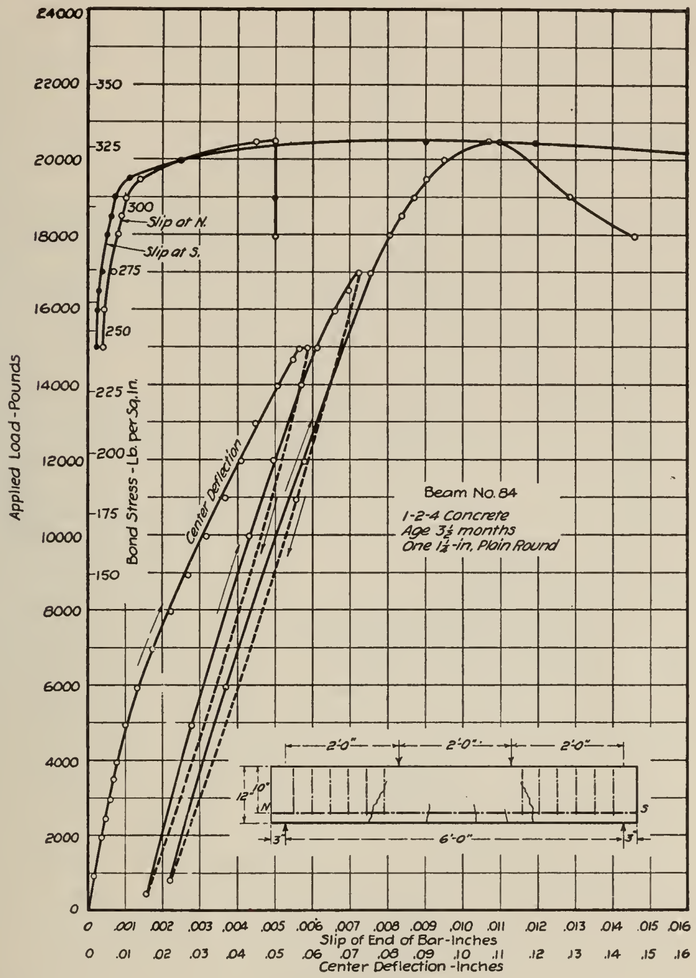


FIG. 51. LOAD-SLIP AND LOAD-DEFLECTION CURVES FOR A BEAM UNDER REPEATED LOADS.

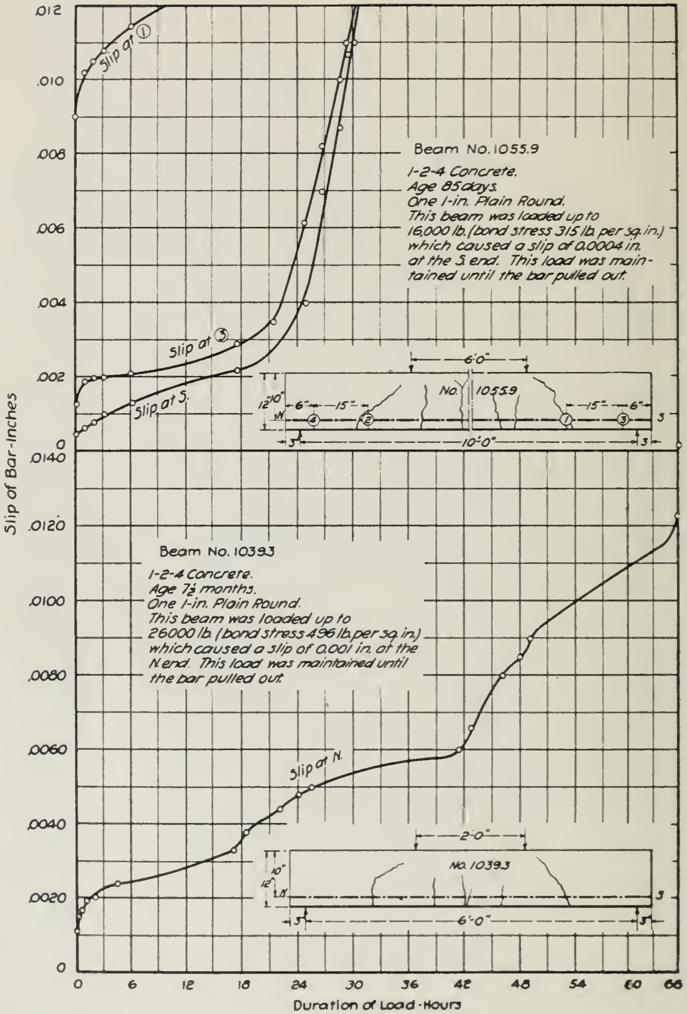


FIG. 52. SLIP OF BAR IN BEAMS UNDER CONSTANT LOADS.

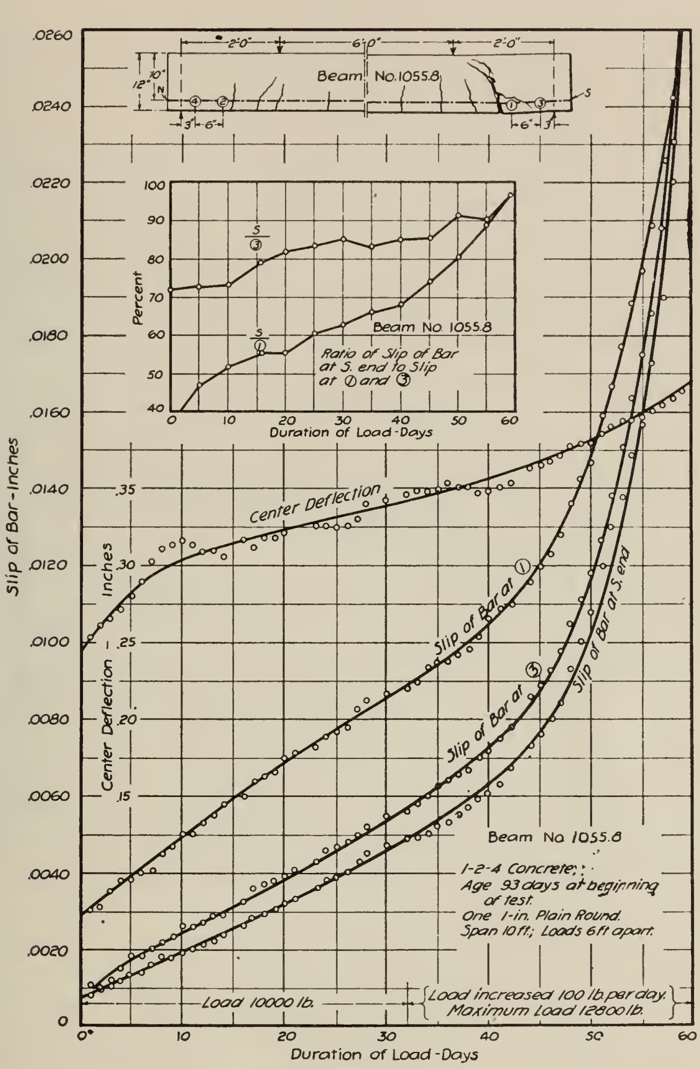


FIG. 53. SLIP OF BAR AND CENTER DEFLECTION OF BEAM NO. 1055.8.

ping continued at a nearly constant rate while the load was maintained at 10 000 lb. There was no marked change in the rate of slip until the load had been increased by about 1000 lb.

Fig. 54 shows the load-slip curves for Beam No. 1055.8. The curves have been plotted from the same origin in terms of the slip at the points where measurements were taken and the computed bond stress during the later stages of the test.

The small diagram in Fig. 53 indicates the ratio of the end slip to the slip at points 6 in. and 12 in. from the end. When the load was first applied the slip of bar at the end was 70% and 40% of the slip at points 6 and 12 in., respectively, from the end; while near failure the slip at inner points was nearly the same as at the end. The slip-of-bar curves indicate that, after slip became general, there was a nearly constant difference between the end slip and the slip at inner points where the concrete was unbroken. It seems probable from the way slip was progressing in this test that failure would have been produced under a load of 10 000 lb., if this load had been maintained for two or three months.

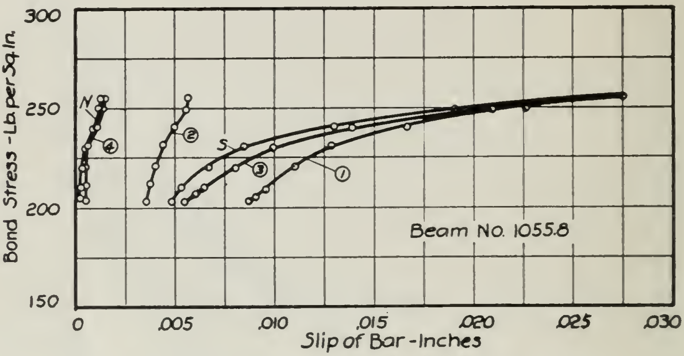


Fig. 54. Load-slip Curves for Beam No. 1055.8.

The center deflection of the beam increased about 0.11 in. during the period of about 40 days in which the load was practically constant. This increase in deflection was probably due principally to the slip of the bar at the end of the beam where bond failure occurred. This slip amounted to about 0.007 in. at the south end after the beam had been under load 40 days.

The tests described above in which the load causing a very small amount of slip at the end of the bar was maintained constant indicate that with beams reinforced with plain bars the load which produces a small end slip will, if acting indefinitely, endanger the permanency of the beam.

91. *Pull-out Tests with 1912 Beam Series.*—A summary of the results of the tests on the pull-out specimens made with the 1912 beams is given in Table 34. The load-slip curves have been plotted in Fig. 55. The tests were made at the age of about 65 days. The number of specimens of each kind is indicated by figures in parentheses. In these curves the average values from all the tests have been used; while for comparison with the bond stresses developed in the corresponding beams the average of the values from the lowest two of each set of three pull-out tests should be used.

The 1-in. plain rounds averaged 448 lb. per sq. in. at the maximum; the ⅝-in. rounds about 11% higher, and the ¾-in. rounds about 25% higher. The 1-in. plain square bars gave values which are about 75% (range 69% to 78%) of those for 1-in. rounds.

In the tests with 1-in. twisted square bars end slip began at a bond stress of 278 lb. per sq. in.—about a mean between the values for the 1-in. plain rounds and the 1-in. plain squares. At an end slip of 0.02 in. the twisted bars gave a bond resistance of 448 lb. per sq. in., which is about the same as given by the plain rounds at their maximum at an end slip of 0.01 in. This is a somewhat better showing than was found for the twisted square bars in the 1909 series of pull-out tests (see Art. 41). After a slip of 0.02 in., there was very little increase in the bond resistance of the twisted square bars, which reached 467 lb. per sq. in. at an end slip of 0.1 in.; this has been considered the maximum bond resistance. The concrete blocks split at a slip of about 0.1; they were not reinforced against bursting.

The 1⅛-in. corrugated rounds gave an average bond resistance of 452 lb. per sq. in. at an end slip of 0.001 in.—about 20% higher than the 1-in. plain rounds at the same slip. At an end slip of 0.01 in., corresponding to the maximum bond resistance of the plain rounds, the corrugated bars gave 595 lb. per sq. in.—33% higher than the 1-in. plain rounds. The blocks split in all the corrugated-bar tests; splitting seemed to have begun soon after a slip of 0.01 in. was developed.

1-in. round bars, with standard threads (8 per inch), showed the highest resistance at beginning of slip; at a slip of 0.001 in. the bond resistance was 505 lb. per sq. in., as compared with 377 for the 1-in. rounds and 452 lb. per sq. in. for the 1⅛-in. corrugated rounds. At an end slip of 0.01 in. the threaded bars gave a bond resistance of 685 lb. per sq. in. These bars split the blocks soon after developing an end slip of 0.01 in.

92. *Bond Resistance in 1912 Beam Tests and Pull-out Tests.*—The summary in Table 34 shows the relation of the bond stresses for the beam and pull-out tests for the 1912 series. The dimensions of the specimen and the embedded length may be expected to affect the bond resistance corresponding to different end slips, hence it will be of interest to compare the results of the beam and pull-out tests at the same amounts of end slip. In this comparison the average values from the lowest two specimens from each set of pull-out tests was used.

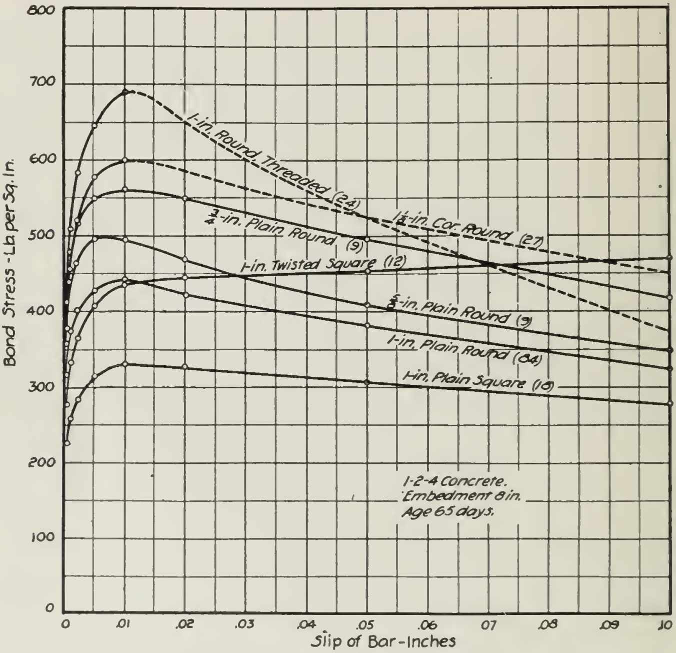


FIG. 55. LOAD-SLIP CURVES FOR PULL-OUT TESTS FROM 1912 BEAM SERIES.

The 6-ft. beams reinforced with 1-in. plain rounds, and loaded in the usual way (Groups 14-16), showed about the same resistance to slip during the early stages of the tests as was found in the pull-out tests. At an end slip of 0.001 in. the values were:—9 beam tests, 345 lb. per sq. in.; 54 pull-out tests, 362 lb. per sq. in. At a slip of 0.002 in. the pull-out tests gave a bond resistance of 389 lb. per sq. in.; the maximum bond resistance in the beams, which came at an end slip of about 0.002 in., was 369 lb. per sq. in. For the tests with 1-in. plain rounds the bond stress at an end slip of 0.001 in. was 85% of the maximum in the pull-out tests and 94% of the maximum in the beam tests.

For the ⅝-in. plain rounds at an end slip of 0.001 in. the beam tests gave a computed bond resistance of 235 lb. per sq. in.; the pull-out tests 394 lb. per sq. in. For the ¾-in. bars the corresponding values were: beam tests, 242 lb. per sq. in.; pull-out tests, 455 lb. per sq. in. For the ⅝-in. bars the maximum computed bond stress in the beam tests was 55% of the maximum bond resistance in the pull-out tests; for the ¾-in. bars, the corresponding value was 51%. The wide differences in the computed bond stresses in the beams reinforced with ⅝ and ¾-in. bars and the corresponding pull-out tests are due to the changes in the distribution of the bond stresses in the beams as the test progresses, as was pointed out in Art. 68, and to the differences in the relative dimensions of the specimens.

The 1-in. plain square bars gave about the same relative values for the beam and pull-out tests as was found for the 1-in. plain rounds; at an end slip of 0.001 in. the bond stresses were: 6 beam tests, 248 lb. per sq in.; 12 pull-out tests, 241 lb. per sq. in. The bond stress at an end slip of 0.001 in. was 75% of the maximum in the pull-out tests and 89% of the maximum in the beam tests.

93. *Variation in the Results of Cube and Pull-out Tests.*—The cube and pull-out tests made of 1-2-4 concrete in the 1912 beam series offer an opportunity to study the variation which may be expected in a large number of tests made under conditions which are fairly uniform. These specimens were made from 44 batches of concrete and covered a period of about 3 months. This discussion includes 129 cube tests and 57 pull-out tests. Table 37 shows the average strength and the variation in strength for the individual cube tests and for the averages of the groups of three tests for hand-mixed and machine-mixed concrete. It will be noted that for the compression tests the mean variations of the individual tests and of the group averages are about the same for both methods of mixing. The higher strength of the machine-mixed concrete is in accordance with the usual experience, but the favorable showing as to the variation of the hand-mixed concrete is contrary to the relation usually assumed.

Table 38 shows values for the pull-out tests of 1-in. plain rounds embedded 8 in. at an end slip of 0.001 in. and for the maximum load. In this table only the machine-mixed specimens have been considered. The values for a slip of 0.001 in. and for maximum load show about the same variation. The pull-out tests show a greater mean variation from the average than the cube tests (16% and 10%, respectively); this justifies the practice of making five pull-out specimens in a set. However, it is instructive to note that the mean variations of the individual tests and of the groups for both cube tests and pull-out tests do not differ materially.

## TABLE 37.

### Variation in the Compressive Strength of 6-in. Concrete Cubes.

1-2-4 concrete from the 1912 beam series.
Age at test about 65 days.

| Item | Hand-mixed Concrete | | | | Machine-mixed Concrete | | | |
|---|---|---|---|---|---|---|---|---|
| | Number of Tests | Average Crushing Strength, lb. per sq. in. | Mean Variation | | Number of Tests | Average Crushing Strength, lb. per sq. in. | Mean Variation | |
| | | | lb. per sq. in. | Per cent | | | lb. per sq. in. | Per cent |
| Individual tests | 36 | 2200 | 204[+] | 9.2 | 93 | 2800 | 294[+] | 10.5 |
| Average of sets of three tests | 36 | 2200 | 194[+] | 8.8 | 93 | 2800 | 281[+] | 10.0 |

[+] Plus or minus.

## TABLE 38.

### Variation of the Values of Bond Resistance in Pull-out Tests.

Includes 57 tests with 1 in. plain round bars from the 1912 beam series.
1-2-4 machine-mixed concrete.
Embedment 8 in. Age at test, about 65 days.
Stresses are given in pounds per square inch.

| Item | At End Slip of 0.001 in. | | | | | At Maximum Bond Resistance | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Highest | Lowest | Average | Mean Variation | | Highest | Lowest | Average | Mean Variation | |
| | | | | lb. per sq. in. | Per cent | | | | lb. per sq. in. | Per cent |
| Individual tests | 546 | 213 | 371 | 55[+] | 14.8 | 690 | 225 | 440 | 73[+] | 16.6 |
| Average of sets of three tests | 509 | 241 | 371 | 49[+] | 13.2 | 599 | 254 | 440 | 66[+] | 15.0 |

[+] Plus or minus.

### *d. General Discussion of Reinforced Concrete Beam Tests.*

94. *Slip of Bar at Internal Points in Reinforced Concrete Beams.* —In many of the beams in the 1911 and 1912 series, measurements were made to determine the amount of slip in the reinforcing bar at points other than the ends. It has been found that these load-slip relations, in view of the information derived from the pull-out tests, give valuable indications as to the bond stresses developed at different points along the length of the bar.

Load-slip curves for a few representative tests are given in Fig. 69 to 76. The curves in the figures are plotted in such a way as to indicate the amount and direction of movement of bar with respect to the adjoining concrete; they are plotted to the right or left with respect to the vertical line on the diagram which lies upon or nearest to the given point. Each horizontal division in the figures represents a slip of 0.001 in. The position of the loads, reinforcement, cracks, and other general features of the beams are indicated in the figures. The numbers within circles in the diagrams and on the photographs of the beams after failure are opposite the positions of the instruments during the test.

A survey of the load-slip curves will confirm the following observations: Slipping of the bar through the concrete was a phenomenon of all the beam tests in which observations were made. The slip was not much influenced by the form of the reinforcing bars or by variations in the points of application of the load. Slip-of-bar was quite pronounced over the middle region of the beams at loads well below those causing the appearance of the first visible cracks in the concrete; this probably indicates that tension cracks were present in the concrete some time before they became visible on the whitewashed surfaces of the beam. Since there is no bond stress due to beam action in this region of the beam, it is apparent that this is what has been termed anti-stretch slip in Art. 68. The load-slip curves for points on the beams outside the region affected by cracks are regular and show that slip increases continuously under an increasing load, or with the lapse of time under load; the curves for points within the region affected by cracks are influenced by the proximity of cracks in the concrete, and show frequent irregularities and reversals in direction. Slip-of-bar is greatest in the immediate vicinity of a crack. The indicated direction of slip in the vicinity of a crack depends upon which side of a crack the instrument was carried. The instruments on the near side of cracks somewhat removed from the middle of the beam and those approximately midway between the cracks near the middle of the beam generally show little or no slip of the bar. In general slip-of-bar is greatest on the far side of cracks near the load points. Slip-of-bar progresses from the load points or from the outermost cracks to the end of the beam at a rate nearly proportional to the increase of load. The rate of movement of the bar depends on various conditions, but it is approximately constant throughout this length after the end of the bar has slipped about 0.001 in.

It was found in these tests that the small openings in the concrete which were made in the bottom of the beams to admit the plug against which the extensometer worked, invited tension cracks, except near the ends of the beams. These cracks frequently formed directly under the plate carrying the extensometer and no doubt affected the character of some of the curves. In some cases it was impossible to tell (except as may be inferred from the form of the resulting curve) which side of the crack was carrying the instrument. It is clear that cracks cannot form in a reinforced concrete beam without more or less slip of bar, and that the instruments near cracks will indicate the greatest amount of slip at any point in their vicinity rather than the average amount over some length of bar. The curves for points near the ends, where the concrete is unbroken, are generally regular, and may be taken to represent the true load-slip relation at these points. Generally slip-of-bar was more pronounced at one end than at the other, although in some of the tests the two ends behaved nearly alike. In many of the tests it could be predicted from the observed differences in the slip measurements, some time before the completion of the test, at which end of the beam bond failure would occur.

Table 39 gives the loads corresponding to beginning of slip at the points where measurement was taken in the tests of 6-ft. beams in the 1912 series. For one-third point loading, slip of bar at the middle became appreciable at about the same load for all the beams, regardless of the kind of bar used; this load averaged about 3700 lb., corresponding to a tensile stress in the concrete of 200 lb. per sq. in., if the concrete be considered as taking the entire stress. For the same beams slip of bar began under the loads at about 4400 lb. Tension cracks in the concrete were not generally visible until the load was more than double this amount.

The load-slip curves for Beam No. 1056.2 (Fig. 69) may be considered typical of those obtained in the tests of 6-ft. beams reinforced with 1-in. plain rounds. In this test, slip was first observed in the middle portion of the beam at a load of about 4000 lb. At the middle, instrument (1), the bar is represented as moving toward the right, but the direction of movement would have been reversed had it happened that the instrument was carried on the other side of the crack. The slip at points south of the middle is greater in every instance than at the corresponding point at the north end of the beam. A general slipping of the bar occurred at the S. end at a load of about 14 000 lb.; this slip varied from about 0.0003 in. at the end to 0.005 in. at the south load point. The computed bond stress at this load was about 270 lb. per sq. in. Slipping at the north end became general at a load of about 16 000 lb. The beam failed by bond at the south end, as shown by the flattening of the curves at (3), (5), (7), and S.

In Beam No. 1052.6 (Fig. 60b and 69), in which auxiliary bars were used outside the load points, slip-of-bar in the outer thirds began at a higher load than in other similar beams without these bars. This was probably due to the auxiliary bars taking a part of the tensile stress during the early stages of the test and thus relieving the tensile stress in the main longitudinal reinforcing bar at the points where slip-of-bar was measured. These auxiliary bars had the effect of reducing the number of large cracks outside the load points (or preventing them entirely) as shown in the photograph in Fig. 60b. The prevention of large cracks seems to have had the effect of causing a more uniform distribution of the bond stress along the bar during the early stages of slip. Slip became general at both ends at a load of about 16 000 lb.; this corresponds to an average bond stress of 308 lb. per sq. in. The slip at this load varied from 0.0005 in. at the north end to 0.0012 in. 1 ft. from the supports. The pull-out tests (average of lowest two from each set) gave a bond resistance of 306 lb. per sq. in. at an end slip of 0.0005 in. The presence of the auxiliary bars had no influence on the maximum bond resistance.

The load-slip curves for the 1-in. square bars given in Fig. 74 show about the same general characteristics as those for beams with round bars. In testing Beam No. 1046.1 measurements of slip of bar were made at eleven points. The low loads at which slipping began at points (4), (6) and (8) is noteworthy; the reversal of direction in the curves for points (4) and (6) was due to the opening of cracks near these points

## TABLE 39.

## Applied Load at First Slip of Bar at Intermediate Points in Reinforced Concrete Beams.

All beams were tested on a 6-ft. span. The loads given in the table caused a slip of about 0.0002 in. at the points indicated.

| Beam No. | Longitudinal Reinforcement | Distance between Loads feet | Maximum Applied Load pounds | Applied Load at First Slip Indicated by Instruments at Different Points. (In Units of 1000 lb.) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | N. End | 33 in. N. of Middle | 24 in. N. of Middle | 12 in. N. of Middle | Middle | 12 in. S. of Middle | 24 in. S. of Middle | 33 in. S. of Middle | S. End |
| 1056.2 | 1 in. plain round | 2 | 19 700 | 16 | 14 | 8 | 4 | 6 | 6 | 8 | 15 | 12 |
| 1052.6 | 1 in. plain round | 2 | 18 800 | 14 | 14 | 12 | .. | .. | .. | 14 | 14 | 14 |
| 1057.2 | 1 in. plain round | 2½ | 19 100 | 13 | 12 | 8 | 4 | 6 | 8 | 4 | 12 | 13 |
| 1058.2 | 1 in. plain round | 3 | 14 000 | 10 | 10 | 8 | 4 | 4 | 4 | 4 | 11 | 11 |
| 1058.3 | 1 in. plain round | 3 | 18 600 | 13 | 10* | 6 | 4+ | .. | 4+ | 6 | 8* | 10 |
| 1059.2 | 1 in. plain round | 3½ | 17 700 | 10 | 10 | 5 | 10 | 2 | 4 | 4 | 8 | 10 |
| 1060.3 | 1 in. plain round | 4 | 30 000 | 19 | 21 | 10 | 10 | .. | 10 | 10 | 17 | 17 |
| 1046.1 | 1 in. plain square | 2 | 19 000 | 14 | .. | 4 | 4 | 3 | 3 | 2 | 7* | 11 |
| 1046.2 | 1 in. plain square | 2 | 19 300 | 12 | 10 | 4 | | .. | 4 | 6 | 10 | 10 |
| 1046.4 | 1 in. plain square | 2 | 20 000 | 11 | 8 | 8 | .. | 4 | 4 | 6 | 8 | 13 |
| 1047.1 | 1 in. twisted square | 2 | 21 800 | 15 | 14 | 6 | 6 | 4 | 6 | 6 | 13 | 13 |
| 1047.3 | 1 in. twisted square | 2 | 20 600 | 12 | 16 | 4 | 4 | .. | 4 | 6 | 19 | 17 |
| 1048.3 | 1⅛ in. cor. round | 2 | 28 300 | 20 | 18 | 2 | 2 | .. | 2 | 6 | 8 | 18 |
| 1048.4 | 1⅛ in. cor. round | 2 | 28 000 | 13 | 15 | 4 | 4 | 2 | 2 | 4 | .. | 10 |

+ Instruments were 18 in. from the middle of the beam.

* Instruments were 30 in. from the middle.

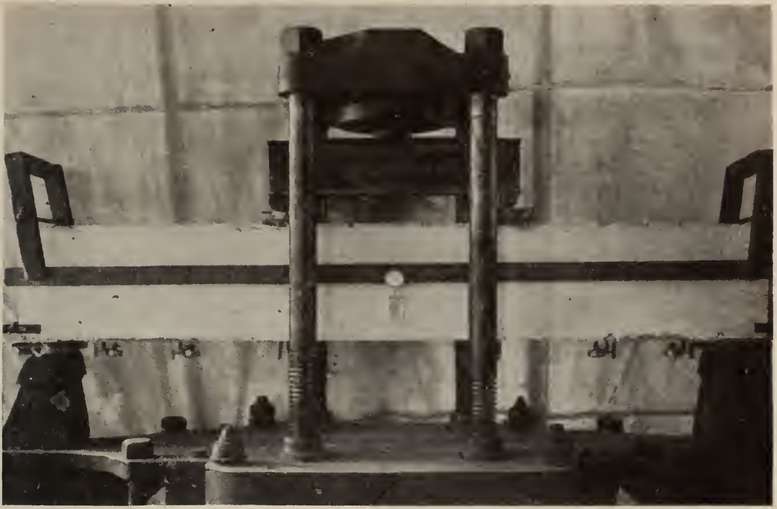
at a load of about 7000 lb. In spite of the large amount of slip at point (8), failure occurred in the beam at the opposite end. The curves for corresponding points at the opposite ends of Beam No. 1046.2 show an unusual uniformity. It will be seen by reference to Fig. 62b that cracks had formed near each of the instruments (2), (3), (4), and (5); the symmetry of the curves is due to the fact that the instruments were in each case carried by the concrete outside the crack. The curves for Beam No. 1046.4 show the bar to have been slipping in one direction throughout nearly two-thirds of its length; however, the presence of a crack at the middle makes the indicated direction of movement of the bar at this point depend upon which side of the crack the instrument happened to be carried.

Slip-of-bar measurements at intermediate points were made on two beams reinforced with 1-in. twisted square bars (see Fig. 75). The bond stresses developed were lower than those found with beams reinforced with 1-in. plain rounds. The curves for these tests exhibit about the same characteristics as in the beams with plain bars. Vertical and diagonal cracks formed during the progress of the tests with about the same frequency as in the other beams. The nearly vertical direction of the curve for instrument (2) in Beam No. 1047.1 is due to the presence of a crack at this point. The load-slip curves for the north ends of Beams No. 1047.1 and 1047.3 and for points 6 in. from the north end showed a large increase of slip at an applied load of about 15,000 lb., immediately after slip at these points became appreciable.

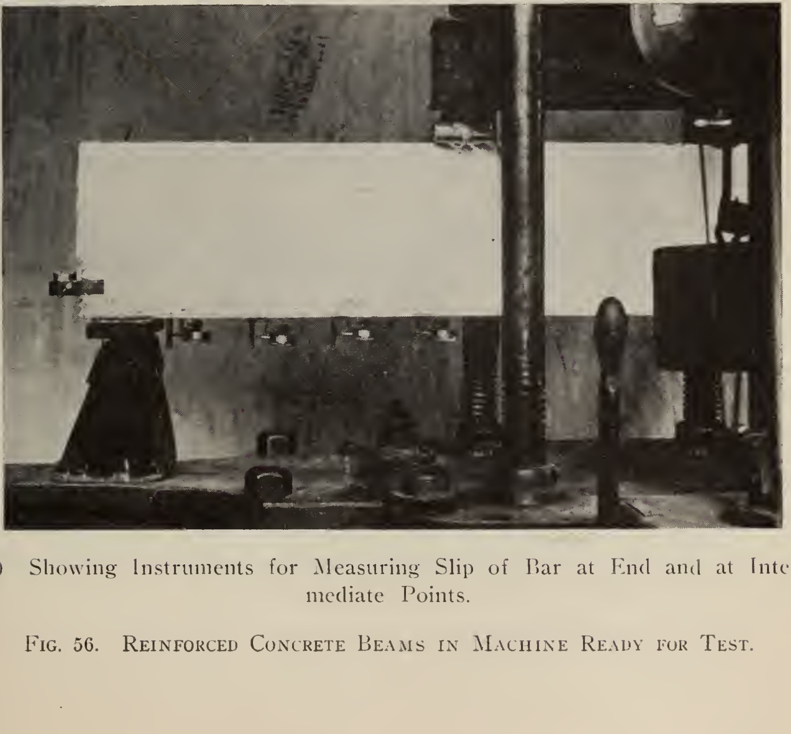
Measurements of slip of bar at intermediate points were taken on four beams in which 1⅛-in. corrugated rounds were used for longitudinal reinforcement. Two beams (1048.3 and 1048.4) were tested on a 6-ft. span and two (1049.2 and 1049.3) were tested on a 10-ft. span. Load-slip curves are given in Fig. 75 and 76. Slip began at points near the middle of the beam at approximately the same loads as in the beams reinforced with plain bars. In the outer thirds, after slip began, the corrugated bar moved at a slower rate under a somewhat higher bond stress than the plain bars. Cracks in the beams reinforced with corrugated bars occurred with about the same frequency as in the beams with plain bars. The curves for points (2), (3), (4), and (5) on Beam No. 1048.3 (see Fig. 76) illustrate the effect of cracks on the amount of movement of the bar at the various points with respect to the adjacent concrete. Instrument (2) was carried by the concrete outside the crack, hence registered a very large movement; instrument (3) was carried by the concrete inside the crack, hence showed practically no slip. The crack near instrument (4) crossed the bottom of the beam in such a way that the instrument was carried by the concrete on both sides of the crack. Instrument (5) was about 3 in. outside a crack until near the maximum load, hence indicated about the normal amount of slip. The instruments at the ends and at points (6) and (7) were undisturbed by cracks and hence more nearly indicate the relation between load and the amount of slip, which may be expected under these conditions.

The influence of span length on the slip of the reinforcing bars at internal points is shown for typical tests in Fig. 71, 72 and 73. The span varied from 5 to 10 ft.; the points of application of the load are shown in the figures. For convenience of reference the curves for Beam No. 1056.2 (6-ft. span) are repeated here to the same scale as used in the beams of other spans. It should be borne in mind that in general the 8 and 10-ft. beams failed in tension. The curves for Beam No. 1051.2 (5-ft. span) are about the same as found for the 6-ft. beams; slip became appreciable near the middle at a load of about 4000 lb. and gradually extended toward each end. The bond stresses developed in the 5-ft. beams at first end slip and at the maximum loads are about the same as for the similar beams of 6-ft. span. Slip began near the middle of the 7-ft. beams at a somewhat lower load than in the 6-ft. beams; slip near the ends of the bars began at a bond stress a little higher than in the 6-ft. beams. In the 8-ft. beams slip began near the middle at about the same load as in the 7-ft. beams. The slip at the ends of the bars did not amount to more than 0.001 in. in any case. In the 10-ft. beams loaded at the one-third points, slip of the bar became apppreciable near the middle at applied loads below 2000 lb. Slip began at points 3 in. inside the supports at about the same load as in the 6-ft. beams. At the ends of the bars slip was only beginning at the maximum load.

The 10-ft. beams loaded at two points 6 ft. apart behaved in much the same way as the 6-ft. beam of the same kind loaded at the one-third points. This is as was expected, since the relation of the dimensions of the beam (as far as bond and web stresses are concerned) were the same in both cases. These tests show that local slip of bar near the middle of the beam is found at loads which are lower than those at which tension cracks appear in the concrete; this probably indicates that cracks were present some time before they became visible, and that beginning of slip was probably coincident with the opening of the most minute tension cracks.

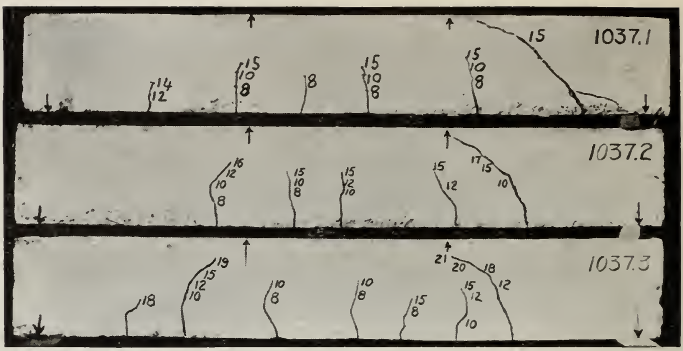


(a) Showing Instruments for Measuring Center Deflection and Slip of Bar at Intermediate Points.

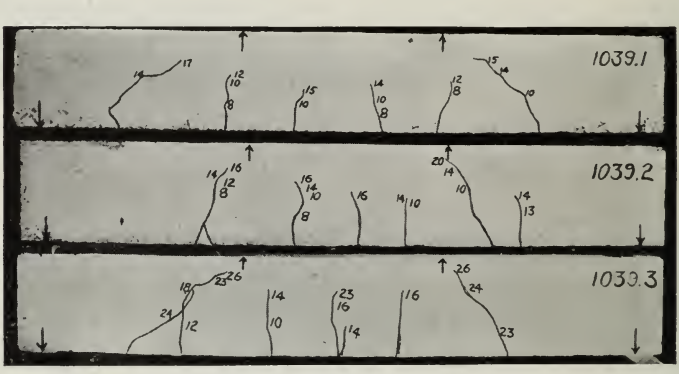


(b) Showing Instruments for Measuring Slip of Bar at End and at Intermediate Points.

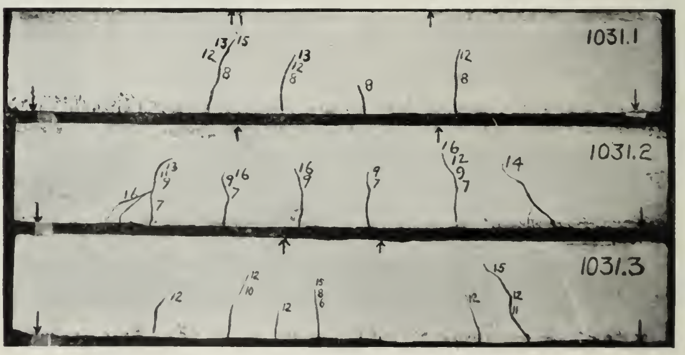
FIG. 56. REINFORCED CONCRETE BEAMS IN MACHINE READY FOR TEST.



(a) Group 1. One 1-in. Plain Round Bar; No Stirrups.

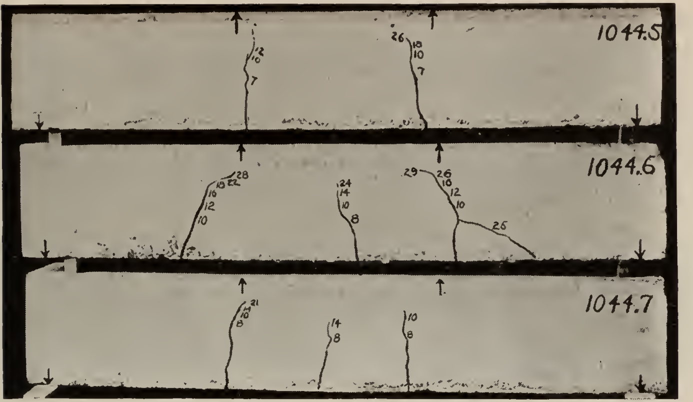


(b) Group 2. One 1-in. Plain Round Bar; ¼-in. Stirrups.

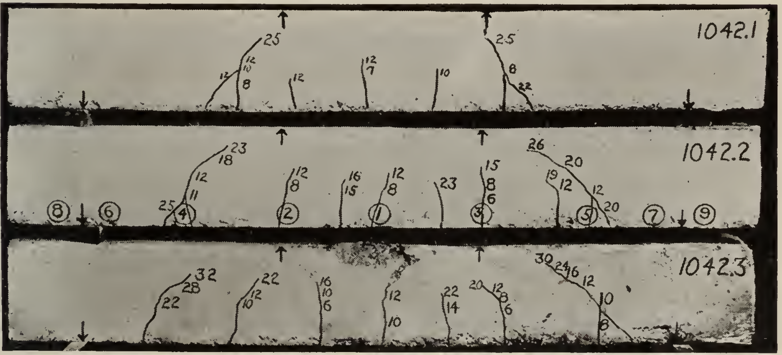


(c) Group 3. One 1-in. Plain Round Bar; ½-in. Stirrups.

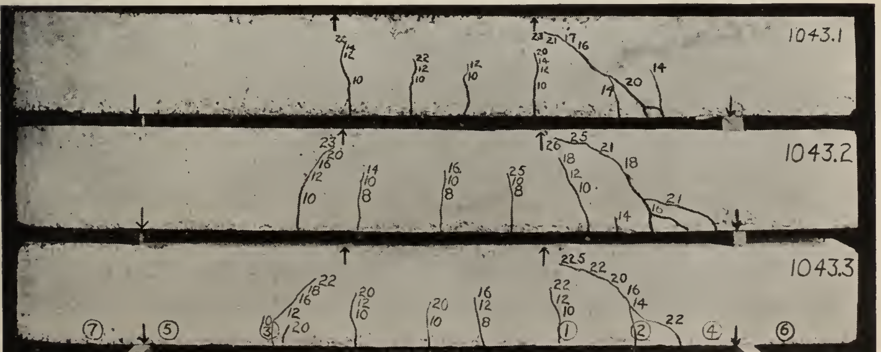
FIG. 57. REINFORCED CONCRETE BEAMS AFTER TEST—1911 SERIES.



(a) Group 6. One 1-in. Plain Round Bar; 4 in. of Concrete Below Bar; No Stirrups.

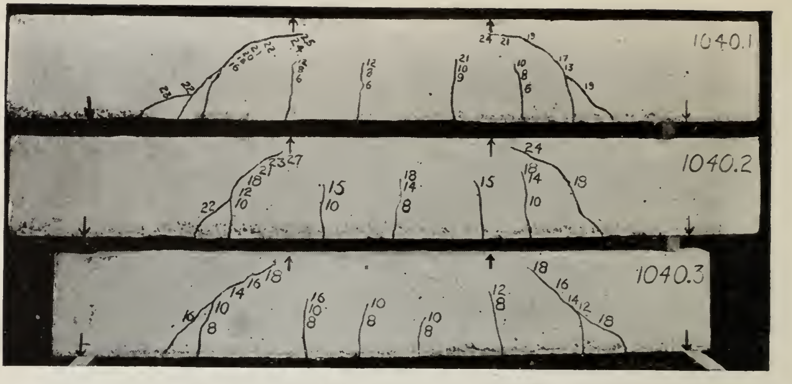


(b) Groups 7 and 13. Ends of Beams Overhanging Supports 9 in.

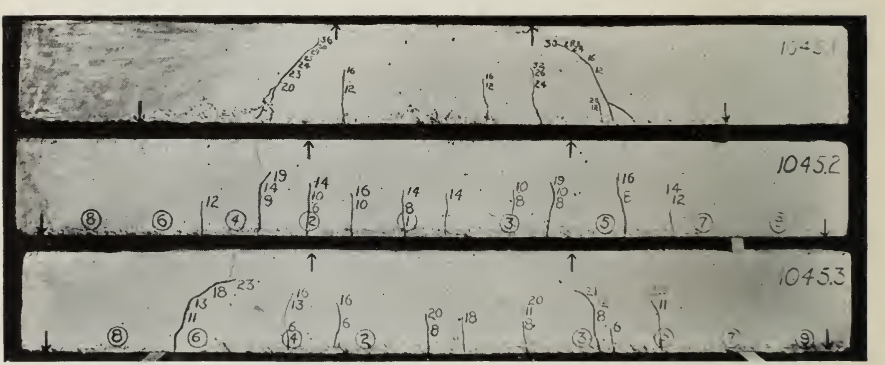


(c) Groups 8 and 10. Ends of Beams Overhanging Supports 15 in.

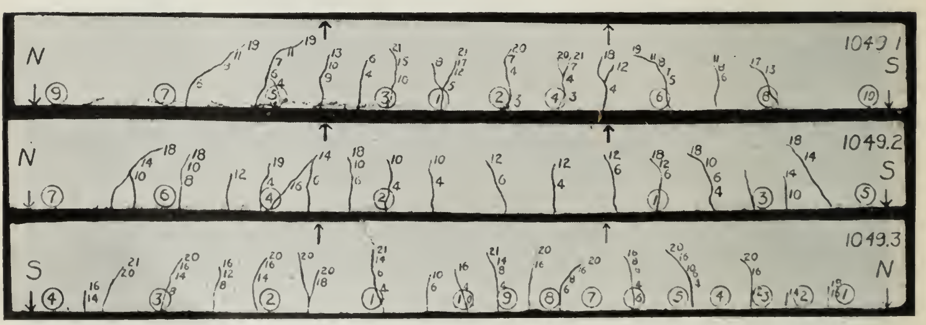
FIG. 58. REINFORCED CONCRETE BEAMS AFTER TEST—1911 SERIES.



(a) Groups 1 and 13. Beams No. 1040.1 and 1040.2 were Each Reinforced with One 1⅛-in. Corrugated Round Bar.

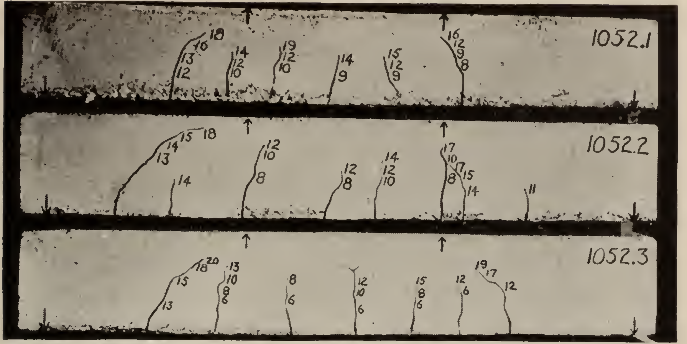


(b) Groups 10 and 11. One 1¼-in. Plain Round Bar.

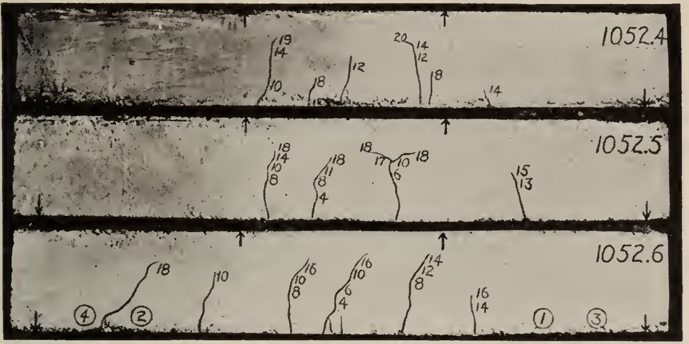


(c) Group 34. One 1⅛-in. Corrugated Round Bar; Span 10 ft.

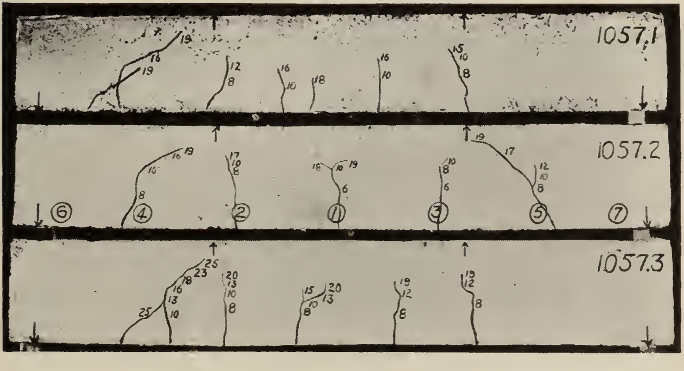
FIG. 59. REINFORCED CONCRETE BEAMS AFTER TEST.



(a) Group 14. One 1-in. Plain Round Bar.

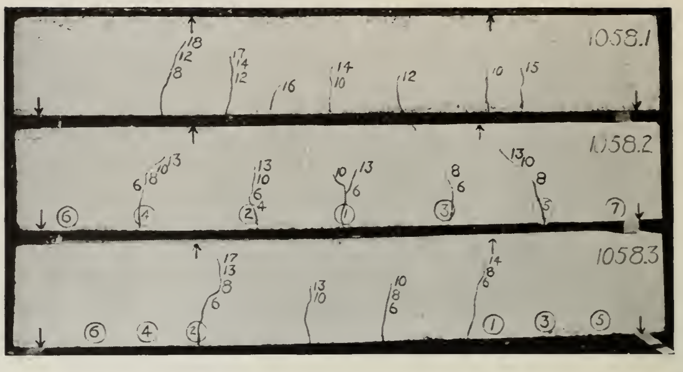


(b) Group 16. One 1-in. Plain Round Bar; Auxiliary Bars at Each End of Beam.

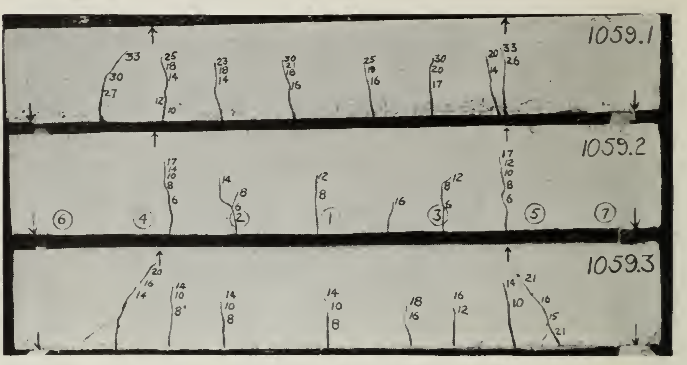


(c) Group 17. One 1-in. Plain Round Bar, Span 6 ft.; Loads 2½ ft. Apart.

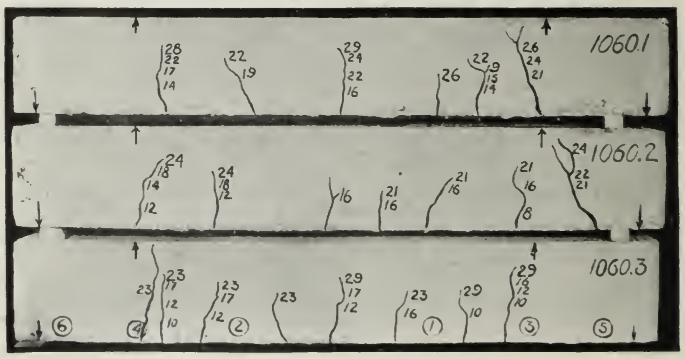
FIG. 60. REINFORCED CONCRETE BEAMS AFTER TEST—1912 SERIES.



(a) Group 18. One 1-in. Plain Round Bar, Span 6 ft.; Loads 3 ft. Apart.

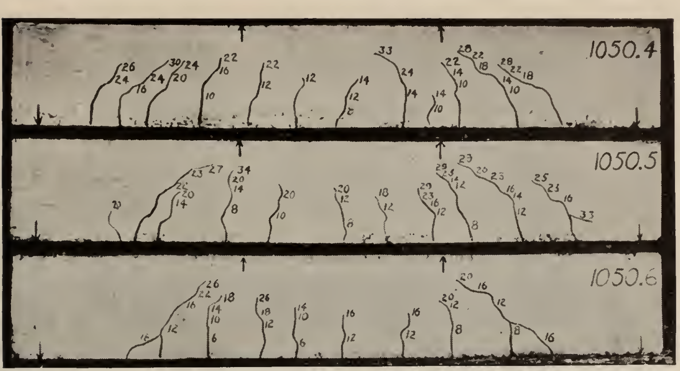


(b) Group 19. One 1-in. Plain Round Bar, Span 6 ft.; Loads 3½ ft. Apart.

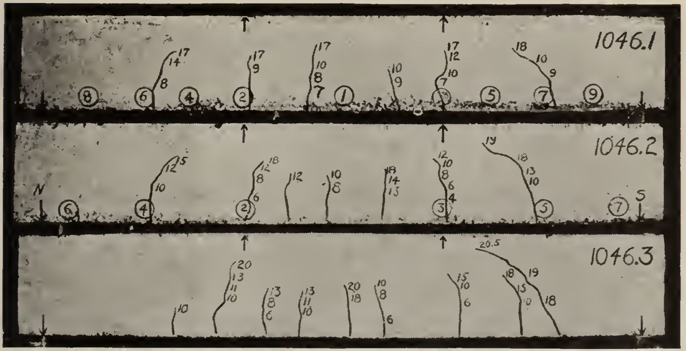


(c) Group 20. One 1-in. Plain Round Bar, Span 6 ft.; Loads 4 ft. Apart.

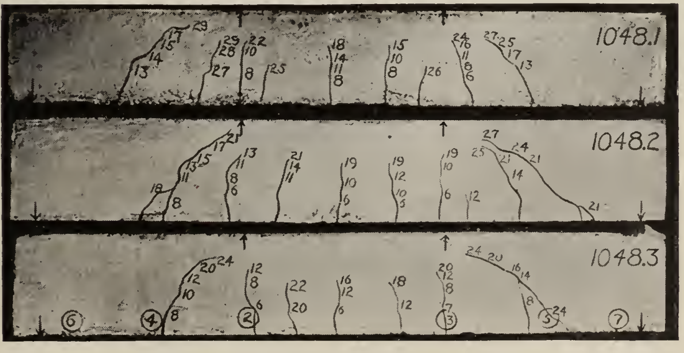
FIG. 61. REINFORCED CONCRETE BEAMS AFTER TEST—1912 SERIES.



(a) Group 27. Four ⅝-in. Plain Round Bars.

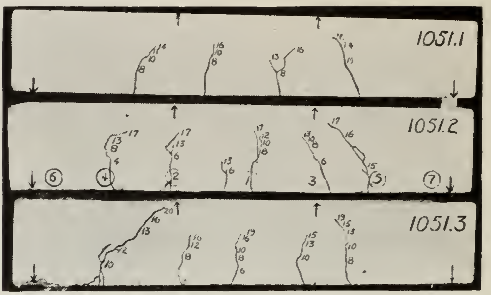


(b) Group 29. One 1-in. Plain Square Bar.

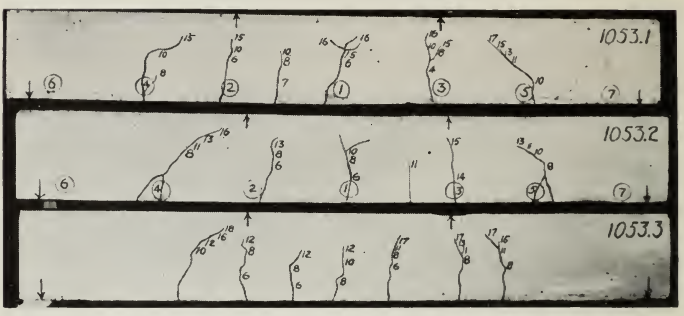


(c) Group 32. One 1⅛-in. Corrugated Round Bar.

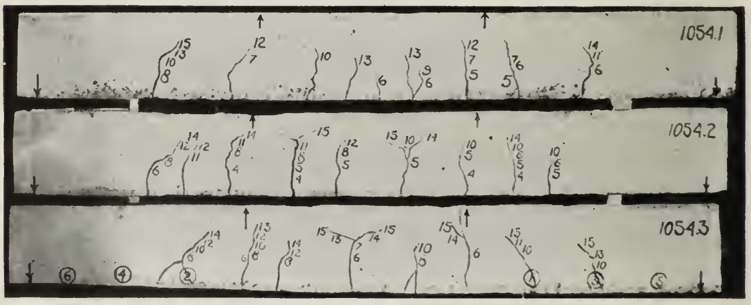
FIG. 62. REINFORCED CONCRETE BEAMS AFTER TEST—1912 SERIES.



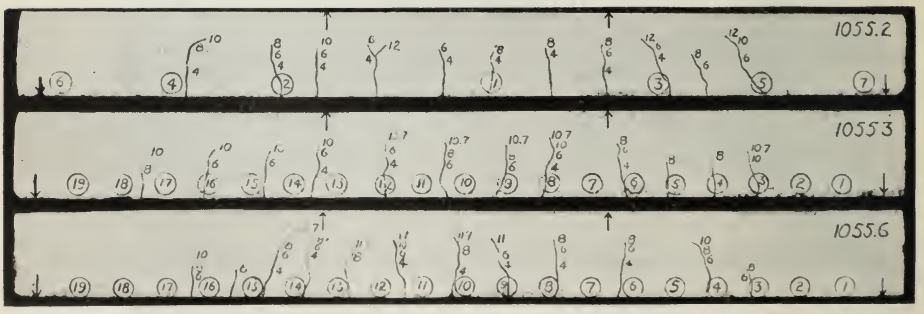
(a) Group 21. One 1-in. Plain Round Bar; Span 5 ft.



(b) Group 22. One 1-in. Plain Round Bar; Span 7 ft.



(c) Group 23. One 1-in. Plain Round Bar; Span 8 ft.



(d) Groups 24 and 25. One 1-in. Plain Round Bar; Span 10 ft.

FIG. 63. REINFORCED CONCRETE BEAMS AFTER TEST—1912 SERIES.

The curves for the groups of 6-ft. beams in which the loads were applied at varying distances apart exhibit some distinctive characteristics (see Fig. 69 and 70). These tests show with unusual distinctness the progress of slip of the bar from the middle toward the ends of the beams. There are many illustrations in this group as to the effect of cracks on the slip of the bar. The curves for point (1) in Beams No. 1056.2 and 1057.2 show the amount of slip which may be expected very close to a crack at the middle of a beam; the curves for points (1) and (2) in Beam No. 1060.3 show that little or no slip occurred at points midway between two cracks in the portion of the beam where vertical shear was absent; the curves for points (2) and (4) in Beam No. 1057.2 and for points (2) and (3) in Beam No. 1056.2 show that slip of bar may be erratic at points on the near side of a crack. Slip at points outside the loads was abnormally large on the far side of the cracks.

95. *Distribution of Tensile Stress along Reinforcing Bar.*—The usual analysis of the stresses in a reinforced concrete beam indicates that in a beam loaded as in these tests (disregarding the weight of the beam) the tensile stress in the reinforcement is uniform between the loads and decreases at a uniform rate from the load points towards the ends, becoming zero at the supports. The slip of bar measurements which were made in many of the reinforced concrete beam tests gave some indication of the bond stresses being developed as long as the amount of slip was small and showed the general variations in bond stresses, but it will be recognized that after the slip of bar has reached an amount approximating that corresponding to the maximum bond resistance, such measurements are of no further value in indicating the amount of bond stress being developed. There are indications that the amount of slip corresponding to the maximum bond resistance of plain bars was not nearly so well defined in the beams as it was in the pull-out tests. In order to learn the amount of bond stress being developed over any given length of bar in a reinforced concrete member, we must first determine the exact stress developed in the reinforcing bar at each point over this length for a given load. If we can accurately determine the variations in the steel stress the bond stress will also be known, since the bond stress developed in any length of bar represents the change in tensile stress over that length.

In the tests of three beams in the 1912 series careful measurements were made to determine the stress in the longitudinal reinforcing bar at frequent intervals throughout the span length. These tests will be discussed in detail in the following paragraphs. All of these beams were 8 by 12 in. in section, 10 in. deep to the center of the reinforcing bar; they were tested by loads applied at the one-third points of a 10-ft. span. Beams No. 1055.3 and 1055.6 were each reinforced with one 1-in. plain round bar; Beam No. 1049.3 was reinforced with one 1⅛-in. corrugated round bar. In Beam No. 1049.3 the tensile stresses were measured at 10 6-in. gauge lengths over only one-half the span length; in the other two tests the stresses were measured at 19 gauge lengths over the entire span. In all the tests the gauge lines formed a continuous series. A non-fixed strain gauge was used in determining the change in stress.* Load was applied in increments of about 2000 lb. In the test of Beam No. 1055.3 the deformations in the reinforcing bar were measured at each increment of load; in the other two tests each load was released and measurements were taken to determine the residual stresses, before applying a higher load. In Fig. 64 to 67 the distribution of bond and steel stresses are shown for different loads on the beams. In these figures the ordinates or abscissas to the curves show the amount of the stress in the reinforcing bar at any point for the loads shown, while the slope of these curves is proportional to the bond stress being developed. In Fig. 64 and 65 the curves for Beams No. 1055.3 and 1055.6 have been plotted by averaging the stresses at the corresponding gauge lines at the two ends of the beam and plotting the resulting values as the stresses for one-half the span length. This probably accounts in some measure for the greater regularity in the curves for beams reinforced with plain bars. It should be pointed out here that the values for steel stresses were determined from measurements over a 6-in. gauge length, and it is evident that the stress may vary considerably over this length of bar, especially at points where the bar has not slipped an amount corresponding to the maximum bond resistance. The stresses given on the diagrams are the averages over the entire gauge length. The plotted points indicate the location of the middles of the gauge lengths. It seems probable that a 6-in. gauge length was too long to accurately locate the points of maximum bond stress in the region affected by cracks, but it was not practicable to use a shorter gauge length. Of course, the usual errors of observation may be expected to affect measurements of this kind. The presence of tensile cracks in the concrete was another disturbing factor.

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*For a detailed discussion of this strain gauge see "Tests of Reinforced Concrete Buildings Under Load," by Arthur N. Talbot and Willis A. Slater, University of Illinois Engineering Experiment Station Bulletin No. 64, 1918. Also, "Use of the Strain Gage in Testing Materials," by Willis A. Slater and Herbert F. Moore; "Proceedings of the American Society for Testing Materials," 1918.

Table 40 gives some of the significant bond stresses for these beams as computed by the usual method and as determined from the steel stress measurements. In this table and in the figures only the stresses due to the applied loads have been considered. The first column of observed stresses are generally the average stresses over a length of about 12 in. in the portion of the beam about 4 to 16 in. outside the load points. The other column shows the average observed stresses over a length of 9 to 15 in. at the ends of the beam. The photographs in Fig. 59 and 63 show the location of cracks in these beams and their growth with increase of load. The numbers inside the circles at the level of the reinforcing bars are opposite the points at which measurements of steel stress were made.

## TABLE 40.

### DISTRIBUTION OF BOND STRESS IN REINFORCED CONCRETE BEAMS.

Beams 8 by 12 in. in section, 10 in. deep to center of reinforcing bar. Loaded at the one-third points of a 10-ft. span.

All beams failed by excessive tensile stress in the reinforcing bars.

| Beam No. | Size and Kind of Bar | Age at Test | Applied Load on Peam pounds | Average Computed Bond Stress lb. per sq. in. | Observed Bond Stress | |
|---|---|---|---|---|---|---|
| | | | | | Over Region just Outside of Load Points* lb per sq. in. | Near Ends of Beam† lb. per sq. in. |
| | | | 2 000 | 38 | 100 | 16 |
| | | | 4 000 | 76 | 125 | 34 |
| 1055.6 | One 1-in. Plain Round | 2 yr. | 6 000 | 114 | 191 | 36 |
| | | | 8 000 | 152 | 226 | 64 |
| | | | 10 000 | 190 | 201 | 117 |
| | | | 11 700 | 222 | 165 | 238 |
| | | | 2 000 | 38 | 48 | 15 |
| | | | 4 000 | 76 | 75 | 54 |
| 1055.3 | One 1-in. Plain Round | 2 yr. | 6 000 | 114 | 155 | 95 |
| | | | 8 000 | 152 | 141 | 100 |
| | | | 10 000 | 190 | 200 | 130 |
| | | | 10 700 | 203 | 140 | 156 |
| | | | 2 000 | 34 | 80 | 20 |
| | | | 4 000 | 68 | 137 | 45 |
| | | | 6 000 | 102 | 226 | 95 |
| | | | 8 000 | 135 | 285 | 135 |
| | | | 10 000 | 170 | 250 | 150 |
| 1049.3 | One 1⅛-in. Corrugated Round | 13 mo. | 12 000 | 204 | 315 | 150 |
| | | | 14 000 | 236 | 350 | 225 |
| | | | 16 000 | 270 | 385 | 260 |
| | | | 18 000 | 306 | 400 | 290 |
| | | | 20 000 | 338 | 450 | 315 |
| | | | 21 000 | 355 | 200 | 360 |
| | | | 21 900 | 370 | ... | 390 |

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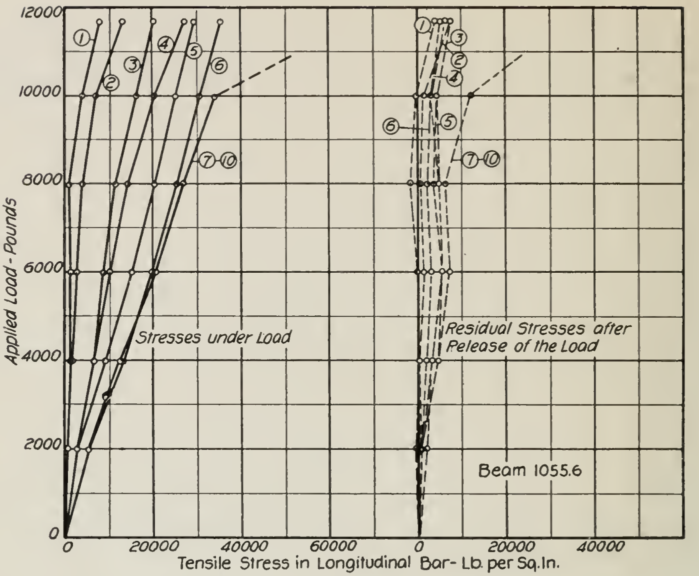
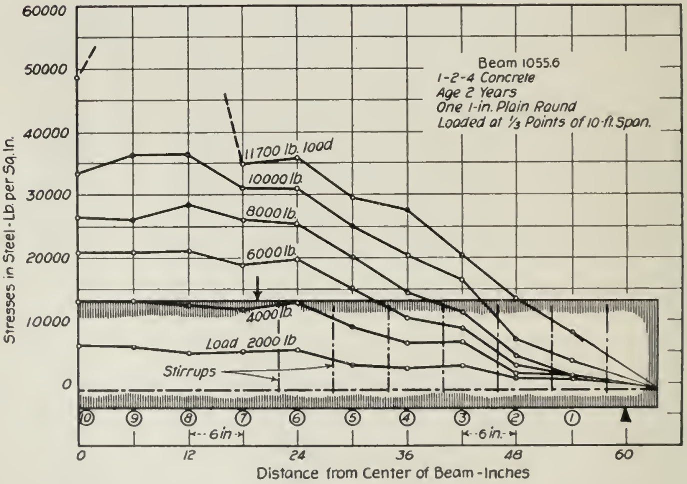
* These stresses are, in general, the average bond stresses developed over a length of about 12 in. in the portion of the beam about 4 to 16 in. outside the load points.

† The average observed stress over a length of 9 to 15 in. at the ends of the beam.



(a) Load-Stress Curves for Longitudinal Bar.



(b) Distribution of Tensile Stress in Longitudinal Bar.

Fig. 64. Diagrams from Test of Beam No. 1055.6. One 1-in. Plain Round Bar; Span 10 ft.

Beam No. 1055.6, reinforced with one 1-in. plain round bar, failed by tension in the steel, although slip at each end of the bar at the maximum load amounted to about 0.002 in. From measurements of steel stress in this test it is seen that there is a wide variation in the bond stresses developed at a given load along the bar through the region where beam bond stress is present. At a load of 2000 lb. the steel stress through the middle third of the beam was nearly constant, with evidence of the highest stress about 6 in. outside the load point. It is significant that the photograph of this beam, Fig. 63d, shows a crack at this point on both ends of the beam. This crack was not observed until a load of 4000 lb. had been applied, but it is evident that the measurements at 2000 lb. load indicate that stresses higher than usual were being developed at these points. At a load of 2000 lb. the average bond stress developed over the 6-in. length between the gauge lines (6) and (5) was about 100 lb. per sq. in.; a stress of about the same value was being developed between gauge lines (3) and (2), and it is significant that cracks opened at these points at low loads. It seems probable that the highest bond stress developed at a load of 2000 lb. was considerably higher than 100 lb. per sq. in., since the measurements over 6-in. gauge lengths probably did not show the most rapid changes in steel stress. Over the 15-in. length from (2) to the end of the beam the bond stress at this load was about 16 lb. per sq. in. At this load the computed bond stress was 38 lb. per sq. in. With a load of 4000 lb. the average bond stress developed over the 12-in. length from (6) to (4) was 125 lb. per sq. in.; from (3) to (2) about 200 lb. per sq. in.; from (2) to the end, 34 lb. per sq. in. The computed bond stress due to an applied load of 4000 lb. was 76 lb. per sq. in. In other words, an applied load of 4000 lb., which developed a steel stress in the middle third of about 13 000 lb. per sq. in., developed a bond stress over certain regions of the span length equivalent to 90% of the computed bond stress at the maximum load carried by the beam. A load of 6000 lb., which produced a tensile stress in the bar at the middle third of the span of about 20 000 lb. per sq. in., developed a bond stress between (6) and (5) of 191 lb. per sq. in.; at the end of the beam the stress at this load was 36 lb. per sq. in. A load of 8000 lb. produced a bond stress from (6) to (4) of 226 lb. per sq. in., and 64 lb. per sq. in. at the end: the computed bond stress at this load was 152 lb. per sq. in. With a load of 10 000 lb. on the beam the average bond stress over the length of 12 in. from (6) to (4) was 201 lb. per sq. in., or lower than was found in this region at a load of 8000 lb.; from (3) to (2) the indicated load stress was 390 lb. per sq. in., but it is possible that the points plotted for steel stress are somewhat erratic. There was a marked shifting of the stresses following a load of 10 000 lb. At the maximum applied load of 11 700 lb. the bond stress at points just outside the load had fallen still lower, while the stress near the ends of the beam was more than double what it was under a load of 10 000 lb. At the maximum load the highest bond stress was developed between points (4) and (2) and amounted to about 294 lb. per sq. in. It will be seen from Fig. 64b that the region of maximum bond stress was thrown from near the load point toward the end of the beam as the loading continued. It should be borne in mind that this beam was 2 years old at the time of test and did not fail by bond. The values in Table 40 show some of the changes which the bond stresses underwent as the load was increased. The bond stress near the load point did not show any marked increase after a load of 6000 lb., while at the end there was a continuous increase up to the maximum applied load. Still higher bond stresses could have been developed near the ends of the beam had the bar been of steel with a higher yield point. Fig. 64a shows that the residual stress in the reinforcing bar upon release of load was about proportional to the stress under load for the lower loads. After a load of 6000 lb. there is no further increase in the residual stresses. At loads near the maximum the residual stresses were nearly equal at all points except over the gauge lines nearest the ends.

Beam No. 1055.3, reinforced with one 1-in. plain round, was similar to No. 1055.6. In this test the loads were not released. Fig. 65 shows the distribution of the tensile stress in the reinforcing bar for different loads; the plotted points are the averages of the measurements at corresponding points in the two ends of the beam. This beam did not show as wide variations of bond stress as were found in the test of Beam No. 1055.6 discussed above, but many features of the tests are similar. It is noteworthy that the bond stresses developed near the ends of the beam were never more than about 75% of the computed bond stress. The bond stresses near the end are shown to be low by the fact that at the maximum load the end slip of the bar was only 0.0003 in. This beam failed by tension in the reinforcing bar. A further examination of the results of these tests on beams of similar make-up, as shown in the tables and diagrams, will indicate how much variation may be expected in the action of beams which are of the same dimensions and made of the same materials.

Beam No. 1049.3 was reinforced with one 1⅛-in. corrugated round bar of high-carbon steel. This beam was tested at the age of 13 mo. on a 10-ft. span. Measurements were made to determine the variation in stress in the reinforcing bar at 10 points along one-half the span length and to determine the amount of slip of bar at five points of the other half of the span. A complete set of observations were taken after the application of each load and another upon releasing each load. The observed values of steel stress and slip of bar are plotted in Fig. 66 and 67. It will be noted that the load-stress curves from this test show much greater irregularities than were found in the other tests. This is probably largely due to the fact that in this test the points are the result of a single measurement, whereas in the tests described above the points represent the average of the values from both ends of the beam. At a load of 4000 lb., about one-fifth the maximum applied load, a bond stress of 137 lb. per sq. in. was being developed over the region (6) to (4) and an average of about 45 lb. per sq. in. over the 21-in. length at the end of the beam. The variation of bond stress throughout the test can be studied from the diagrams in Fig. 67 and from the values in Table 40. The observed bond stress over the region just outside the load point continued to rise until it reached 450 lb. per sq. in. at a load of 20 000 lb. Near the end of the beam the bond stress continued to rise until it reached 390 lb. per sq. in. at the maximum load of 21 900 lb. The highest bond stress observed over a considerable length of the bar in this test occurred be-

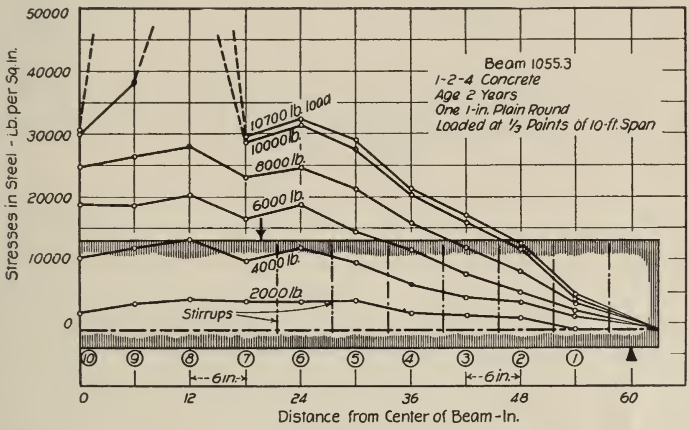
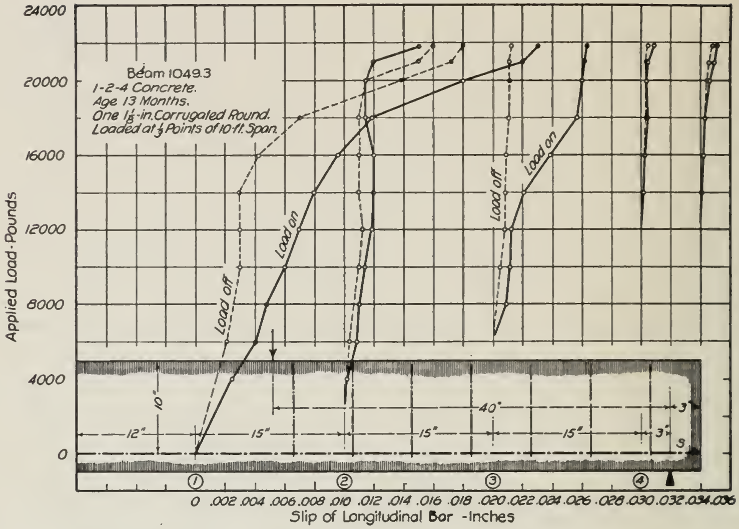
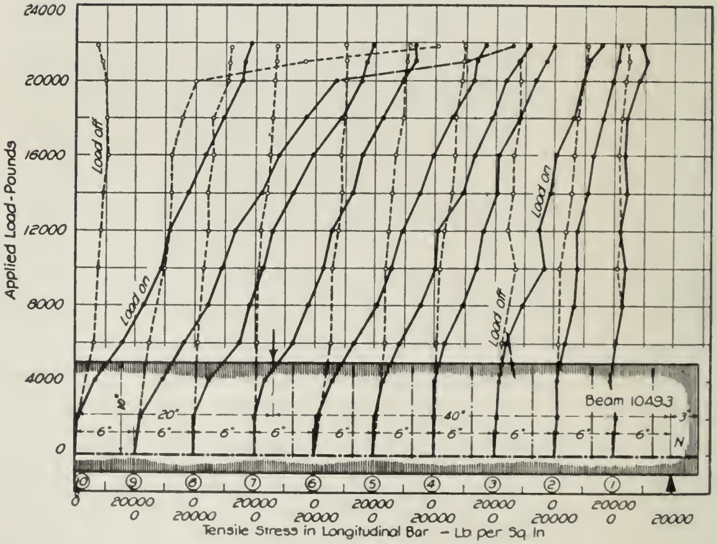


FIG. 65. DISTRIBUTION OF TENSILE STRESS IN BAR IN BEAM NO. 1055.3. ONE 1-IN. PLAIN ROUND BAR; SPAN 10 FT.

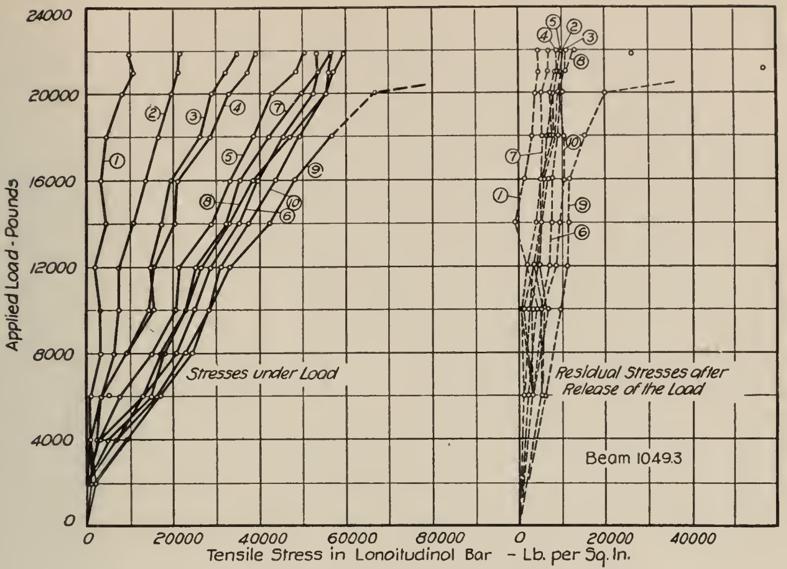


(a) Load-slip Curves for Longitudinal Bar.

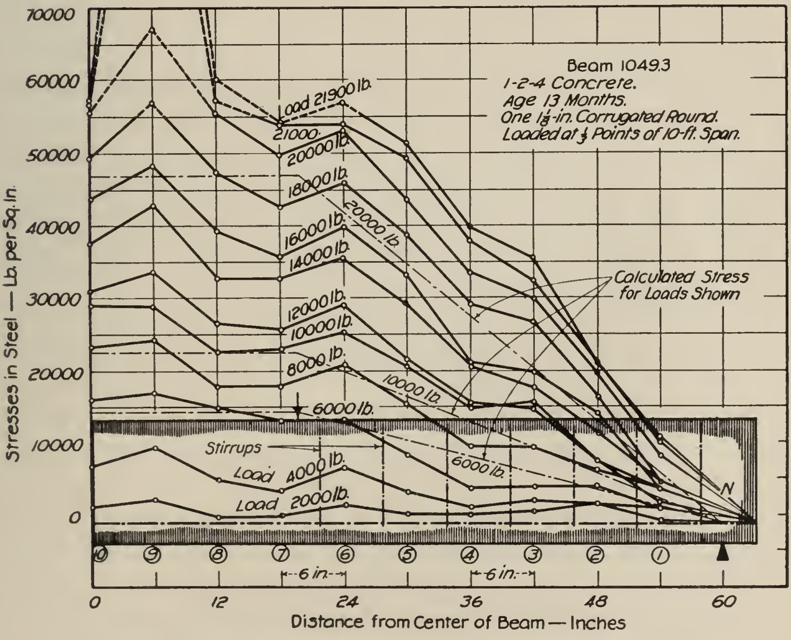


(b) Load-stress Curves for Longitudinal Bar.

Fig. 66. Diagrams from Test of Beam No. 1049.3. One 1⅛-in. Corrugated Round Bar; Span 10 ft.



(a) Load-stress Curves for Longitudinal Bar.



(b) Distribution of Tensile Stress in Longitudinal Bar.

FIG. 67. DIAGRAMS FROM TEST OF BEAM NO. 1049.3. ONE 1⅛-IN. CORRUGATED ROUND BAR; SPAN 10 FT.

tween (3) and (1) at the maximum load, and amounted to 580 lb. per sq. in. It is evident that stresses as high as this must have occurred at all points along the bar from the load point to (3) at some stage of the test, but the measurements were not sufficiently refined or the observations taken at sufficiently frequent intervals to detect such changes. This makes it apparent that there must be very rapid changes in the distribution of bond stress along the bar in a test of this kind. Fig. 66a shows the slip of bar at five points in Beam No. 1049.3. The amount of slip measured at any point depends to a large extent upon the proximity of cracks in the concrete and the load-slip curves may be expected to show considerable variation due to this cause. The residual slip after release of load is equal to about one-half the total slip up to a load of about 16 000 lb. The residual tensile stresses in the bar exhibit about the same characteristics as the residual slip-of-bar measurements. It is noteworthy that the residual tensile stresses over the larger portion of the length under observation did not increase much with increase of load after a load equal to about one-half the maximum had been applied. It is probable that a period of rest following the release of a load would have shown a material reduction in the residual stresses measured.

These tests show that the actual bond stresses developed in beams of this kind vary widely at all stages of the tests and that the bond stress calculated in the usual way represents the average stress, but does not indicate the extremes of stress in different portions of the span where beam bond stresses are present. The actual bond stresses developed varied from less than one-half to more than twice the calculated bond stress as determined in the usual manner.

As was pointed out in Art. 68, "Phenomena of Beam Tests," these tests indicate that at the early loads which develop the maximum beam bond resistance over a short length of bar outside the load points in beams in which the distance from the load to the support is as much as four times the effective depth, the bond stress developed near the ends of the bar may not be more than, say, 15% of the maximum bond resistance. The ratio of these stresses was not definitely determined, but the measurements of the tensile stress in the steel and the slip of bar indicate that the ratio given is approximately correct. It is probable that for longer beams the value would be found to be lower; in shorter beams the bond stress at the end of the bar when the maximum bond resistance was first developed outside the load points was probably as high as 40% of the maximum bond resistance.

In Fig. 68 is shown the distribution of bond and tensile stresses which may be expected in a reinforced concrete beam which fails by bond under two symmetrical loads. The curves indicate the changes which the stresses at different points undergo as the load is increased. In the region between the loads the tensile stress in the bar would be constant (disregarding the weight of the beam and the effect of anti-stretch slip) as indicated by the horizontal lines in the left portion of the figure. Between the load point and the support, the tensile stress at any point would be represented by the ordinates to the curves and the oblique lines, and the bond stress by the slope of these lines. The diagram indicates that the maximum beam bond resistance is first developed a short distance outside the load point at a load of 40% of the maximum load. The region over which the maximum bond resistance is being developed by a given load is indicated by the heavy solid lines. Since a bond stress much higher than the average is developed over a portion of the span, it is evident that at other points the bond stress must be less than the average. For a load 40% of the maximum the computed tensile stress outside the load point would be indicated by the ordinates to the dotted line KS, and the bond stress by the slope of this line, while the actual distribution of the tensile and bond stresses would be represented somewhat as shown by the curved solid line KS. This curve indicates that for a short distance outside the load point and near the support the bond stress is small, and that over a short length the maximum bond resistance is being developed. As the load is increased the region just outside the load point over which the bond stress is small is gradually extended and at the same time the portion of bar over which the maximum bond resistance is being developed is lengthened and pushed nearer the support. At the load causing failure by bond the actual stress in the bar is somewhat as shown by the line TLOS. In other words, the maximum bond resistance developed is represented to scale by MR instead of by LR as computed; and at failure this maximum stress was being developed over the length of bar PS, instead of over the length RS, as assumed by the usual methods of calculation. The parabolic line TS represents the theoretical stresses for a uniformly distributed load which produces the same maximum tensile stress at the mid-span as that produced by the concentrated loads shown. This curve suggests that a uniform load gives a more favorable condition for bond than concentrated loads. This is also evident from other considerations.

It is not intended that Fig. 68 should give a quantative indication of the distribution of stresses in any particular reinforced concrete beam. The presence of anti-stretch slip and the bond stresses due to this cause will greatly modify this distribution of stresses over certain regions of the span. However, it is clear that this somewhat idealized sketch does suggest the explanation of the discrepancy between the computed bond stresses in beam tests and the bond resistance found in other ways. The more accurate determination of the actual distribution of these stresses for beams of different make-up is a proper subject for further experimental study.

96. *Relation of Slip of Bar to Diagonal Tension Cracks.*—About 80 per cent of the beams reported in this bulletin failed in bond or an obvious combination of bond and diagonal tension. In many tests it was difficult to definitely assign the primary cause of failure, since the bar had shown considerable slip and at the same time there were evidences of failure from excessive diagonal tensile stresses in combination with the large slip. In assigning the manner of failure as shown in the tables, all the evidence of the test was considered—the slip of bar at the ends and at intermediate points, the size and position of cracks in the beam and the calculated stresses in the steel and concrete. In some of the tests the measurements gave indications of bond failure, but the calculated tensile stress in the bar and the cracks in the beam showed that the longitudinal steel was over-stressed at the middle of the span.

At failure nearly all of the beams except those of the longer spans showed one or two prominent diagonal cracks at one or both ends about midway between the load and the support. In Tables 28 and 34 the applied loads on the beam at the time of the observation of the first outer crack and at the first slip of the ends of the bar have been given in parallel columns. Reference to Fig. 69 to 76 and to Table 39 will show that slip of bar becomes appreciable at points about midway between the loads and the supports at loads about one-third the maximum for the beams reinforced with plain bars and loaded at the one-third points. For 25 tests on such beams the first visible crack outside the load points appeared at loads averaging 57% of the maximum and slip at the end began at 70% of the maximum load. These tests show that there is a considerable slip at the point where the diagonal cracks appear before these cracks became visible and indicated that the opening of these outer cracks was probably due primarily to slip of bar. The proportion of this slip due to beam bond stress and to anti-stretch slip will depend upon the dimensions and reinforcement of the beam. The bars used in these test beams were relatively large. The differences between the conditions in the middle third of the beam length and in the outer thirds should be kept in mind. In the middle region beam bond stress is not brought into action and the slip is wholly of the nature of what has been termed anti-stretch slip. In the outer thirds there is a combination of beam bond stress and of bond stress due to anti-stretch slip. The first outer crack generally became visible at a slip of bar of 0.002 to 0.005 in. It is evident that diagonal cracks may open at a very small slip of bar and that they open at loads which give very small end slip and even before end slip is noted. In the beams of Group 6 in which 4 in. of concrete was placed below the center of the bar, the cracks were restricted to the region within the load points or to a short distance outside (see Fig. 58a). The vertical shearing stresses developed were considerably higher than in other beams which were not reinforced with vertical stirrups.

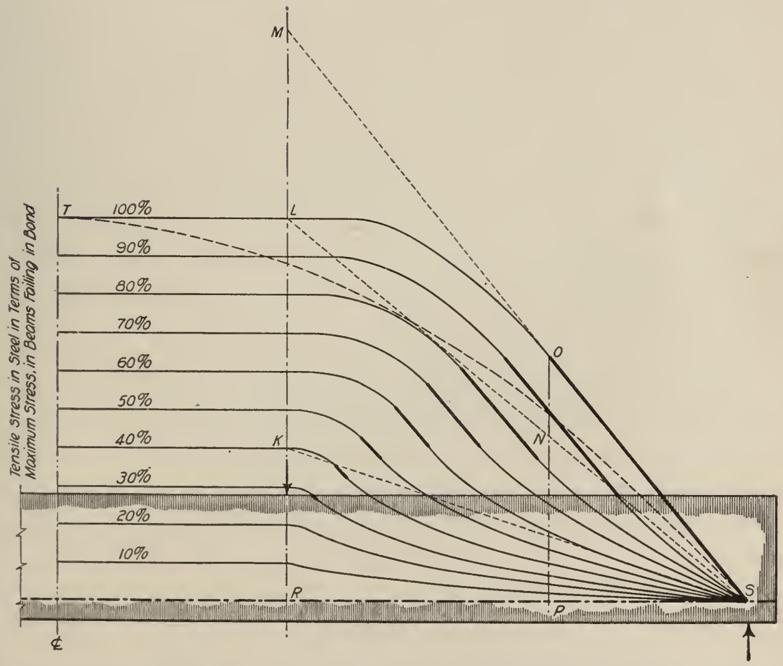

M
T
100%
L
90%
80%
70%
60%
50%
40%
K
30%
20%
10%
O
N
R
P
S
℄
Tensile Stress in Steel in Terms of Maximum Stress in Beams Failing in Bond

FIG. 68. IDEALIZED DIAGRAM SHOWING THE DISTRIBUTION OF TENSILE AND BOND STRESSES IN A SIMPLE REINFORCED CONCRETE BEAM.

97. *Influence of Slip of Bar upon Beam Deflection.*—Several treatises on reinforced concrete develop formulas which are intended to give expressions for the center deflection of a reinforced concrete beam, when the dimensions of the beam, the condition of the ends and the loading are known. These formulas will not be considered here, except to say that they are based on the usual deflection formula for homogeneous beams but modified to involve the assumptions that are commonly made in the analysis of reinforced concrete beams. These formulas show that beam deflection is a function of the loading, condition of the ends of the beam, span length, the elastic properties of the material and the dimensions of the section. With a combination of two materials having such different properties as concrete and steel and owing to the variation in the modulus of elasticity of concrete as the compressive stress increases, the deflection of a reinforced concrete beam is an extremely complex function. However, formulas have been devised which give fairly accurately the value of the deflection up to working loads if the proper value is assigned to the modulus of elasticity of the concrete.

One of the assumptions referred to above is that a plane section before flexure is a plane section after flexure. We have seen that during the greater part of the test of a reinforced concrete beam, slip of bar is an important phenomenon, and we may expect slip of bar to exert a marked influence on the deflection. Turneaure and Maurer,* in an investigation of deflection of reinforced concrete beams found that their formula gave values for deflection which were close to the true ones up to about one quarter of the maximum load carried by the beam. This is about the load at which slip of bar became pronounced through the middle portion of the span and we may expect that from this point the deflection is considerably influenced by slip of bar. As long as slip of bar is not general, there is no distinct change in the course of the load-deflection curve due to this cause, although its effect must be present from the time slipping begins. In homogeneous beams the additional deflection due to shear is neglected. In reinforced concrete beams the deflection due to slip of bar is an analogous quantity which has an important influence on the total deflection of the beam. As soon as slip of bar becomes general, as indicated by slip at the ends, we may expect the deflection to respond more rapidly to this cause. Many of the load-deflection curves in Fig. 77 to 86 show a decided increase in deflection immediately following the beginning of slip at the ends of the bar. Noteworthy examples of this action are found in Beam No. 1052.6, Fig. 81; Beam No. 1050.1, Fig. 84; Beam No. 1048.4, Fig. 86.

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*Principles of Reinforced Concrete Construction, 1909.

98. *Critical Bond Stress in Reinforced Concrete Beams.*—The foregoing discussions of beam tests in which slip of bar was measured indicate that slip of bar is found at relatively low loads. It was seen that slip of bar occurred first in the middle portion of the span at early loads. With increase of load slip of bar progresses through the outer thirds toward the ends. With the appearance of slip of bar at the ends of the beam, slip had become general through the region of beam bond stress and the beams were not able to permanently withstand a load appreciably greater than that causing first end slip. The tests indicate that a very small end slip may correspond to a critical bond stress.

The average bond stresses computed on the basis of the vertical shear are not the actual stresses being developed. As has already been pointed out, the tests show that maximum bond resistance is developed at certain points of the bar at loads much below that causing bond failure in the beam. Certain observations indicate that the bond stress first reaches the value of the maximum bond resistance in the portion of the span in which beam bond stresses are developed, at points a short distance outside of the load points. Anti-stretch slip may act to increase the bond stress developed over certain regions of the span. While slip of bar gives an indication of the bond stress being developed at any point as long as the amount of slip is small, it should be noticed that after the slip of bar has exceeded an amount corresponding to the maximum bond resistance further slip gives no indication of the amount of bond stress being developed.

99. *Working Stresses for Bond.*—The Committee on Concrete and Reinforced Concrete,* appointed by four of the leading American engineering societies (commonly known as the "Joint Committee") recommended the use of a bond stress equivalent to 4% of the compressive strength of the same concrete at 28 days of age for plain bars of ordinary mill surface and values one-half as great for drawn wire. The compressive strength of the concrete is to be determined from tests on 8 by 16-in. cylinders. For concrete giving a compressive strength of 2000 lb. per sq. in., at 28 days, the allowable bond stress is 80 lb. per sq. in. for

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*See report in Proceedings of the American Society of Civil Engineers, Feb., 1913. Also Proceedings of the American Society for Testing Materials, 1913, p. 228.

plain bars. These values have been generally accepted by American designers. The British Joint Committee on Reinforced Concrete† recommended the use of a bond stress of 100 lb. per sq. in. in concrete having a compressive strength of 1800 lb. per sq. in. when tested in the form of cubes not less than 4 in. on a side or cylinders not less than 6 in. in diameter and of a length not less than the diameter. Tests recently made at the University of Illinois on compression specimens of different forms show that the compressive strength of 6-in. concrete cubes is about 27% higher than that of 8 by 16-in. cylinders. On this basis the British Committee's concrete would have a compressive strength about 70% of the American Committee's concrete and the allowable working bond stress would be about 80% higher than the corresponding values of the American Committee, considering the quality of the concrete.

It was seen in Art. 48 that the average bond stress in the pull-out tests when end slip began was about 17% of the compressive strength of 6-in. cubes made from the same concrete. Reducing this to the basis used above, we may say that slip began in the pull-out tests at a bond stress equal to about 13% of the compressive strength of 8 by 16-in. cylinders. In the same way the ultimate bond resistance in the pull-out tests discussed in Art. 48 was about 19% of the compressive strength of 8 by 16-in. cylinders. Certain of the beam tests indicate that the actual bond resistance of a bar embedded in a reinforced concrete beam was not materially different from that of pull-out specimens made in the same position. The working stress recommended by the American Joint Committee is equal to about ⅓ the bond stress corresponding to first slip of bar and to about 1/5 the ultimate pull-out resistance of plain bars. Apparently this indicates a factor of safety of about 3 against slip of bar, if we consider first slip of bar in the region where beam bond stresses are developed as the critical stress. However, we have seen that on account of the unequal distribution of bond stress in beams loaded at the one-third points, the bond stresses actually developed over a short length outside the loads is probably near the ultimate bond resistance even for working loads at the age of the beams tested. This in reality may not be a serious consideration since there should be a sufficient reserve of bond resistance in the embedded portion of the bar where the bond stress is low, and an additional reserve in the increasing bond resistance with the increase in the age of the concrete. From the way in which the steel stress is distributed in Fig. 64 to 67, it would seem that the stress conditions of a beam under a uniformly distributed load are more favorable to bond resistance than the system of concentrated loads used in the tests. It would appear that the values of working stresses for bond recommended by the British Committees are entirely too high considering the grade of concrete. The values of the American Committee are as high as should be used under average conditions of workmanship.

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†See Second Report Joint Committee on Reinforced Concrete, published by the Royal Institute of British Architects, 1911.

The data for beams reinforced with deformed bars are not as complete as for plain bars, but on account of the undesirable secondary stresses which are introduced in the concrete as a result of high bond stresses, it seems the part of wisdom to use the same values for deformed as for plain bars. However, it should be recognized that properly designed deformed bars may be expected to give a greater uniformity of steel stress than plain bars, and they may be counted upon to guard against local deficiencies in bond resistance due to lack of care in placing the concrete around the bars or to other defects due to poor workmanship.

## IV. SUMMARY.

100. *Summary.*—The tests covered a wide range of conditions and the results have a significant bearing on the nature of bond resistance, the action of bars of different forms under bond stress, and the behavior of beams subjected to high bond stresses. The load-slip determinations have given definite information on the nature and distribution of bond resistance. The following is a resumé of the principal observations and conclusions which have been stated and discussed in the text. Paragraphs 2 to 34, inclusive, refer primarily to the results of the pull-out tests.

(1) Bond between concrete and steel may be divided into two principal elements, adhesive resistance and sliding resistance. The source of adhesive resistance is not known, but its presence is a matter of universal experience with materials of the nature of mortar and concrete. Sliding resistance arises from inequalities of the surface of the bar and irregularities of its section and alignment together with the correspond-

ing conformations in the concrete. The adhesive resistance must be overcome before sliding resistance comes into action. In other words, the two elements of bond resistance are not effective at the same time at a given point. Many evidences of the tests indicate that adhesive resistance is much the more important element of bond resistance.

(2) Pull-out tests with plain bars show that a considerable bond stress is developed before a measurable slip is produced. Slip of bar begins as soon as the adhesive resistance is overcome. After the adhesive resistance is overcome, a further slip without an opportunity of rest is accompanied by a rapidly increasing bond stress until a maximum bond resistance is reached at a definite amount of slip.

(3) The true relation of slip of bar to bond stress can best be studied by considering the action of a bar over a very short section of the embedded length. The difficulties arising from secondary stresses made it impracticable to conduct tests on bars embedded very short lengths. The desired results were obtained by varying the forms of the specimens in such a way that the effect of different combinations of dimensions could be studied.

(4) Pull-out tests with plain bars of the same size embedded different lengths furnish data which suggest the values of bond resistance over a very short length of embedment, or indicate values of bond resistance which are independent of the length of embedment. Tests with bars of different size which were embedded a distance proportional to their diameters give the true relation when the effect of size of bar is eliminated. Two series of tests of this kind on plain round bars of ordinary mill surface gave almost indentical values for bond resistance after eliminating the effect of length of embedment and size of bar, and we may consider that these values represent the stresses which were developed in turn over each unit of area of the embedded bar as it was withdrawn by a load applied by the method used in these tests. These tests showed that for concrete of the kind used (a 1-2-4 mix, stored in damp sand and tested at the age of about 60 days) the first measurable slip of bar came at a bond stress of about 260 lb. per sq. in., and that the maximum bond resistance reached an average value of 440 lb. per sq. in. If we conclude that adhesive resistance was overcome at the first measurable slip, it will be seen that the adhesive resistance was about 60% of the maximum bond resistance. This ratio did not vary much for a wide range of mixes, ages, size of bar, condition of storage, etc.

(5) Sliding resistance reached its maximum value for plain bars of ordinary mill surface at a slip of about 0.01 in. The constancy in the amount of slip corresponding to the maximum bond resistance for a wide range of mixes, ages, sizes of bar, conditions of storages, etc., is a noteworthy feature of the tests. With further slip the sliding resistance decreased slowly at first, then more rapidly, until with a slip of 0.1 in. the bond resistance was about one-half its maximum value.

(6) Pull-out tests with plain round bars show end slip to begin at an average bond stress equal to about one-sixth the compressive strength of 6-in. cubes from the same concrete; the maximum bond resistance is equal to about one-fourth the compressive strength of 6-in. cubes. These values were about the same for a wide range of mixes, ages and conditions of storage. In terms of the compressive strength of 8 by 16-in. concrete cylinders these values would be about 13% for first end slip and 19% for the maximum bond resistance.

(7) The tests indicate that bond stress is not uniformly distributed along a bar embedded any considerable length and having the load applied at one end. Slip of bar begins first at the point where the bar enters the concrete, and the bond stress must be greater here than elsewhere until a sufficient slip has occurred to develop the maximum bond resistance at this point. Slip of bar begins last at the free end of the bar. After slip becomes general, there is an approximate equality of bond stress throughout the embedded length.

(8) Small bars gave a bond resistance somewhat higher than the large bars during the early stages of the test. This was probably on account of greater irregularity of section and alignment of the smaller bars. The maximum bond resistance was not materially different for bars of different diameters.

(9) Computations based on the elastic properties of the materials indicate that in the pull-out tests the tensile deformation in the bar had a much greater effect on the amount of bond stress which permitted a given slip of bar than had the compressive deformation in the concrete block in which the bar was embedded.

(10) Rusted bars gave bond resistances about 15% higher than similar bars with ordinary mill surface.

(11) The tests with flat bars showed wide variations of bond resistance and were not conclusive. Square bars gave values of unit-stress about 75% of those obtained with plain round bars.

(12) T-bars gave lower unit bond resistance than plain round bars, but gave about double the bond resistance per unit of length that was found for the plain round bars of the same sectional area.

(13) With polished bars the bond resistance is due almost entirely to adhesion between the concrete and steel. Numerous tests with polished bars embedded in 1-2-4 concrete and tested at 60 days indicated a maximum bond resistance of about 160 lb. per sq. in., or about 60% of the bond resistance of bars of ordinary surface at small amounts of slip. This value agrees closely with tests reported elsewhere, and apparently represents the value of the tangential adhesion between any clean steel and concrete of this quality. The sliding resistance of polished bars was very low.

(14) Tests with polished bars with wedging and non-wedging tapers showed that adhesion was broken for both types of bar at about the same bond stress as in the polished bars of uniform section.

(15) The tests with polished bars with wedging taper showed that after the adhesion was broken a considerable movement of the bar (as much as ¼ in. with the smallest tapers) was required before the bond resistance again reached the amount which was at first carried by the adhesive resistance. The amount of movement necessary to restore the bond stress to the value of the original adhesive resistance was inversely proportional to the amount of taper. This indicates that a definite normal compression must be developed in the surrounding concrete before a longitudinal component equivalent to the original tangential adhesion is produced.

(16) It was noted in the tests with plain bars that sliding resistance was due to inequalities of the surface of the bar and to irregularities of its section and alignment. The projections on a deformed bar give an exaggerated condition of inequality of surface or irregularity of section. Adhesive resistance must be destroyed and the usual sliding resistance largely overcome and the concrete ahead of the projections must undergo an appreciable compressive deformation before the projections on a deformed bar become effective in taking bond stress. The tests indicate that the projections do not materially assist in resisting a force tending to withdraw the bar until a slip has occurred approximating that corresponding to the maximum sliding resistance of plain bars. As slip continues a larger and larger portion of the bond stress is taken by direct bearing of the projections on the concrete ahead.

(17) In determining the comparative merits of deformed bars, the bar which longest resists beginning of slip should be rated highest, other considerations being equal. The bond stresses developed at an end slip of 0.001 inch furnished the principal basis of comparison for the different types of deformed bars. At an end slip of 0.001 in. 12 sets of deformed bars of ¾-in. and larger sizes embedded 8 in. in 1-2-4 concrete, tested at about 2 months, developed an average bond resistance of 318 lb. per sq. in., 4% higher than the corresponding value for plain bars. At this stage of the test, two sets of deformed bars gave practically the same bond resistance, five sets gave lower values, and five sets higher values than the plain rounds. At an end slip of 0.01 in., corresponding to the maximum bond resistance of plain bars, the average bond resistance of the 12 sets of deformed bars was 445 lb. per sq. in., 10% higher than plain rounds. At this stage of the test two sets gave about the same values, two sets gave lower values, and eight sets gave higher values than the plain bars. The hooping used in these specimens had a marked effect in increasing the bond resistance even at small amounts of slip.

(18) The concrete cylinders of the pull-out specimens with deformed bars were reinforced against bursting or splitting, because it was desired to study the load-slip relation through a wide range of values. The bond stresses corresponding to an end slip of 0.1 in. are the highest stresses reported for the deformed bars. In only a few tests was the maximum bond resistance reached at an end slip less than 0.1 in. It should be recognized that, in general, the bond stresses reported for deformed bars at end slip of 0.05 and 0.1 in., could not have been developed with bars embedded in unreinforced blocks. These high values of bond resistance must not be considered as available under the usual conditions of bond action in reinforced concrete members. In the tests in which the blocks were not reinforced, evidence of splitting of the blocks was found at end slips of 0.02 to 0.05 in.

(19) The normal components of the bearing stresses developed by the projections on a deformed bar may produce very destructive bursting stresses in the surrounding concrete. The bearing stress between the projections and the concrete in the tests with certain types of commercial deformed bars was computed to be from 5800 to 14 000 lb. per sq. in. at the highest bond stresses considered in these tests. For bars having projections of different heights and spacing, the bearing stresses on the projections at the highest bond stresses considered were inversely proportional to the bond stress which had been developed by the bar at an end

slip of 0.01 in., the slip at which the projections were beginning to be effective. These considerations show that the ratio of the area of the projections measured at right angles to the bar to the superficial area of the bar in the same length is the proper criterion for judging of the effective bond resistance of a deformed bar. In some forms of bar the bearing stresses must have been much higher than the values given above. The large slip and the high bearing stresses developed in the later stages of the tests show the absurdity of seriously considering the extremely high values that are usually reported to be the true bond resistance of many types of deformed bars.

(20) Round bars with standard V-shaped threads gave much higher bond resistance at low slips than the commercial deformed bars. The average bond resistance at an end slip of 0.001 in. was 612 lb. per sq. in. The maximum bond resistance was 745 lb. per sq. in. These were the only deformed bar tests in which failure came by shearing the surrounding concrete.

(21) In a deformed bar of good design the projections should present bearing faces as nearly as possible at right angles to the axis of the bar. The areas of the projections should be such as to preserve the proper ratio between the bearing stress against the concrete ahead of the projections and the shearing stress over the surrounding envelope of concrete. Failure by shearing of the concrete should be avoided. The tests indicate that the areas of the projections measured at right angles to the axis of the bar should not be less than, say, 20% of the superficial area of the bar. A closer spacing of the projections than is used in commercial deformed bars would be of advantage. Advocates of the deformed bar would do well to recognize the fact that in a deformed bar which may be expected to develop a high bond resistance, a certain amount of metal must be used in the projections which probably will not be available for taking tensile stress.

(22) The 1-in. twisted square bars gave a bond resistance per unit of surface at an end slip of 0.001 in., only 88% of that for the plain rounds. Following an end slip of about 0.01 in., these bars showed a decided decrease in bond resistance, and a slip of 5 to 10 times this amount was required to cause the bond resistance to regain its first maximum value. After this, the bond resistance gradually rose as the bar was withdrawn. Some of the bars were withdrawn 2 or 3 in. before the highest resistance was reached. The apparent bond stresses at these slips were very high; but, of course, such stresses and slips could

not be developed in a structure and could not have been developed in the tests had the blocks not been reinforced against bursting. Such values are entirely meaningless under any rational interpretation of the tests.

(23) The load-slip curves for twisted square bars are similar to those for polished bars with wedging taper. The twisted bar is essentially a combination of the wedging and non-wedging taper. As the bar is drawn through the concrete the wedging tapers are drawn more firmly against the concrete ahead, while at the same time the non-wedging tapers are separated from the concrete with which they were originally in contact. The drop in the load-slip curves after an end slip of about 0.01 in. shows that the separation of about one-half of the surface of the bar from its original contact and the continued sliding of the flatter portions of the bar, until a large slip has occurred, have a greater influence in reducing the average bond resistance than the increased bearing of the wedging tapers has in raising the bond resistance. The results found with the twisted square bar do not justify its present widespread popularity as a reinforcing material.

(24) The tests with plain round bars anchored by means of nuts and washers and with washers only showed that the entire bar must slip an appreciable amount before these forms of anchorage come into action. Anchorages of the dimensions used in these tests did not become effective until the bar had slipped an amount corresponding to the maximum bond resistance of plain bars. With further movement the apparent bond resistance was high, but was accompanied by excessive bearing stresses on the concrete.

(25) The load-slip relation for bars anchored by means of hooks and bends was not determined. The high resistance given in these tests was probably a result of the bearing stresses developed in the concrete ahead of the bends.

(26) Tests on specimens stored under different conditions indicate that concrete stored in damp sand may be expected to give about the same bond resistance and compressive resistance as that stored in water. Water-stored specimens gave values of maximum bond resistance higher in each instance than the air-stored specimens; the increase for water storage ranged from 10 to 45%. The difference seemed to increase with age. The presence of water not only did not injure the bond for ages up to three years, but it was an important factor in producing conditions which resulted in high bond resistances. However, it was found that

specimens tested with the concrete in a saturated condition gave lower values for bond than those which had been allowed to dry out before testing. The bars in specimens which had been immersed in water as long as three and one-half years showed no signs of rust or other deterioration.

(27) Specimens made out-doors in freezing weather, where they probably froze and thawed several times during the period of setting and hardening, were almost devoid of bond strength.

(28) Pull-out tests made at early ages gave surprisingly high values of bond resistance. Plain bars embedded in 1-2-4 concrete and tested at 2 days did not show end slip of bar until a bond stress of 75 lb. per sq. in. was developed. Bond resistance increases most rapidly with age during the first month. The richer mixes show a more rapid increase than the leaner ones. The tests on concrete at ages of over one year showed that the bond resistance of specimens stored in a damp place may be expected ultimately to reach a value as much as twice that developed at 60 days.

(29) The load-slip relation of leaner and richer mixes was similar to that for 1-2-4 concrete. For a wide range of mixes the bond resistance was nearly proportional to the amount of cement used. This relation did not obtain in a mix from which the coarse aggregate had been omitted.

(30) When the application of load was continued over a considerable period of time or when the load was released and reapplied, the usual relation of slip of bar to bond resistance was considerably modified. The few tests which were made indicate that the bond stress corresponding to beginning of slip is the highest stress which can be maintained permanently or be reapplied indefinitely without failure of bond. The effect of continued and repeated load, impact, etc., may well be the subject of further experimental study.

(31) Little difference was found in the pull-out tests whether the load was distributed over the entire face of the block or over a narrow ring at the center of the block or around the edge of the face of the block.

(32) Specimens molded in a horizontal position gave lower bond resistance than those molded in a vertical position; when settlement of the bar with the settlement of the concrete was entirely prevented, the bond resistance was reduced to about 60% of that found for similar specimens which were molded with the bars in a vertical position. Plain

bars tested by being pulled in the same or the opposite direction from the settlement of the concrete during setting gave about the same bond resistance, but in the tests of certain deformed bars this was not true.

(33) The term "autogenous healing" is used to designate phenomena observed in pull-out tests and in compression tests of concrete cylinders in which the hardening of the concrete was interrupted by loading the specimen at early ages to its ultimate resistance. Up to an age of one year the bond resistance of specimens stored in damp sand was not affected by as many as four loadings at intervals during the period of storage up to the ultimate resistance. For specimens stored in air and tested in the same way, the bond resistance was less than for damp-sand storage, but the tests showed a steady increase in bond resistance with each loading up to three months. Specimens which had been stored in air for two months before the first test and in water thereafter showed a decrease in bond with each subsequent loading, although the bond resistance in the last test was fairly high. The presence of water apparently permits the continuation of the hydraulic action of the cement for several months after the mixing of the concrete.

(34) Bond resistance of plain bars is greatly increased if the concrete is caused to set under pressure. With a pressure of 100 lb. per sq. in. on the fresh concrete for five days after molding, the maximum bond resistance was increased 92% over that of similar bars in concrete which had set without pressure. The greater density of the concrete and its more intimate contact with the bar seems to be responsible for the increased bond resistance. Light pressures gaves an appreciable increase in bond resistance. With polished bars the effect of pressure was slight.

(35) As might have been expected, the compressive resistance of concrete setting under pressure was increased in much the same ratio as the bond resistance. At the age of 80 days the initial modulus of elasticity in compression for concrete which set under a pressure of 100 lb. per sq. in. was about 37% higher and the compressive strength was increased by about 73% over that of concrete which had set without pressure. The density of the concrete, as determined by the unit weights, was increased about 4% by a pressure of 100 lb. per sq. in. on the fresh concrete. The increase in strength and density was relatively greater for the low than for the high pressures. A pressure continued for one day, or until the concrete had taken its final set and hardening had begun, seems to have produced the same effect in increasing the

strength and elastic properties of the concrete as when the pressure was continued for a much longer period.

(36) Concrete cylinders tested in compression at age of 80 days after having been loaded to failure at 7 days gave compressive strengths nearly as high as those tested for the first time at the same age. Retests of cylinders which had set under pressure gave similar results.

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(37) Beams of comparatively short span reinforced with bars of large size were used in order to develop high bond stresses and give bond failures. Most of the beams failed in bond; a few failed by a combination of bond and diagonal tension or by tension in the steel.

(38) The usual method of computing the bond stress in a reinforced concrete beam does not take account of all the phenomena of bond action. Slip of bar due to beam bond action and the presence of anti-stretch slip may be expected to greatly modify the distribution of bond stress over the length of the bar, and otherwise to affect resistance to beam bond stresses. However, the nominal values for bond resistance, computed by the usual formula, form a useful basis for comparison in beams in which the dimensions and general make-up are similar.

(39) Slip of bar was a phenomenon in all beam tests in which careful slip observations were made. These load-slip relations give important indications as to the bond stress developed at points along the length of the beam.

(40) Slip was first observed in the middle region of the span at loads producing a tensile stress in the steel of about 6000 lb. per sq. in. In this region the shear is zero and hence beam bond action, as usually understood, is absent. As the load was increased, slip of bar progressed through the outer thirds toward the ends of the beam at a rate nearly proportional to the increase of load. After slip occurred at the ends, the outer thirds of the length of the bar moved toward the middle of the span relative to the adjacent concrete. Slip of bar was probably partly responsible for the opening of outer cracks, since slipping was observed in the outer thirds of the beams before the cracks became visible.

(41) The mean computed values for bond stresses in the 6-ft. beams in the series of 1911 and 1912 were as given below. All beams were of 1-2-4 concrete, tested at 2 to 8 months by loads applied at the one-third points of the span. Stresses are given in pounds per square inch.

| | Number of Tests | First End Slip of Bar | End Slip of 0.001 in. | Maximum Bond Stress |
|---|---|---|---|---|
| 1 and 1¼-in. plain round | 28 | 245 | 340 | 375 |
| ¾-in. plain round | 3 | 186 | 242 | 274 |
| ⅝-in plain round | 3 | 172 | 235 | 255 |
| 1-in. plain square | 6 | 190 | 248 | 278 |
| 1-in. twisted square | 3 | 222 | 289 | 337 |
| 1⅛-in. corrugated round | 9 | 251 | 360 | 488 |

(42) In the beams reinforced with plain bars end slip begins at 67% of the maximum bond resistance; for the corrugated rounds this ratio is 51%, and for the twisted squares, 66%.

(43) The bond unit resistance in beams reinforced with plain square bars, computed on the superficial area of the bar, was about 75% of that for similar beams reinforced with plain round bars of similar size.

(44) Beams reinforced with twisted square bars gave values at small slips about 85% of those found for plain rounds. At the maximum load, the bond-unit stress with the twisted bars was 90% of that with plain round bars of similar size.

(45) In the beams reinforced with 1⅛-in. corrugated rounds, slip of the end of the bar was observed at about the same bond stress as in the plain bars of comparable size. At an end slip of 0.001 in., the corrugated bars gave a bond resistance about 6% higher and at the maximum load, about 30% higher than the plain rounds.

(46) The beams in which the longitudinal reinforcement consisted of three or four bars smaller than those used in most of the tests gave bond stresses which, according to the usual method of computation, were about 70% of the stresses obtained in the beams reinforced with a single bar of large size. The progressive opening of cracks with increase in load was well shown in these tests. These beams showed cracks nearer the ends than usual. The distances of the outermost cracks from the ends of the beams suggest that the unbroken length of embedment has an important bearing on the maximum loads which the beams may be expected to carry before failing by bond. It seems probable that the lower computed bond stresses in these tests are due to errors in the assumptions made as to the distribution of bond stress and not to actual differences of bond resistance in the bars of different size.

(47) The tests on beams with the loads placed in different positions with respect to the span gave little variation in bond resistance during the early stages of the tests. The maximum bond resistances increased rapidly as the load approached the supports. These tests indicate that the variation in the maximum bond stresses must be due to the presence of other than normal beam action.

(48) Nearly all the beams tested on span lengths of 7 to 10 ft. failed by tension in the steel and did not develop the maximum bond resistance, although high bond stresses were obtained. The bond stress corresponding to first end slip of bar did not vary much with the span length.

(49) The bond stresses developed in the beam tests indicate that with beams of the same cross-section the bond stresses are distributed in the same way during the early stages of the test in beams varying widely in span length and loading. During the later stages of the test, the distribution of bond stress seems to depend largely upon the conditions of stress in the concrete through the region of the span where beam bond stresses are high. The distribution of bond stresses in beams of different cross-section apparently varies with the relative dimensions of the beam and the reinforcing bars.

(50) The use of auxiliary tensile reinforcement in the outer thirds of the beam served to modify the distribution of bond stress during the early stages of the test, but did not have any influence on the maximum bond resistance. While the auxiliary bars seemed to prevent the opening of outer cracks, the tests indicate that interior cracks which did not appear on the surface of the beam may have opened to an extent that permitted the same distribution of bond stress as was found in other tests.

(51) Increasing the thickness of the concrete below the reinforcing bars beyond the depth usually employed caused a very large increase in the resistance to bond and web stresses. The added stiffness of the beam and the increased flexural strength through the outer thirds of the span, prevented the formation of cracks in these regions. In the other beam tests such cracks were found to interrupt the continuity of bond action and to be an important factor in producing lower average bond resistances.

(52) Increasing the length of overhang of the ends of the beam beyond the support did not increase the resistance to web stresses as indicated by the opening of outer cracks, but it had an influence on the bond resistance. The bond resistance at first end slip was greater in the beams with the longer overhang. The maximum bond resistance was materially increased by the additional overhang.

(53) In the reinforced concrete beams it was found that very small amounts of slip at the ends of the bar represented critical conditions of bond stress. For beams failing in bond the load at an end slip of 0.001 in. was 89% to 94% of the maximum load found in beams

reinforced with plain bars, and 79% of the maximum load for similar beams reinforced with corrugated bars. As soon as slip of bar became general, other conditions were introduced which soon caused the failure of the beam.

(54) The bond stresses developed in a reinforced concrete beam by a load applied as in these tests varies widely over the region in which beam bond stresses are present. High bond stresses are developed just outside the load points at comparatively low loads. The load which first developed a bond stress nearly equal to the maximum bond resistance in the region of beam bond stresses produced a stress near the support which was not more than about 15 to 40% of the maximum bond resistance. As the load is increased, the region of high bond stress is thrown nearer and nearer the support, and at the same time the bond stress over the region just outside the load point becomes steadily smaller. This indicates a piecemeal development of the maximum bond stress as the load is increased. The actual bond stresses in certain tests varied from less than one-half to more than twice the average bond resistance computed in the usual manner.

(55) Slip of bar in a reinforced concrete beam has a marked influence in increasing the center deflection during the later stages of loading.

(56) The comparison of the bond stresses developed in beams and in pull-out specimens from the same materials is of interest. Such a comparison should be made for similar amounts of slip. In the pull-out tests the maximum bond resistance came at a slip of about 0.01 in. for plain bars. The mean bond resistance for the deformed bars tested was not materially different from that of the plain bars until a slip of about 0.01 in. was developed; with a continuation of slip the projections came into action and with much larger slip high bond stresses were developed. The beam tests showed that about 79 to 94% of the maximum bond resistance was being developed when the bar had slipped 0.001 in. at the free end; hence the bond stress developed at an end slip of 0.001 in. was used as a basis of the principal comparisons in the pull-out tests. However, it is recognized that, under certain conditions, the stresses developed at larger amounts of slip may have an important bearing on the effective bond resistance of the bar.

(57) The pull-out tests and beam tests gave nearly identical bond stresses for similar amounts of slip in many groups of tests, but it seems that this was the result of a certain accidental combination of dimensions in the two forms of specimens and did not indicate that the com-

puted stresses in the beams were the correct stresses. However, it is believed that a properly designed pull-out test does give the correct value of bond resistance, and gives values which probably closely represent the bond stresses which actually exist in a beam or other member as slipping is produced from point to point along the bar. The relative position of the bar during molding may be expected to influence the values of bond resistance found in the tests.

(58) A properly made pull-out test on a specimen of correct design is a valuable aid in determining the bond resistance of reinforcing steel in concrete, if due consideration is given to the load-slip relation. The tensile stress in the bar should be kept well below the elastic limit. Best results will be obtained by using a relatively short embedment. An embedment of 8 diameters is recommended.

(59) A working bond stress equal to 4% of the compressive strength of the concrete tested in the form of 8 by 16-in. cylinders at the age of 28 days (equivalent to 80 lb. per sq. in. in concrete having a compressive strength of 2000 lb. per sq. in.) is as high a stress as should be used. This stress is equivalent to about one-third that causing first slip of bar and one-fifth of the maximum bond resistance of plain round bars as determined from pull-out tests. The use of deformed bars of proper design may be expected to guard against local deficiencies in bond resistance due to poor workmanship and their presence may properly be considered as an additional safeguard against ultimate failure by bond. However, it does not seem wise to place the working bond stress for deformed bars higher than that used for plain bars.

101. *Concluding Remarks.*—The tests described in this bulletin have thrown considerable light on the value of bond resistance and the distribution of bond stress for a wide range of conditions in both beam and pull-out tests. It may not be expected that all of the results indicated can be applied without modification to members in which the conditions of stress differ widely from those present in the tests.

Most of the foregoing discussions and conclusions are based on comparisons involving the load-slip relations. In a few of the tests the bond stress was determined from a study of the variations in the tensile stress in the reinforcing bar. The latter method furnishes a much more direct means of measuring the bond stress, but it has been available only since the recent development of a non-fixed extensometer. Additional tests are planned which are expected to give further information on this subject.

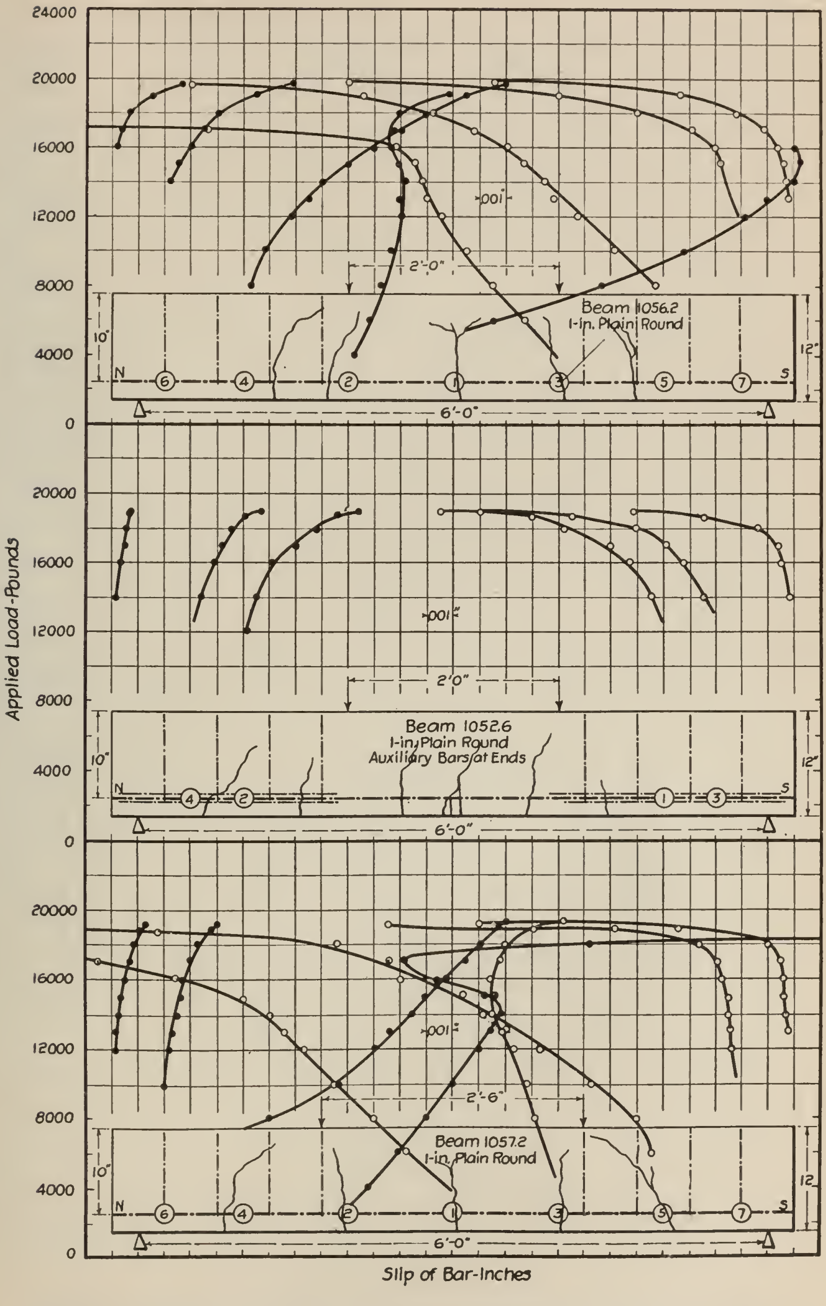

FIG. 69. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

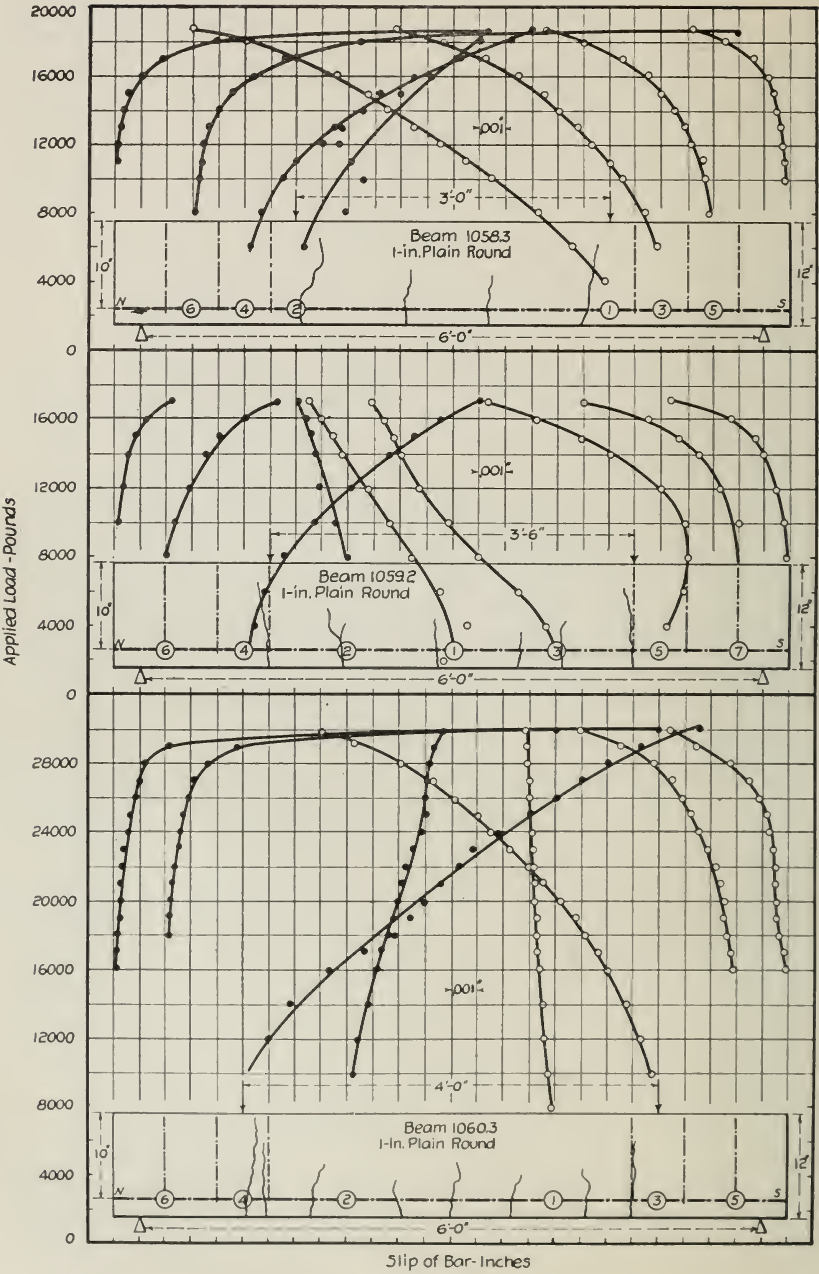

FIG. 70. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

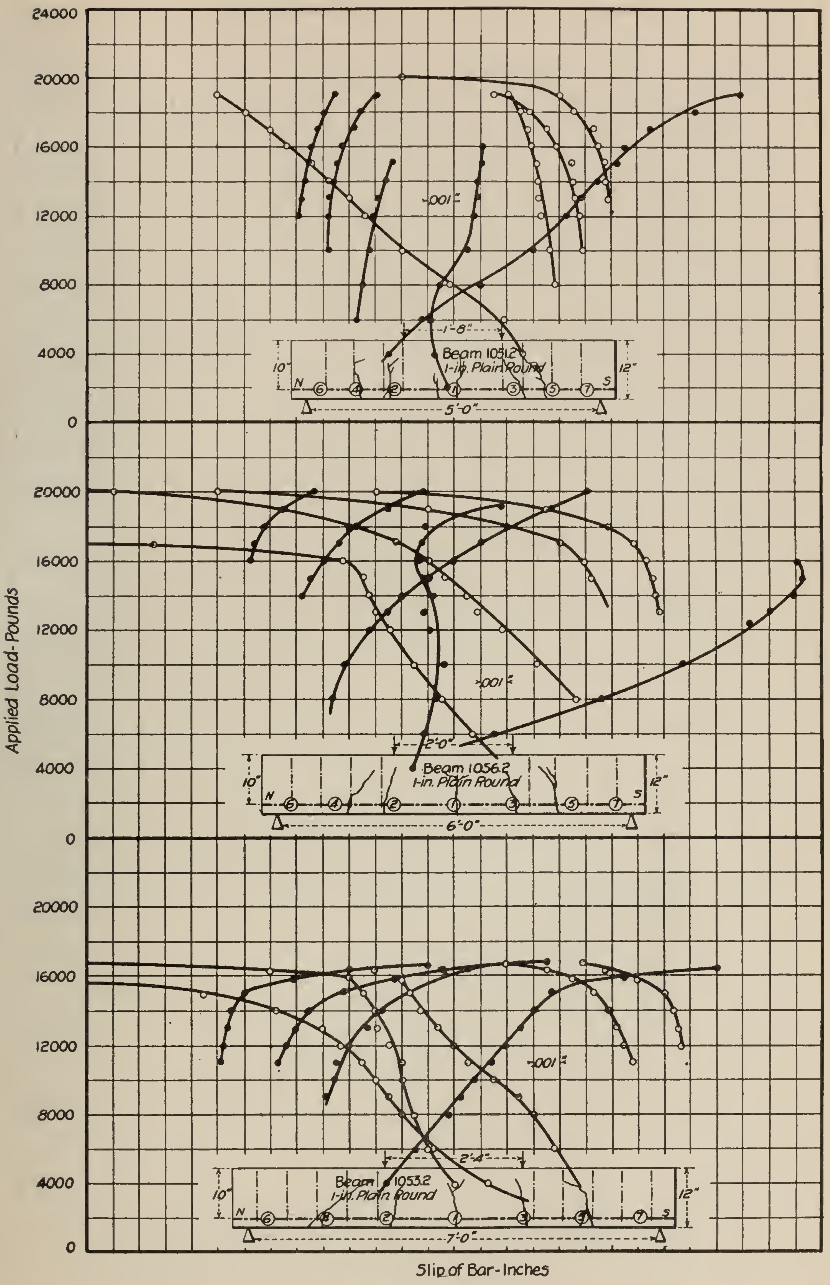

FIG. 71. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

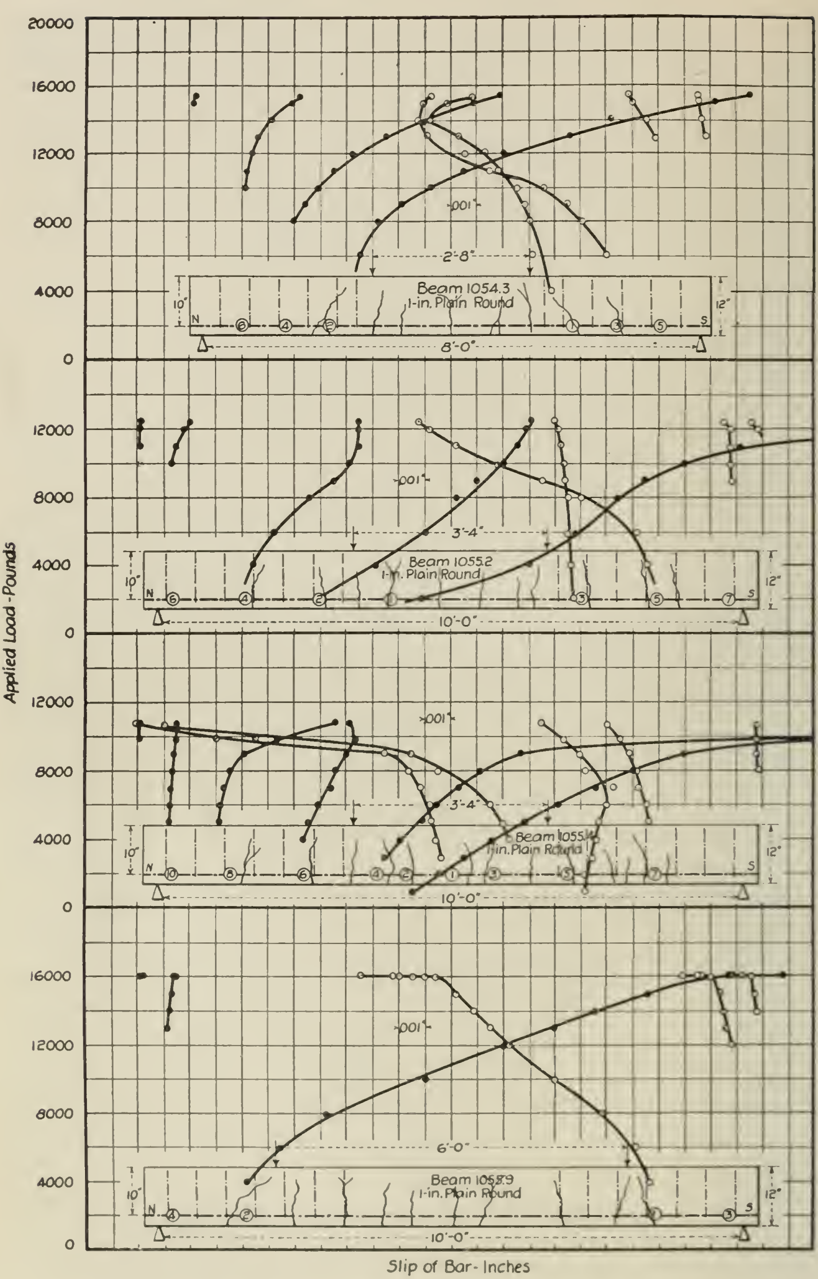

FIG. 72. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

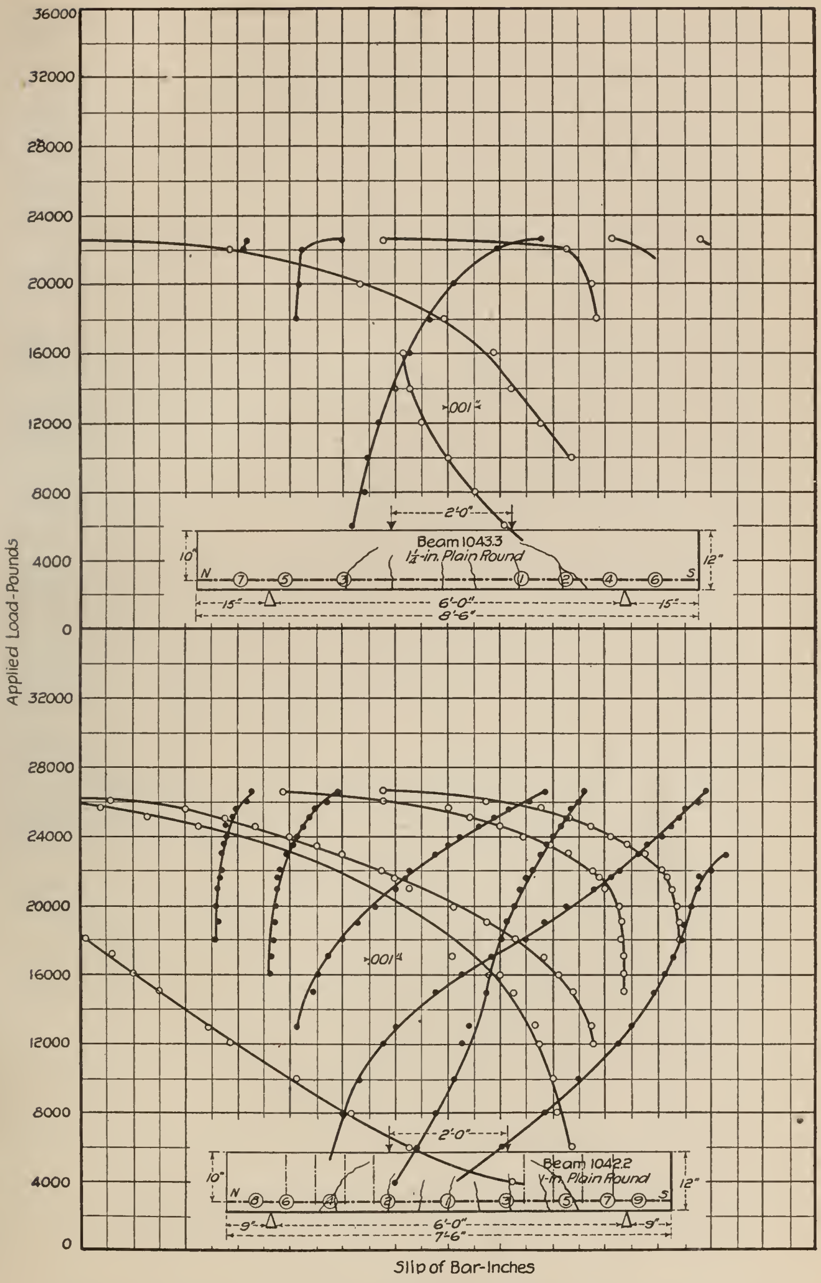

FIG. 73. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

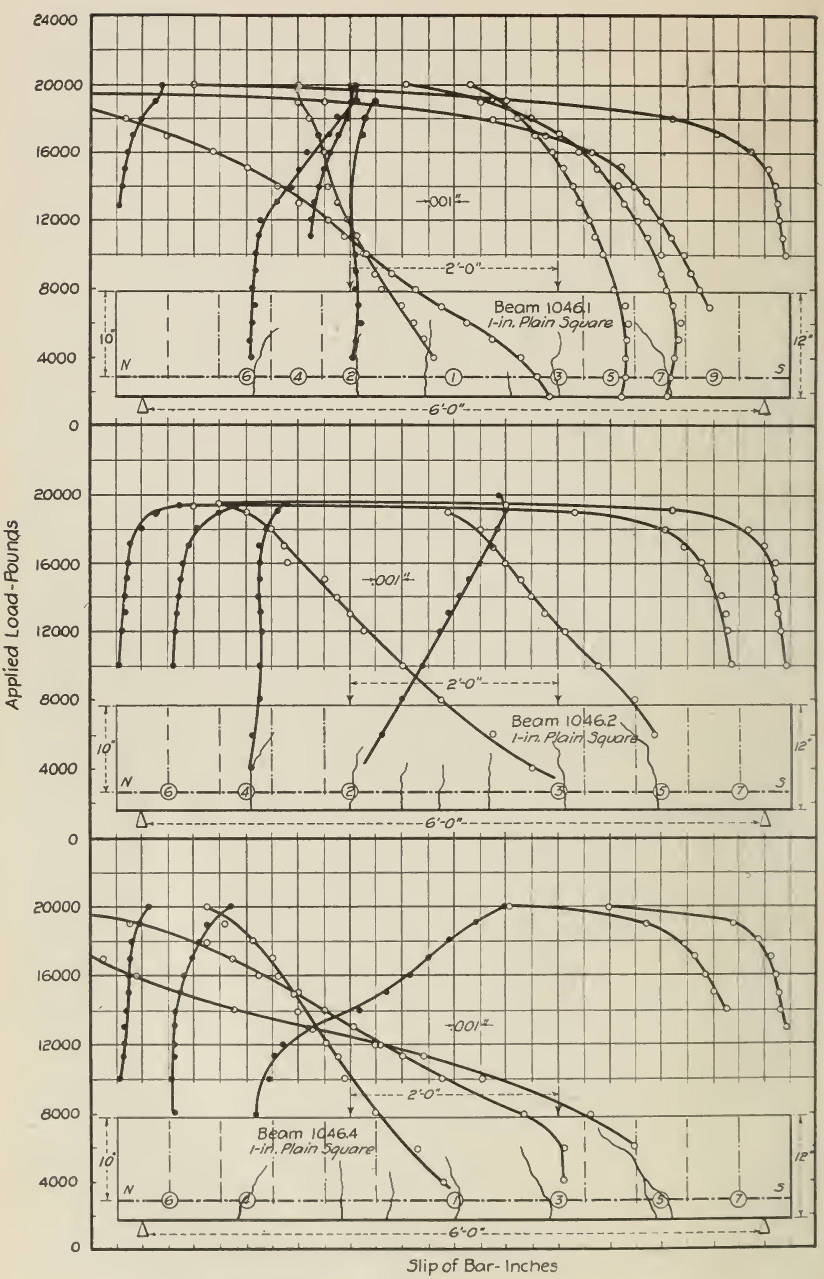

FIG. 74. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

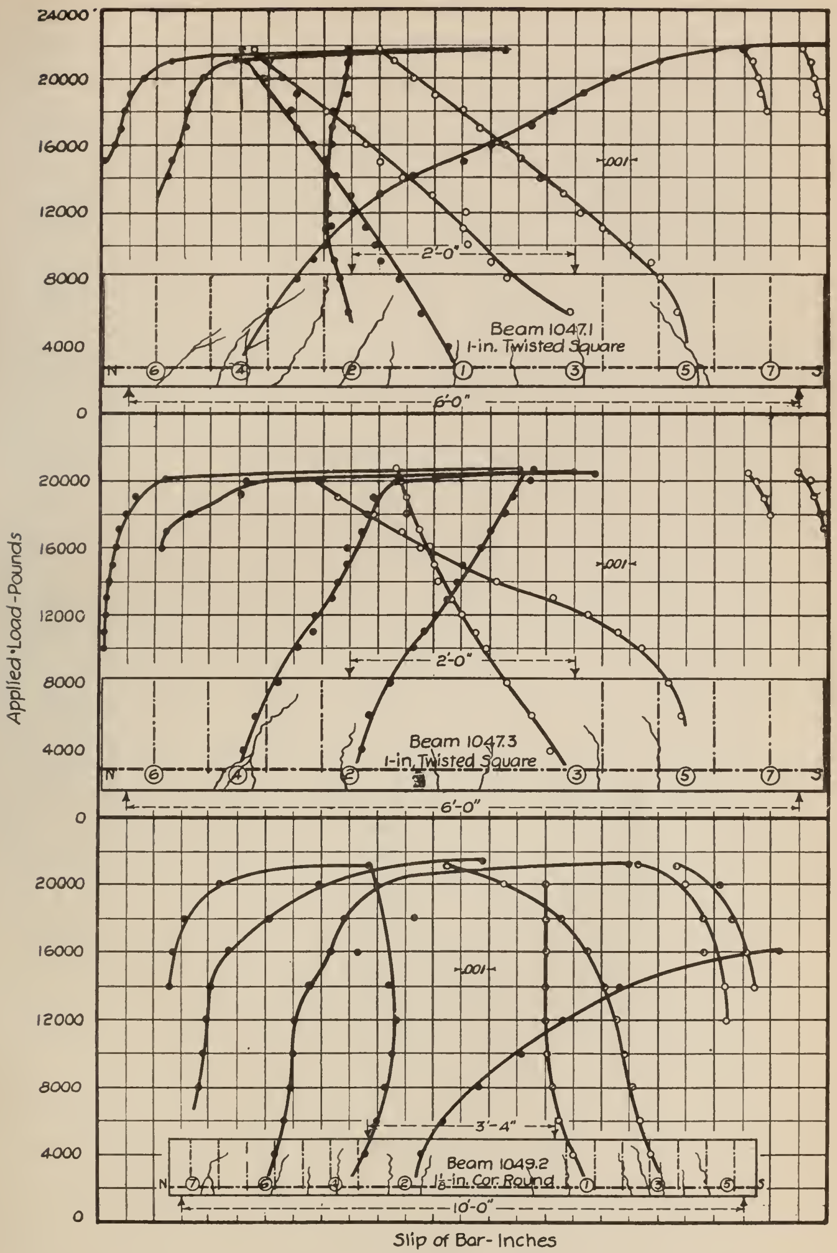

FIG. 75. LOAD-SLIP CURVES FOR BARS IN REINFORCED CONCRETE BEAMS.

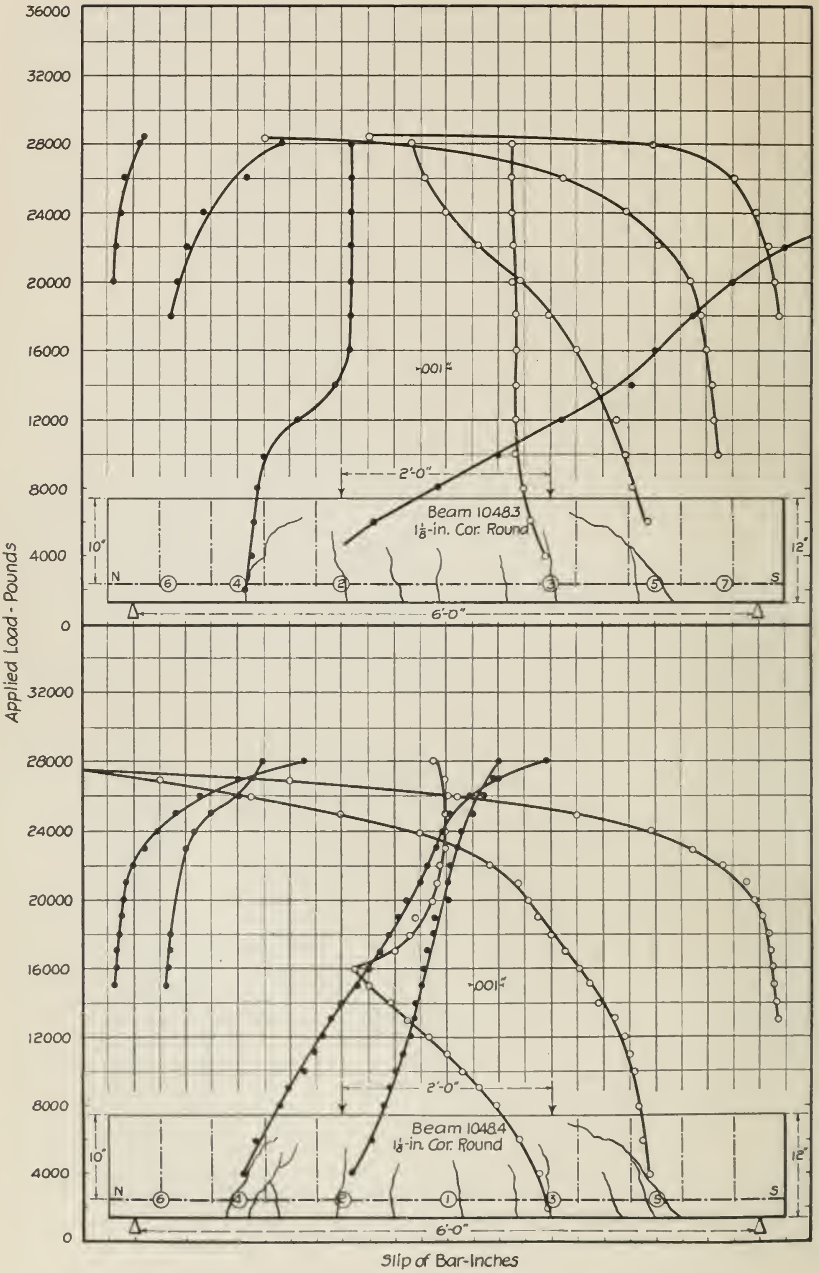

Fig. 76. Load-slip Curves for Bars in Reinforced Concrete Beams.

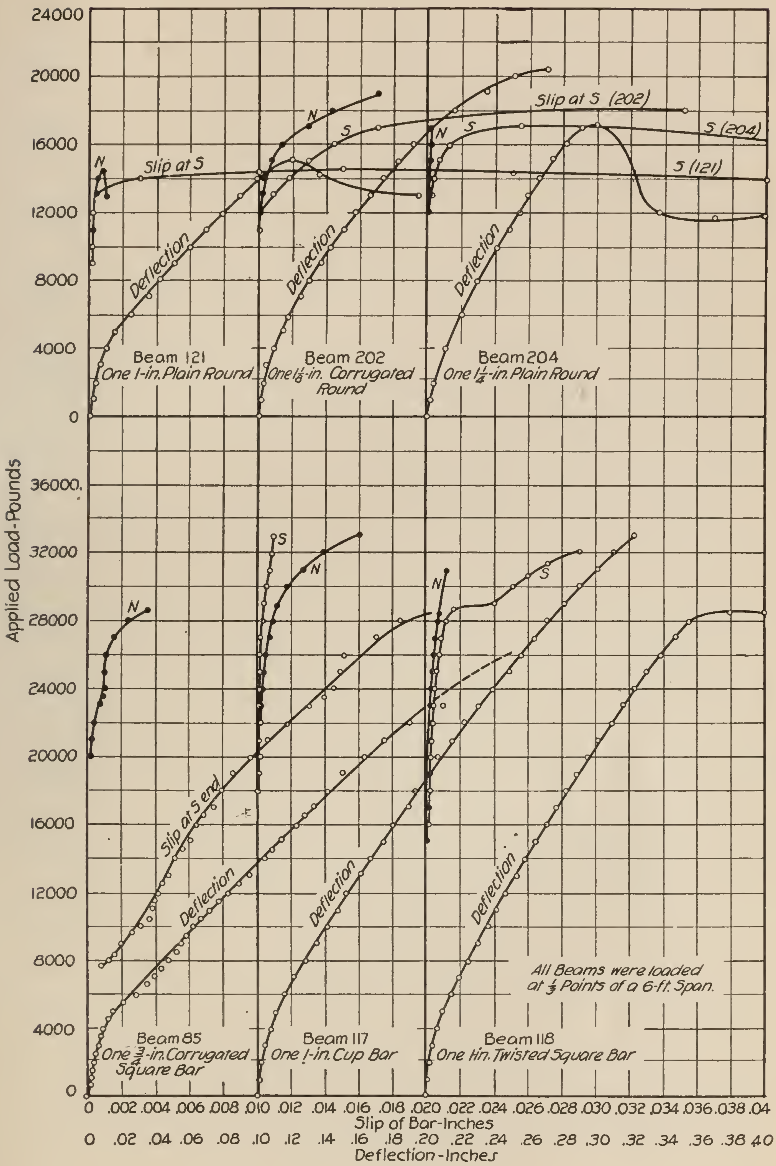

FIG. 77. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

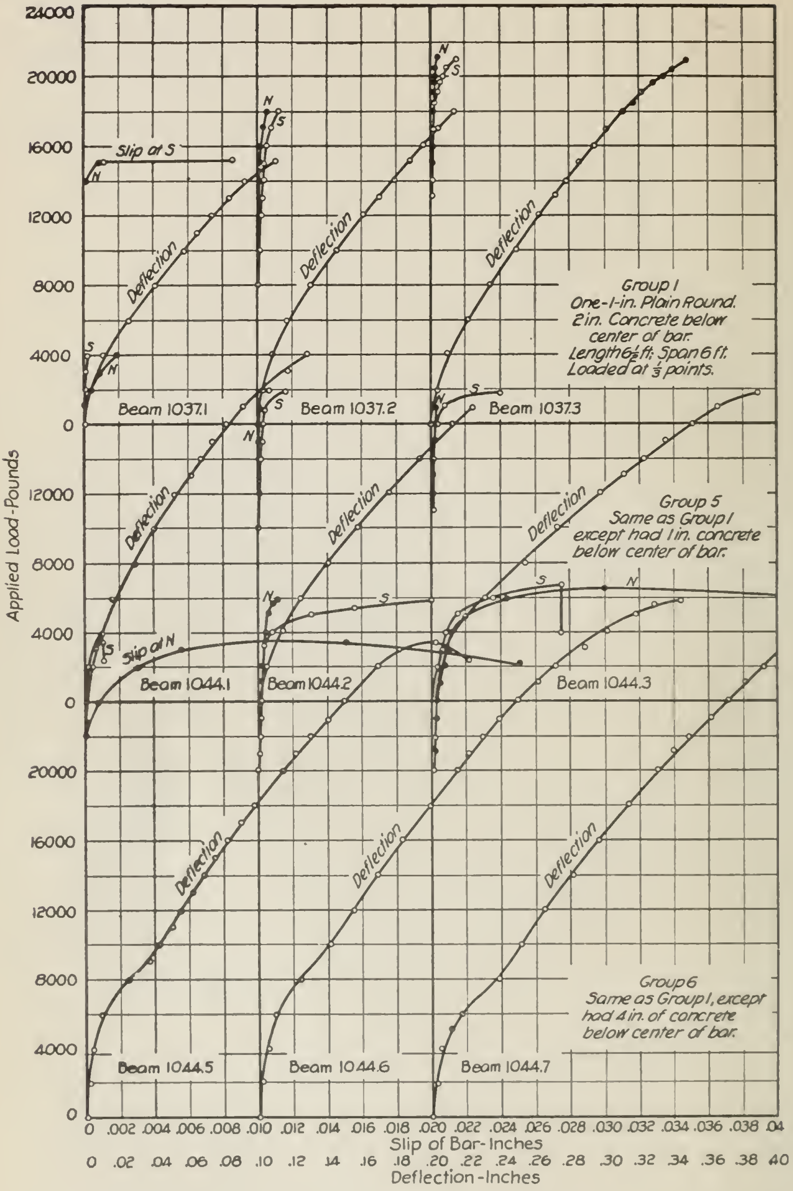

FIG. 78. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

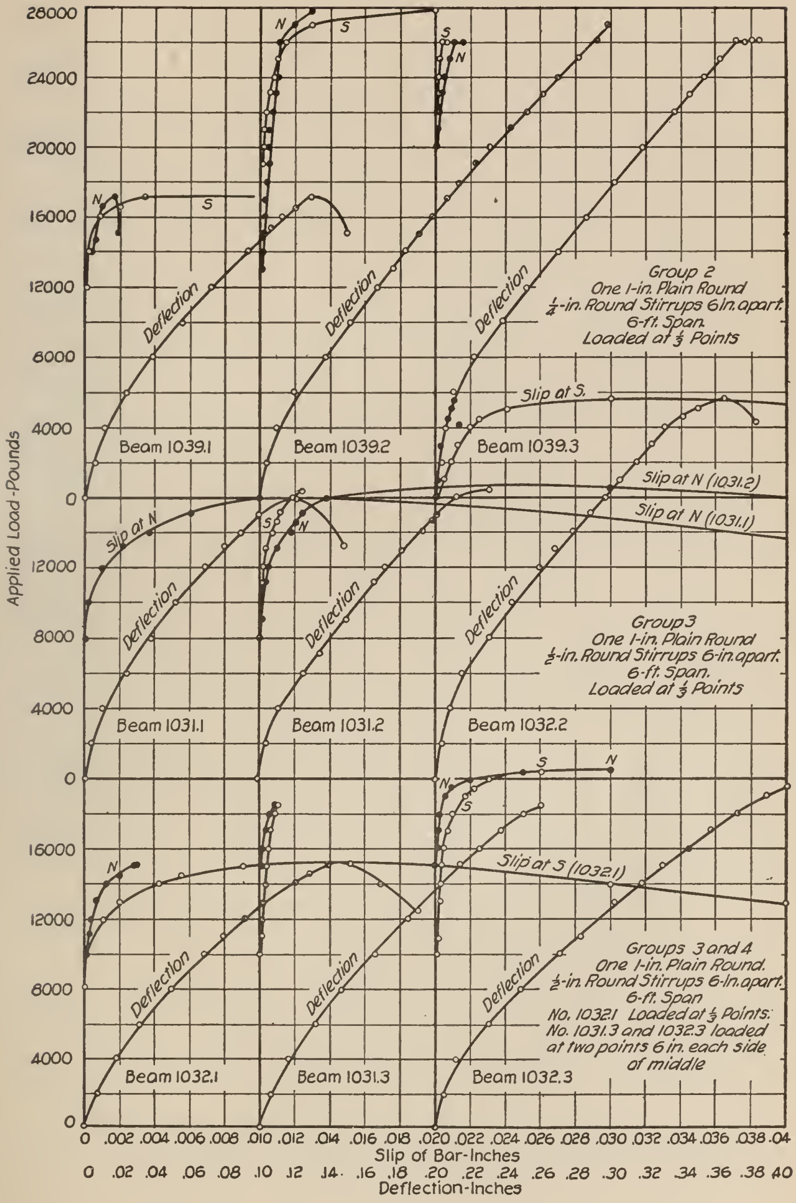

FIG. 79. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

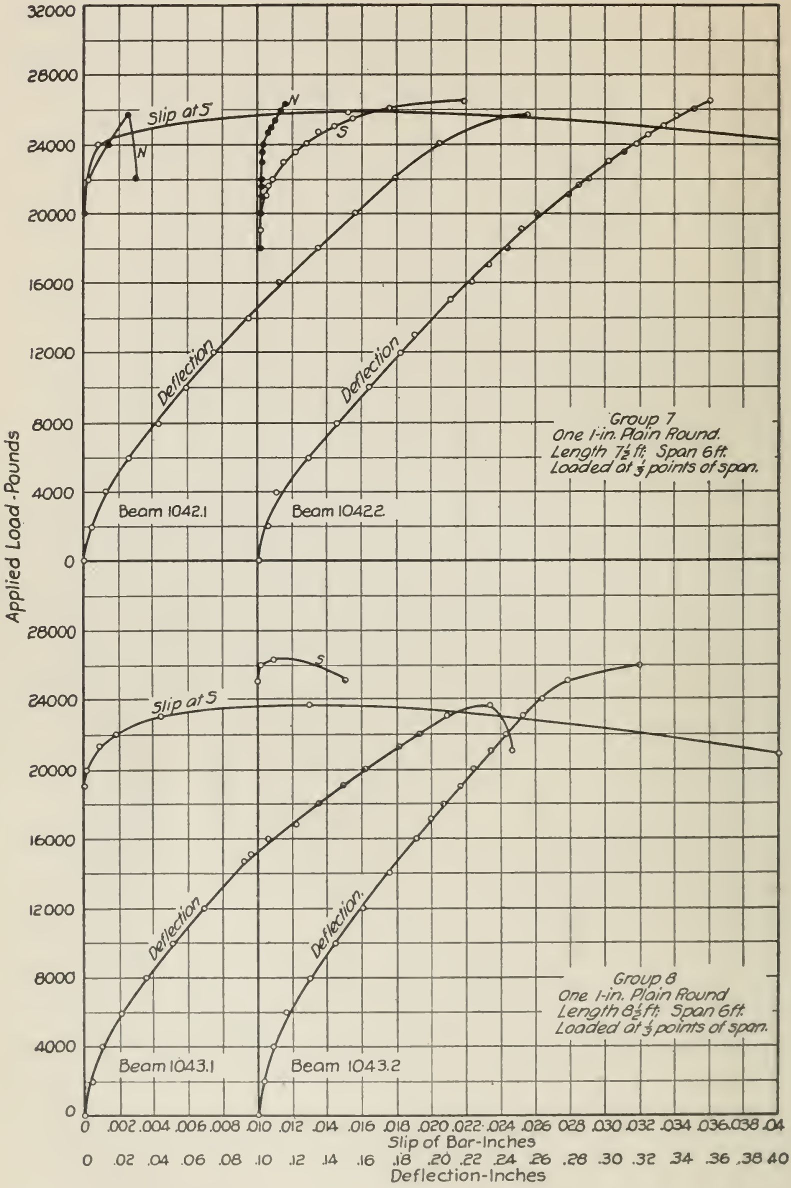

FIG. 80. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

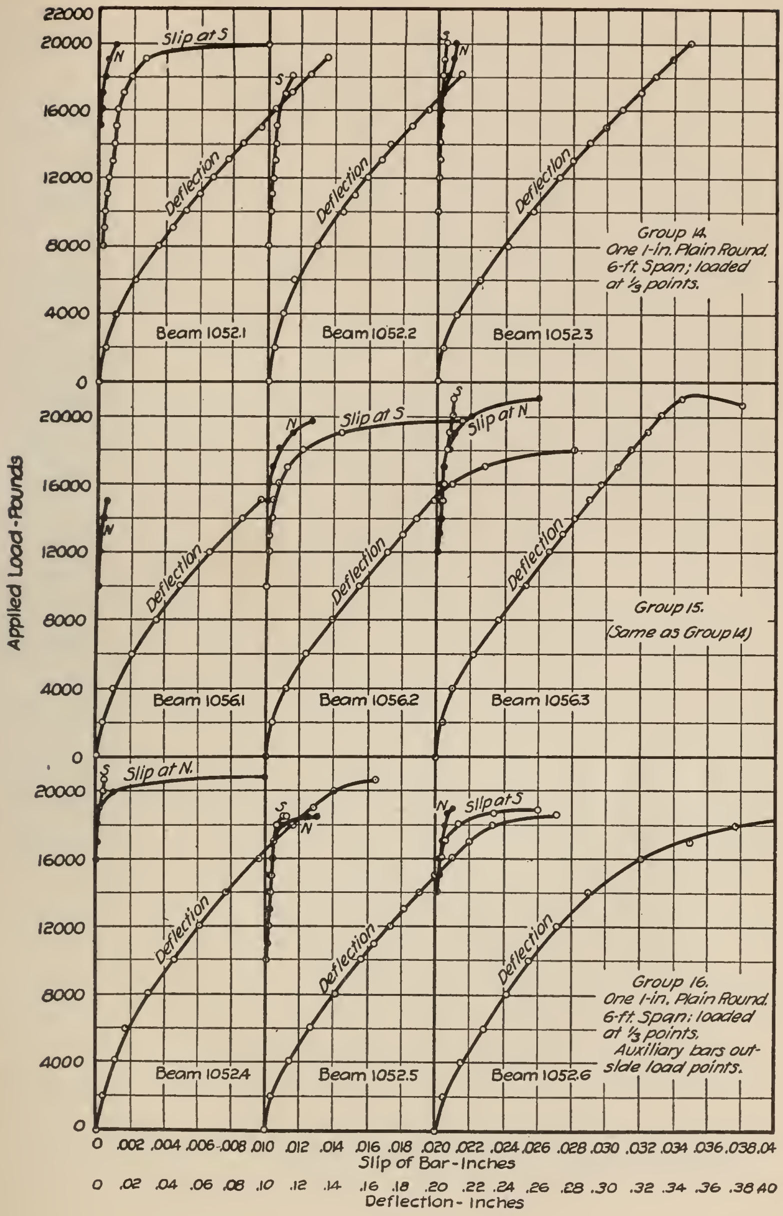

FIG. 81. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

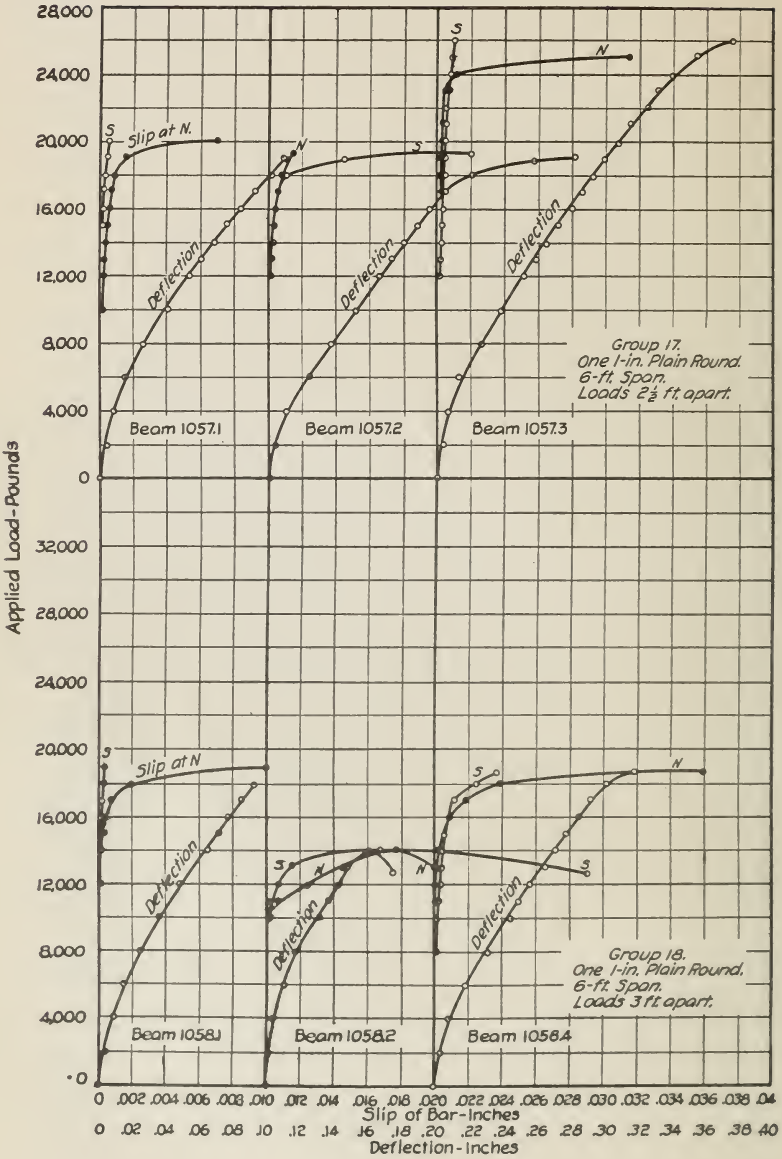

FIG. 82. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

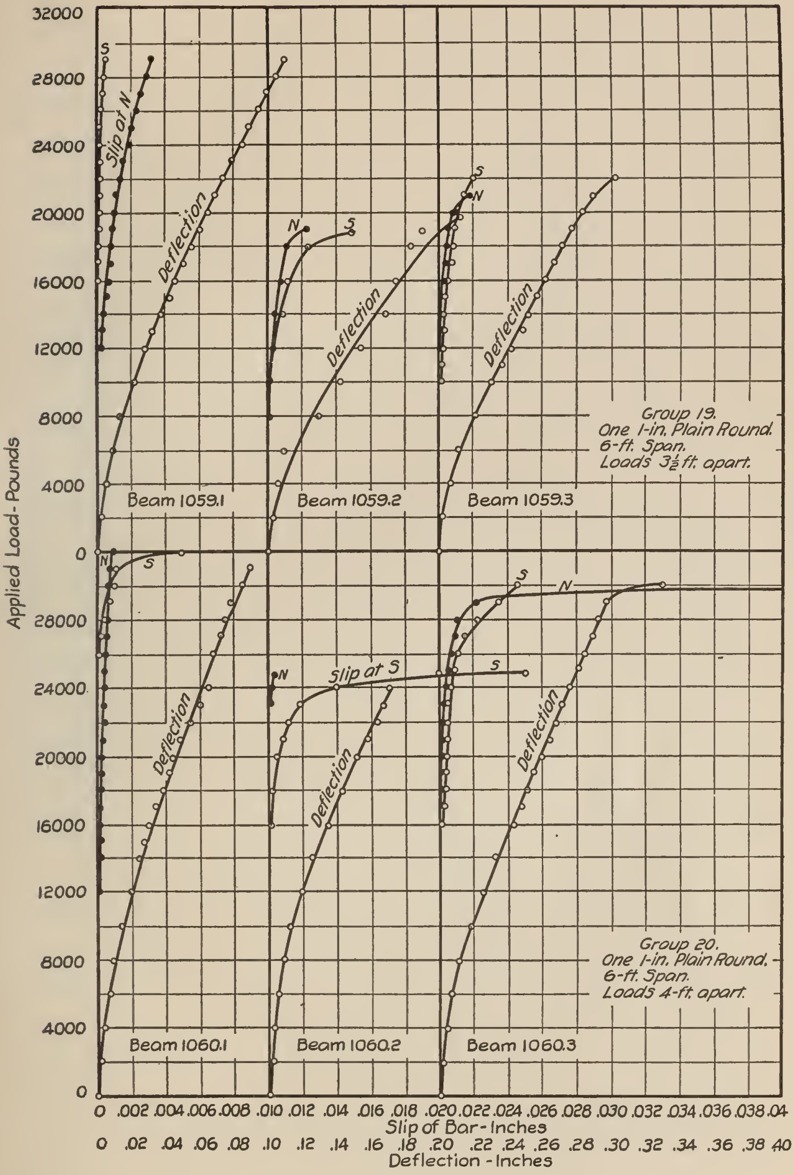

FIG. 83. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

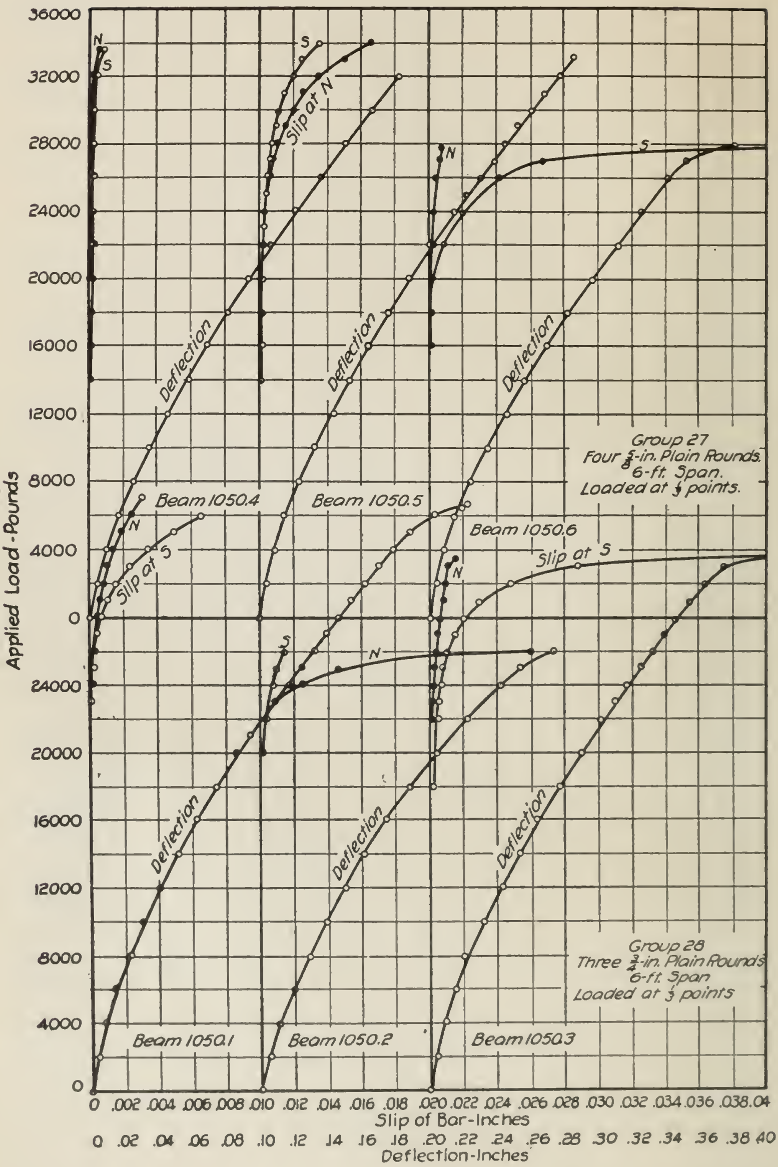

FIG. 84. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

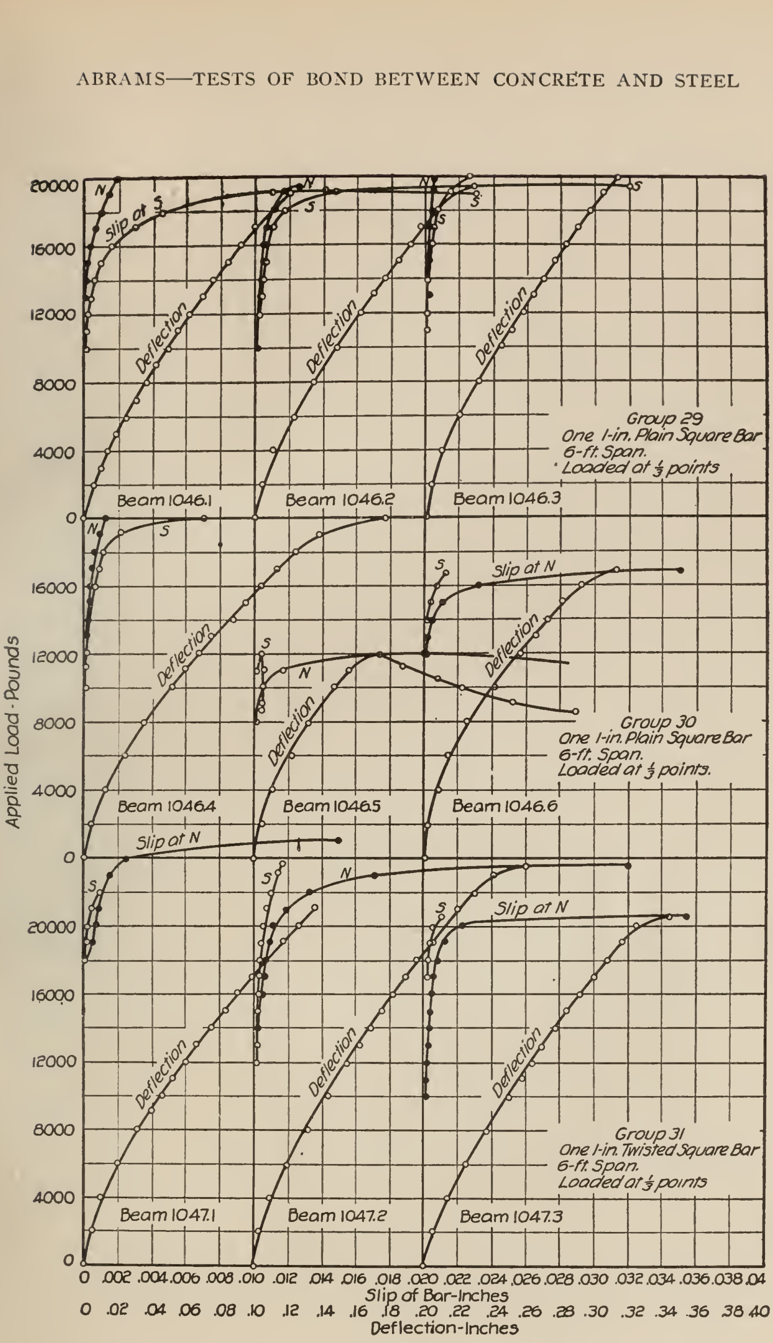

FIG. 85. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

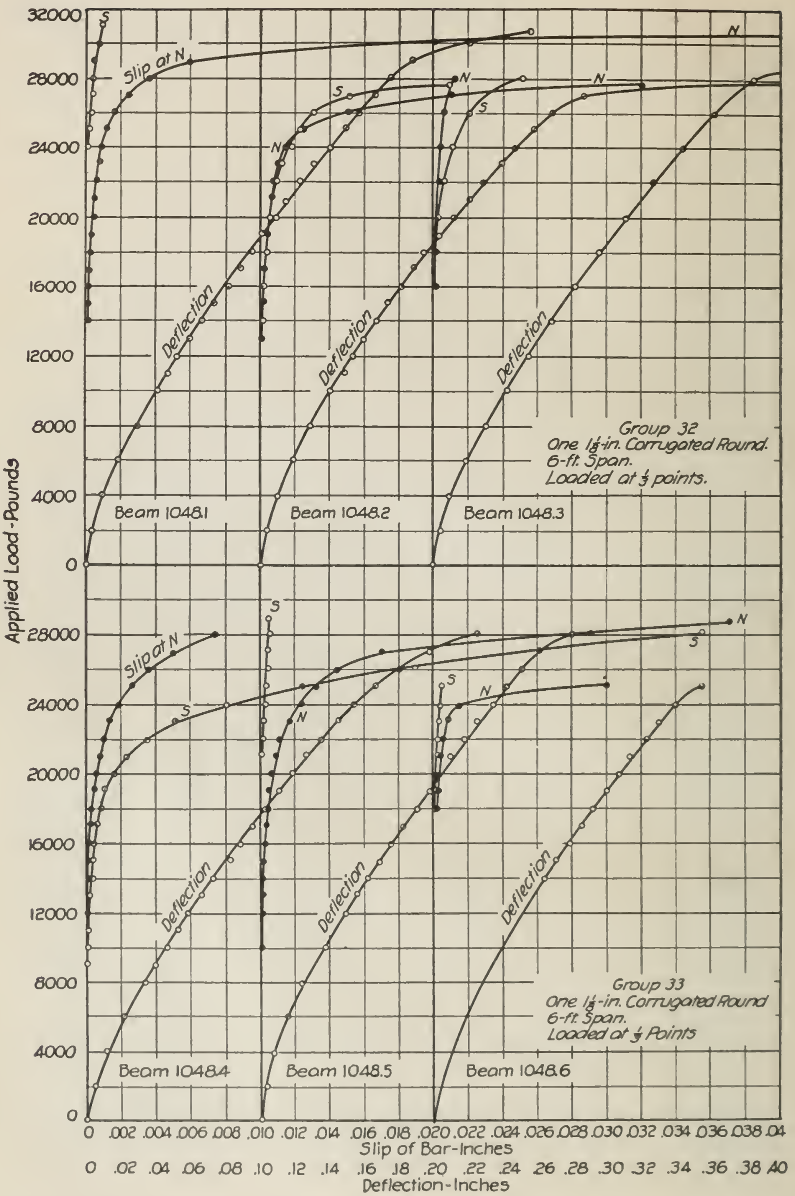

FIG. 86. LOAD-DEFLECTION AND END-SLIP CURVES FOR REINFORCED CONCRETE BEAMS.

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# MAGNETIC AND OTHER PROPERTIES OF ELECTROLYTIC IRON MELTED IN VACUO

BY
TRYGVE D. YENSEN

No. 72
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 72 MARCH, 1914

# MAGNETIC AND OTHER PROPERTIES OF ELECTROLYTIC IRON MELTED IN VACUO

BY TRYGVE D. YENSEN, ASSISTANT, ELECTRICAL ENGINEERING DEPARTMENT, ENGINEERING EXPERIMENT STATION

## CONTENTS

| | | Page |
|---|---|---|
| I. | INTRODUCTION | 3 |
| | 1. Scope of Bulletin | 3 |
| | 2. Historical Review | 4 |
| | 3. Early Experiences | 11 |
| | 4. Acknowledgments | 12 |
| II. | MATERIAL AND APPARATUS | 13 |
| | 5. Electrolytic Iron | 13 |
| | 6. Crucibles | 13 |
| | 7. Vacuum Furnace | 13 |
| | 8. Reheating Furnace | 13 |
| | 9. Pyrometer | 14 |
| | 10. Permeameter | 14 |
| | 11. Conductivity Bridge | 17 |
| III. | DETAILS OF THE EXPERIMENTS | 17 |
| | 12. Cleaning the Iron | 17 |
| | 13. Melting the Iron | 17 |
| | 14. Forging the Ingots into Rods | 18 |
| | 15. Preparing the Test-pieces | 18 |
| | 16. Annealing and Quenching | 19 |
| | 17. Magnetic Tests | 20 |
| | 18. Metallurgical Analyses | 20 |
| | 19. Mechanical Tests | 21 |
| | 20. Chemical Analyses | 21 |
| IV. | RESULTS | 22 |
| V. | DISCUSSION OF RESULTS | 38 |
| VI. | SUMMARY AND CONCLUSION | 47 |
| Appendix I. | Crucibles and Arc Furnace | 49 |
| Appendix II. | Vacuum Furnace | 53 |
| Appendix III. | Reheating Furnace | 57 |
| Appendix IV. | Permeameter | 59 |
| Appendix V. | Bibliography | 70 |

# MAGNETIC AND OTHER PROPERTIES OF ELECTROLYTIC IRON MELTED IN VACUO.

## I. Introduction.

1. *Scope of Bulletin.*—Numerous attempts have been made in recent years to determine the properties of pure iron and the influence of other elements upon these properties. Electrolytically deposited iron of a high degree of purity and the finest grades of Swedish charcoal iron were used in these experiments. On account of the high affinity of pure iron for other elements, especially at high temperatures, it is very difficult to expose it to such temperatures without having it contaminated by the substances with which it comes in contact. Among the elements exerting the most influence upon the properties of pure iron in proportion to the quantities added are carbon and oxygen. It has been shown that pure iron in the molten state will even reduce carbon monoxide and combine with the carbon. Other reducing and neutral gases, such as hydrogen and nitrogen, are partly dissolved by iron in the molten state, and, although again partly dispelled in cooling, are liable to be entrapped and cause blowholes.

On account of the difficulties above enumerated, the only means of melting pure iron without contamination appears to be to melt it in vacuo. Consequently, after numerous attempts with other furnaces, the author constructed a vacuum furnace for melting the electrolytic iron and its alloys with other elements.

It was originally intended to include in this bulletin the results obtained not only with pure iron, but also with certain alloys of iron with other elements. The results obtained with pure iron seem, however, sufficiently interesting to justify devoting one bulletin to its properties alone. There has been included for the sake of comparison: 1, two samples of Swedish charcoal iron, one as cut from the original plate and the other as remelted in the vacuum furnace; 2, three samples of electrolytic iron to which have been added small amounts of carbon; and 3, two samples of commercial steel used for electrical purposes and supplied by the manufacturer. It is expected that other bulletins containing the results of experiments upon various iron alloys will appear in the future. The magnetic, electrical, mechanical and metallurgical properties have been studied. No attempt has been made to study the magnetic properties in very strong fields.

2. *Historical Review.*—To mention only briefly all the researches upon the magnetic properties of iron and other ferromagnetic substances would be beyond the scope of this bulletin. However, it seems desirable to review briefly the experiments, the results of which have a definite bearing upon the present investigation. In Appendix V will be found a bibliography relating to the magnetic and allied properties of iron. While this bibliography is not complete, it covers the field quite thoroughly.

Considering first the researches made upon electrolytic iron as deposited, it is found that electrolytic iron was used as a basis for magnetic investigations as early as the middle of the nineteenth century. Beetz[1]* in 1860 deposited iron under the influence of a powerful magnet, and the plates thus produced were found to be very hard and have a large permanent magnetism. Holz[6] in 1871 made a statement to the same effect. Other investigators, Kramer[2] in 1861 and Klein[3] in 1868, claimed that electrolytic iron is soft and possesses no permanent magnetism. Leick[10] in 1896 deposited iron on brass rods from solutions of ferrous sulphate and ferrous chloride with or without sal ammoniac, and obtained perfectly uniform deposits. The rods were measured as deposited and were found to have a residual magnetism of above 70 per cent of the temporary magnetism, the thinner the deposit the higher the percentage. The coercive force was found to be 20 to 30 c. g. s. units, and the saturation value about the same as for soft iron. For $H = 90$† he obtained an induction of 18,900. Burgess and Taylor[20] in 1906 tested the iron as deposited after being cut into rings. They found the deposit hard and brittle. For $H = 100$ they found $B = 15{,}750$, the retentivity $= 10{,}300$, and the coercive force $= 11$. Schild[21] in 1908 confirmed the results of Burgess and Taylor. In 1910 Terry[226], in an excellent paper that will be referred to later, in the case of iron as deposited and unannealed, found for $H = 100$, $B = 17{,}400$, the retentivity $= 9{,}560$, and the coercive force $= 7.53$.

---

*Numerals refer to the bibliography given in Appendix V.

†The following notation is used throughout this bulletin:

$H$ = magnetizing force = $.4\,\pi n\,I$—gilberts per cm.
$H_c$ = coercive force—gilberts per cm.
$B$ = flux density—gausses.
$B_r$ = retentivity—gausses.
$\mu$ = permeability = $\frac{H}{B}$—gausses per gilbert per cm.

Turning now to the experiments upon soft commercial iron and electrolytic iron after being melted, we find one of the earliest investigators to be Stoletow of Moscow, who in 1873, using a ring of soft iron with two coils wound on it, found that the permeability of iron first increased and then decreased as the magnetizing force increased. At the same time Rowland[5] gave a most important contribution to the knowledge of magnetism of iron. He expressed his results in terms of absolute units, and discovered the shape of the magnetization curve as it is known today. He found for a ring of Norway iron a maximum permeability of 5,515, at that time considered a remarkably high value. In 1885 J. Hopkinson[7] published the results of his investigations on 35 different samples of iron and iron alloys, giving chemical analyses. The specific electrical resistance he found to vary from 13.78 microhms for wrought iron to 100 microhms for cast iron. For wrought iron he found for a maximum induction of 18,251: the retentivity = 7,248, the coercive force = 2.30, the energy loss = 13,356 ergs per c. c. per cycle. He employed the double bar and yoke method of testing, but instead of reversing the magnetizing current, as the practice is today, he jerked the rod to be tested out of its solenoid and obtained a kick of the galvanometer needle proportional to the magnetism in the rod. In 1896 Parshall[9] published additional data upon the magnetic properties of commercial iron. In 1900 and 1902 Barrett, Brown and Hadfield[15] published what will probably be regarded as the most important results ever published on the magnetic and electrical properties of iron and iron alloys. They used as their standard iron Swedish charcoal iron, containing the following impurities:

C — .028%
Si — .070%
S — .005%
P — .044%
Mn — trace

Considering first the specific resistance, these investigators found that their pure iron standard had a resistance of 10.2 microhms per c.c. and that 1 per cent of any element added to pure iron increased its specific resistance by an amount approximately proportional to the specific heat or inversely proportional to the atomic weight of the alloying element. This relation was first suggested by Le Chatelier[13] in 1898, and was confirmed by Barrett[19] in 1902, who published the following table:

### TABLE 1.

### EFFECT OF DIFFERENT ELEMENTS UPON THE ELECTRICAL RESISTANCE OF PURE IRON. (FROM BARRETT.)

| Alloy of Iron with | Increase in Spec. Resist. for 1% Added of Element of Col. 1. microhms | Spec. Heat of Alloying Element | Atomic Weight of Alloying Element |
|---|---|---|---|
| Tungsten | 2.0 | .035 | 184 |
| Cobalt | 3.0 | .107 | 59 |
| Nickel | 3.5 | .109 | 59 |
| Cromium | 5.0 | .100(?) | 52 |
| Carbon | 5.0 | .160* | 12 |
| Manganese | 8.0 | .122 | 55 |
| Silicon | 13.0 | .183 | 28 |
| Aluminum | 14.0 | .212 | 27 |

*For graphite

Considering next the magnetic properties of the 100 or more rods investigated by Barrett, Brown and Hadfield, it is found that the only alloys that are more magnetic than the purest commercial iron are those containing small percentages of silicon and aluminum. Carbon appears to be the element most detrimental to magnetic quality. The magnetization curves for the iron standard together with their best silicon alloy, containing 2½ per cent Si., and their best aluminum alloy, containing 2¼ per cent Al. are shown in Fig. 1. These curves were obtained after careful annealing. In Table 2 is given some additional data for these rods.

### TABLE 2.

### SUMMARY OF THE MAGNETIC AND ELECTRICAL PROPERTIES OF HADFIELD'S BEST MAGNETIC IRON. (FROM BARRETT, BROWN & HADFIELD.)

| Rod No. | Description of Iron | Maximum Permeability $\mu_{max}$ | B for $\mu_{max}$ | Hysteresis Loss for $B_{max}$ =9,000. Ergs per c. c. per cycle | Hysteresis Loss for $B_{max}$ =4,000. Ergs per c. c. per cycle | Retentivity for $B_{max}$ =17,700 | Coercive Force for $B_{max}$ =17,700 |
|---|---|---|---|---|---|---|---|
| S. C. I. | Stand. | 2,100 | 4,000 | 2,334 | 638 | 10,800 | 1.10 |
| 898E | 2½% Si | 5,000 | 4,000 | 1,549 | 436 | 8,000 | 0.80 |
| 1,167H | 2¼% Al | 5,400 | 5,000 | 1,443 | 386 |  | 0.80 |
|  | Lohy.* |  |  |  | 537 |  |  |

*From Messrs. J. Sankey & Sons.

In 1901 Gumlich and Schmidt[16] published results obtained at the Physikalische Teknische Reichsanstalt. They investigated a large number of commercial irons. Their best iron, magnetically, had a maximum permeability of 8,350 with the retentivity = 10,300, the coercive force = 0.6, and the electrical resistance = 11.3 microhms.

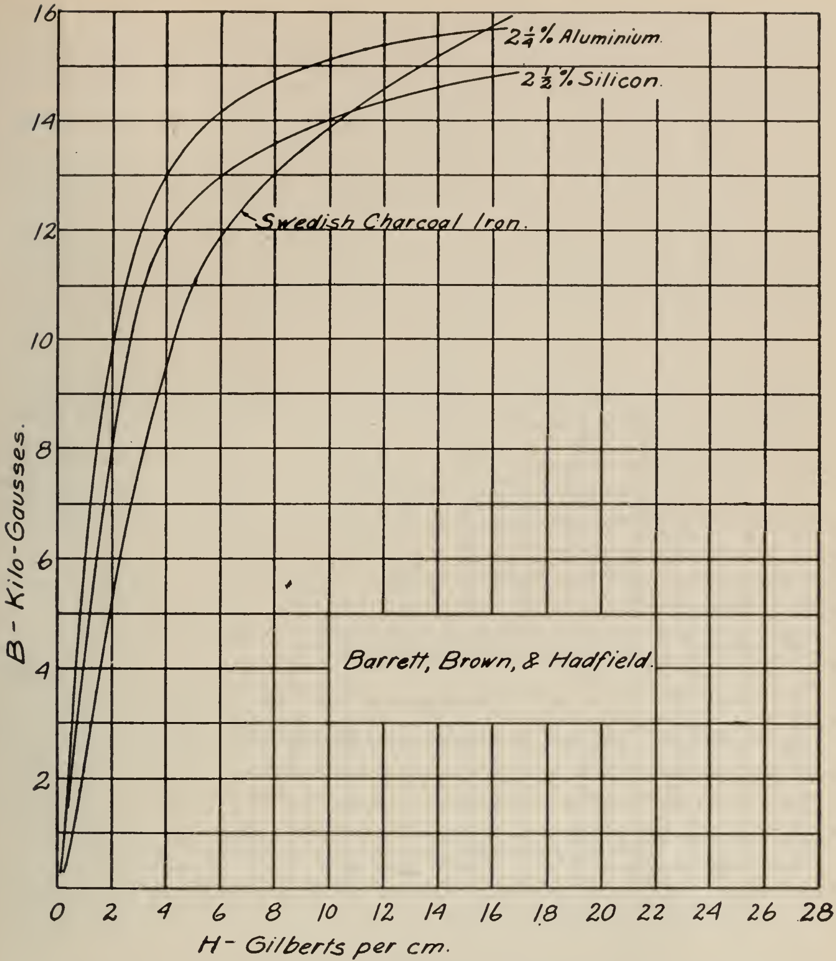

FIG. 1. HADFIELD'S BEST MAGNETIC IRON. THOROUGHLY ANNEALED.

In 1909 and 1910 Burgess and Aston published a series of articles on the magnetic properties of electrolytic iron after being melted[28], and on the influence on these properties of arsenic[22-23], antimony[28], tin[23], bismuth[22], silicon[27], copper[26] and nickel[25]. They also published an article comparing the magnetic properties of electrolytic iron with those of some commercial steels[28]. The iron used by them was doubly refined electrolytically of the following composition: C =.012 per cent, Si = .01 per cent, Fe = 99.97 per cent. This iron was melted in a resistance furnace in a covered magnesia crucible placed inside a covered graphite crucible, buried in crushed carbon. The ingots thus produced were forged into rods and turned. After being melted and forged, the carbon content was found upon analysis to vary from .012 per cent in some specimens to 0.12 per cent in others, showing that the iron had absorbed carbon in the process. On account of this variation in absorbed carbon, and because also of the variation in mechanical treatment, which can hardly be altogether avoided, the magnetic properties varied considerably, even for samples with no alloying element added to them. Table 3 gives the magnetic properties of their best electrolytic iron rod, No. 117A.

### Table 3.

### Summary of the Magnetic Properties of Burgess' Best Electrolytic Iron (From Burgess and Aston). Rod No. 117A.

| Heat Treatment | Values of B in Gausses for | | | | Coercive Force | Retentivity | Specific Elec. Res. microhms |
|---|---|---|---|---|---|---|---|
| | H=10 | H=20 | H=50 | H=100 | | | |
| Unannealed. | 13,100 | 15,750 | 17,600 | 18,850 | 5.5 | 12,300 | 12.1 |
| Ann. 675° C. | 10,000 | 15,950 | 17,700 | 18,850 | 6.2 | 13,800 | |
| Ann. 1000° C. | 13,350 | 15,450 | 17,050 | 18.200 | 3.1 | 10,000 | |
| Quenched 900° C. | 11,800 | 14,600 | 16,950 | 18,150 | 2.5 | 8,000 | |

This result shows an improvement over the Swedish charcoal iron.

Of the various alloying elements investigated, arsenic (3 to 4 per cent), tin (1 to 2 per cent), and silicon (up to 5 per cent) were found to increase the permeability slightly at low densities and bismuth at high densities, while copper, manganese, antimony and nickel were found to decrease the permeability to a greater or less extent. The electrical resistance of the electrolytic standard measured 12.1 microhms.

Guggenheim[29] in 1911 has shown, for iron containing 0.2 per cent carbon, that silicon in quantities up to 1.8 per cent decreases the permeability, but that from 1.8 to 5.0 per cent silicon improves the permeability and decreases the hysteresis loss. He found for $B_{\max} = 10{,}000$ in sheets $\frac{1}{2}$ mm. thick the following values for the hysteresis loss:

For ordinary sheet iron....6,000 ergs per c. c. per cycle
For best silicon steel......2,910 ergs per c. c. per cycle

Later, Hamlen and Rossiter[30] gave some data for Stalloy, the trade name for a commercial transformer steel. The maximum permeability was found to be:

$\mu_{\max}$ for unannealed Stalloy........3,100 at $B = 4{,}500$
$\mu_{\max}$ for annealed Stalloy..........4,200 at $B = 7{,}000$

The hysteresis loss = 2,500 ergs per c. c. per cycle for $B = 10{,}000$.

In 1912 De Nolly and Veyret[31] summarized their results in the following recommendation for dynamo iron:

Composition: Carbon < 0.1 per cent, silicon = 4 per cent, manganese < 0.3 per cent, sulphur and phosphorus as little as possible. The final iron should be annealed at 800° C. and slowly cooled.

Besides the researches upon iron at ordinary temperatures in low and medium fields, a large amount of work has been done to determine the properties of iron in intense fields and at temperatures ranging from —200° C. to +1,300° C. Among the earliest investigators working with iron in intense fields are found Ewing and Low, who, in 1887[101] and 1889[102], published the results of their investigations on the properties of iron in fields ranging from 3,000 to 25,000 gilberts per cm. To obtain these fields they employed test-pieces machined into the shape of double cones with a short "isthmus" at the apex. They found the following values for the saturation intensity of magnetization, I:

For "Lowmoor" iron $I = 1{,}680$ for $H = 6{,}000$
For Swedish iron $I = 1{,}700$ for $H = 6{,}000$

DuBois[103] in 1890 obtained 1,700 to 1,750 for $H = 2{,}500$. Gumlich[104] in 1909, using electrolytic iron, obtained 1,725 for $H = 6{,}000$. The most important investigation in this field is that by Hadfield and Hopkinson[105], who investigated the saturation values for I (they termed this value the "Magnetism" or the "Specific Magnetism" of the substance) for a large number of alloys. Their results are summarized as follows:

1. Every alloy has a definite saturation intensity ("specific magnetism"), in most cases reached for $H = 5{,}000$. Alloys behave as if consisting of a mixture of magnetic and non-magnetic substances.
2. No alloy has a higher specific magnetism than pure iron.
3. The specific magnetism for pure iron of density 7.80 is 1,680.
4. The effect of carbon is to decrease the specific magnetism by a percentage equal to six times the percentage of carbon. As the percentage of iron carbide is 15.5 times the percentage of carbon, the specific magnetism of iron carbide is 2/3 that of pure iron.
5. Quenching iron-carbon alloys from a high temperature decreases the specific magnetism by a large but uncertain amount.
6. Additions of silicon or aluminum reduce the specific magnetism as if they are inert materials. Silicon seems to neutralize the effect of carbon to some extent.

The effect of temperature upon the magnetic properties of iron was studied as early as 1874 by Rowland[201], followed by a large number of investigators (see Appendix V). Fleming and Dewar[213] in 1896 found the permeability to be lower at —200° C. than at ordinary temperatures. For Sankey's best transformer iron they found for a temperature of — 185° C. $\mu_{max} = 2,800$, and for a temperature of + 15° C. $\mu_{max} = 3,300$ for $B = 7,000$. The hysteresis loss was found to be the same at both temperatures, namely 2.1 watts per pound per 100 cycles (= 3,000 ergs per c. c. per cycle), for $B_{max} = 10,000$.

Morris[217] in 1897 used Swedish charcoal iron in the form of rings. For Sankey's best transformer iron he found that for small values of H the permeability increased as the temperature was raised, reaching a maximum of 15,000 at a temperature of about 765° C. When the temperature was raised still further the permeability was found to drop off very rapidly and become nearly 1 at 780° C. For higher values of H, $\mu$ decreased steadily, until it made a sudden drop at the same critical temperature. The maximum value of $\mu$ at ordinary temperatures was found to be 4,580. Wills[222] in 1900 confirmed the results of Morris. He found a maximum value of permeability of 17,228 for $H = 0.172$ at a temperature of about 730° C., dropping almost to unity at 780° C. Honda and Shimizu[224] in 1905, using ovoids, determined the critical points for iron, cobalt and nickel, and found the magnetism in iron to disappear at 785° C., in cobalt at 1,100° C. and in nickel at 350° C.

In 1910 Terry published the results of a very extended investigation upon electrolytic iron as deposited. He summarized his results as follows:

1. Freshly prepared iron is very hard magnetically.
2. Different samples show marked dissimilarities at low fields which disappear upon annealing at 1,000° C.
3. For large fields, all samples are quite similar, the values obtained for susceptibility being intermediate between those of Leick and Schild.
4. Plunging into liquid air produces no permanent hardening.
5. The retentivity has a maximum in the neighborhood of room temperature.
6. Marked softening occurs magnetically at the hydrogen transformation points.
7. Ferro-magnetism reappears on cooling at the same temperature at which it disappears on heating. This temperature is 785° C.

8. The depression in the permeability-temperature curves obtained by Morris does not exist for this iron.

9. The best temperature for annealing is 1,100° C.

10. Although when properly annealed it has a lower coercive force and a higher permeability than Swedish iron, its high retentivity causes a large hysteresis loss.

From this brief review the following general conclusions may now be drawn:

1. No better agency than pure iron exists for conveying magnetism.

2. The purest iron obtainable is electrolytic iron. This iron as deposited is, however, magnetically hard but may be cured by certain heat-treatments.

3. As pure iron at high temperatures is very readily contaminated by elements that decrease its magnetic properties, it is necessary to protect it, by some means or other, from contamination, if subjected to high temperatures, and particularly if melted.

4. As it is almost impossible with ordinary methods to melt pure iron without having it slightly oxidized, it may be deoxidized by adding to it some deoxidizing agent such as silicon or aluminum, elements that have a higher affinity for oxygen than iron.

5. It is highly desirable that the electrical resistance of iron for magnetic purposes be as high as possible. It is, therefore, very fortunate that the deoxidizing agents mentioned under 4 have the property of increasing the electrical resistance to a very large extent, thus lowering the eddy current losses in the iron when subjected to alternating currents.

As already mentioned, it is not the intention to take up in this bulletin the effect of these deoxidizing agents upon pure iron. This will be deferred to some later date. This bulletin will show to what extent iron may be protected without the use of these agents, namely by melting it in vacuo.

3. *Early Experiences.*—The author of this bulletin took up the investigation of the magnetic properties of iron and iron alloys during the summer of 1911 and, with a short interruption, has continued it since that time. Electrolytically deposited iron has been the basis for the investigation. At first a Hoskins resistance furnace was used for the melting of the iron. The crucibles, after being well protected by two covers, were placed in the heating chamber, heated until the iron was melted and left until cool enough to handle. It was supposed that the atmosphere in the furnace was sufficiently reducing to prevent oxida-

tion of the iron, as the resistor consisted of carbon plates, but it was soon discovered that the iron was so badly oxidized that it went to pieces in the attempt to forge the ingots at forging temperatures. Little trouble was had in forging ingots heated to dull redness.

After this first experience the crucibles were covered with crushed carbon following substantially the method of Burgess and Aston. While no serious oxidation took place under these conditions, it was impossible to obtain uniform results. This was the case both with the electrolytic iron and with the Swedish charcoal iron. Attempts were made to use a definite fineness of carbon and to determine what effect changing the fineness had upon the results, but no relation could be discovered. Fifteen electrolytic and fifteen Swedish iron rods were made, and there are hardly two rods that have the same B-H curve either as forged or after being annealed at 900° C. The magnetic and electrical data for eight of the electrolytic iron rods are given in Table 4 for the sake of comparison. From these results it is seen that relatively large quantities of carbon were absorbed by the iron. On account of this difficulty it was decided to construct a vacuum furnace for the melting of the iron. While the results obtained with this furnace are not entirely uniform, the non-uniformity must be attributed chiefly to the unavoidable differences in the mechanical treatment of the rods.

4. *Acknowledgments.*—The author wishes to express his gratitude to a large number of persons connected with the University of Illinois, who have rendered very valuable assistance in carrying on this investigation.

Among these he wishes particularly to mention Prof. E. J. Berg and Prof. E. B. Paine for their interest in the work as heads of the Electrical Engineering Department, Dr. Grinnell Jones, now of Harvard, for assistance in the production of the electrolytic iron, Prof. S. W. Parr for opening his laboratory for the chemical determinations, and Mr. J. M. Lindgren for making the determinations, Dr. O. F. McFarland for the metallurgical part of the investigation, Mr. Rudolph McDermott for the critical temperature determinations and for assistance in general, and Mr. J. H. Belt for electrical resistance determinations.

The Ceramics Department has been of valuable assistance in connection with making the crucibles in which to melt the iron. Finally, a number of electrical manufacturing companies have shown their willingness to co-operate in the investigation by going to great trouble and expense in preparing rods from various grades of iron for the purpose of comparison.

## II. MATERIAL AND APPARATUS.

5. *Electrolytic Iron.*—The electrolytic iron used in this investigation was obtained by methods developed by Burgess[301]. As a considerable amount of work has been done recently by Watts and Li[302] to improve upon the methods originally used by Burgess, the details of the refining plant constructed by the author will be omitted, and only the chief features will be mentioned. The electrolyte used consisted of ferrous sulphate and ammonium chloride. Swedish charcoal iron was used as anodes, having the following impurities:

Si — .032%
S — .0002%
C — .163%
P — none
Mn — none

The Swedish iron was first deposited upon lead cathodes, and these were used as anodes in the second refining. The chemical analysis of the doubly refined iron shows that it contains C — .006 per cent, Si — .01 per cent. This iron, of a purity of 99.97 to 99.98 per cent, is the material that has been used as the basis for the present investigation. The iron, as deposited, was very rough and nodular, but for the purpose intended this condition was of no importance as the iron had to be broken into small pieces.

6. *Crucibles.*—The crucibles for melting the electrolytic iron were made from electrically fused magnesia, practically free from iron but containing about 2 per cent silica. They measure 3¼ in. (8.3 cm.) outside diameter and 4 in. (10 cm.) in height, holding 500 to 600 grams of crushed electrolytic iron. The arc furnace for producing the magnesia and the method of making the crucibles are described in Appendix I.

7. *Vacuum Furnace.*—As already mentioned in the introduction, a Hoskins resistance furnace was at first employed for melting the iron, but on account of the contamination of the iron with oxygen and carbon, this was superseded by a vacuum furnace. The details of this furnace are given in Appendix II. Suffice it to say here that it is of the Arsem type and was constructed in the shop of the Electrical Engineering Department. It is capable of melting 500 to 600 grams of pure iron in half an hour with a vacuum of less than 0.5 cm. of mercury.

8. *Reheating Furnace.*—The reheating furnace employed for annealing the test-pieces is described in detail in Appendix III. It was modeled after the Hoskins carbon plate resistance type furnace and was

constructed in the department shop. For the critical temperature investigation of the iron a small Hoskins muffle furnace was employed.

9. *Pyrometer.*—All temperature determinations were made with a standard platinum-platinum + 10 per cent rhodium pyrometer. This was calibrated at intervals by means of the melting points of copper (1,083° C.), aluminum (658° C.), and tin (232° C.) in a reducing atmosphere and the boiling points of sulphur (444.7° C.) and water (100° C.), all of standard purity. The calibration curve was found to be practically constant, uniformly low by 6 to 8° C. In the early part of the investigation the cold junction was kept at room temperature and the galvanometer readings corrected by adding one-half of this, but later a vacuum bottle was provided for keeping this junction at 0° C. Electroquartz protecting and insulating tubes were used throughout.

10. *Permeameter.*—A number of different instruments for the magnetic measurements were tried and discarded. As it was desirable to make a large number of test pieces, Rowland's ring method was impracticable, both on account of the difficulty of construction and because of the difficulty of electrical resistance measurements. An instrument was needed that could measure accurately the magnetic properties of relatively short rods. The one finally decided upon was constructed by the author in accordance with recommendations made by the Bureau of Standards[402]. While this instrument requires considerable time for the actual measurements, no corrections need ordinarily to be made, and the meters read directly in B and H. The apparatus with all its accessories is fully described in Appendix IV. Briefly stated, it consists of a double bar and yoke, with one main and one auxiliary solenoid, separately operated, and four compensating coils in series, next to the yokes. By means of three secondary, or search coils, next to the bars, the magnetic flux can be investigated at different points of the magnetic circuit and can be equalized by adjusting the currents in the various magnetizing coils. With no leakage the magnetizing force for the rod measured is

$$H_T = .4\pi n_T I_T$$

where $n_T$ = number of turns per cm. length of the main solenoid, and $I_T$ = current in amperes of the main solenoid.

It is shown in Appendix IV that the correction to $H_T$ on account of the end effects of the various magnetizing coils is within + 1 per cent of $H_T$ for ordinary iron, and is within + 4 per cent for the highest permeability found for electrolytic iron melted in vacuo. These corrections have not been made in the results reported in this bulletin for reasons to be shown below.

For the determination of "B" a Leeds and Northrup ballistic galvanometer was used in the early part of the work. The period of this galvanometer was adjusted to 15 seconds, and its resistance was 1,800 ohms. It could be calibrated without changing the galvanometer circuits by means of a mutual inductance (also described in Appendix IV) consisting of a primary coil 90 cm. long by 10 cm. diameter and a secondary coil about 8 cm. long, wound over the middle portion of the primary. This galvanometer served the purpose very well, until the vacuum iron was to be measured. This iron was found to have such high permeability, particularly after being annealed, and such low electrical resistance that the eddy currents produced in the rod as well as the high inductance of the coils made the change of magnetism too sluggish for accurate determinations. This was particularly true for hysteresis measurements. in changing from $B_{max}$ to some point on the descending part of the loop, such as $B = -\frac{1}{2} B_{max}$, the change would require as much as five or six seconds before being completed. To remedy this difficulty a Grassot fluxmeter was obtained for the final measurements. In this instrument the suspension effect has been practically eliminated, and the deflection is independent of the time required by the flux to complete the change. All the different connections for operating the permeameter are made through rocking mercury switches operated by means of keys similar to piano keys. In this manner any number of switches up to ten may be operated simultaneously, and comparatively little time is occupied in making the adjustments.

In Fig. 2 is shown the magnetization curves for the standard rod No. 3-33B, as obtained, first, by the Bureau of Standards, second, by means of the author's permeameter using the Leeds and Northrup galvanometer, and third, by using the Grassot fluxmeter. The second curve was obtained by first making adjustments by means of the fluxmeter and then by making the final measurement with the galvanometer. The discrepancy between the first and third curves at low and medium densities may be partly explained in this way: The rod as measured by the Bureau of Standards was covered with a thin film of oxide, preventing perfect contact between the rod and the yokes. On this account a comparatively large compensating current was necessary to obtain equality of flux for low values of H. The author tested the bar upon its return from the Bureau of Standards and obtained figures somewhat lower than but

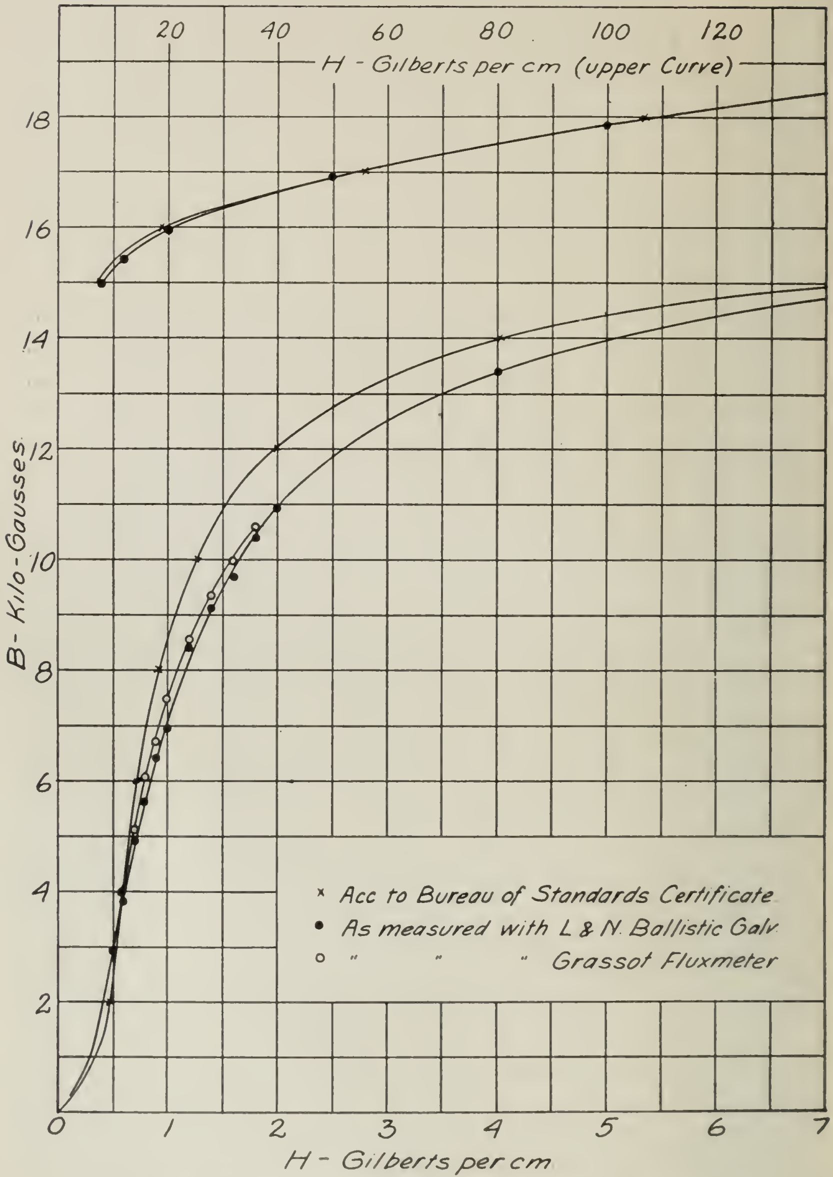

FIG. 2. MAGNETIZATION CURVES FOR STANDARD ROD. No. 3-33B.

quite close to those submitted by the Bureau. However, when it later on was found necessary to clean the contacts of the rods in order to keep the compensating currents within proper limits, the standard rod was also cleaned and remeasured. The curves shown in Fig. 2 are the results of this final test. It may be that these results are due to the effect of the cleaning process upon the magnetic properties of the iron. But this is doubtful, as the cleaning was done by means of light hand rubbing with rather fine emery cloth. Again it may be, as will be discussed later, that the discrepancy is due to some slight straining of the rod as clamped between the yokes. At any rate the author's curve, for low and medium densities, is below that of the Bureau of Standards, and he feels, therefore, that the results given in this bulletin, if they be slightly in error, are on the safe side. For this reason no corrections of $H_T$ due to the end effects, referred to above, have been made.

11. *Conductivity Bridge.*—The electrical resistance was measured on a Thompson double bridge using 13 cm. of the rods between the contacts. The bridge was checked by means of a potentiometer and standard cell and the discrepancy found to be less than 1 per cent.

## III. Details of the Experiments.

12. *Cleaning the Iron.*—The electrolytic iron before being placed in the crucible to be melted was thoroughly cleaned as follows: The iron was crushed into small pieces and the requisite amount for one melt, from 500 to 600 grams, weighed out and placed in a bath of 1:3 hydrochloric acid solution until all rust had disappeared. The acid was then removed by introducing boiling distilled water through a glass tube that at the same time served as a stirrer. This process was continued until the solution was perfectly clear and free from acid. The water was then poured off and any water adhering to the iron dissolved by means of two baths of denatured alcohol from which the moisture had been removed by quicklime. After such a treatment the iron might be left exposed to the air for a day or more without rusting. However, during these experiments it was immediately transferred to a crucible, covered by means of a magnesia cover, and placed in the furnace.

13. *Melting the Iron.*—As mentioned in the introduction, an ordinary resistance furnace was used in the early part of the investigation. The covered magnesia crucible was placed inside a covered graphite crucible and buried in crushed graphite. The melting was completed in about three hours, the current was then cut off and the furnace left to cool.

In the vacuum furnace the magnesia crucible was placed in a low graphite receiver as shown in Appendix II. The furnace was exhausted, first by means of a water aspirator capable of reducing the pressure to 4 or 5 cm. of mercury, and then by means of a Geryck double cylinder pump. A vacuum of 0.5 cm. was deemed sufficient for this work, although it was less at times. As soon as this vacuum was reached the furnace was heated up, slowly at first, so as to give the absorbed gases time to get away without too much loss of vacuum. When vacuum was regained and the resistor had become dull red, requiring about one hour, full load was put on and left on for the length of time required to thoroughly melt the charge. The time required was ascertained by means of a preliminary test, in which a cover with a small hole in the center was used on the crucible. The condition of the iron could then be watched through the window in the top of the furnace. About 15 kw. for 30 minutes was usually sufficient to melt the charge, but in order to insure homogeneity about 12 kw. was left on for another 15 minutes. The furnace was then left to cool.

14. *Forging the Ingots into Rods.*—When removed from the furnace the ingots were as bright as silver and so soft that they could be cut with a knife. They were reheated in a coke forge and forged under a steam or trip hammer. No difficulty was experienced in the process, but care had to be used not to heat the ingots too hot, as the iron burned very readily, and after overheating it was liable to go to pieces under the hammer. An ordinary forging temperature, dull to bright orange, has been found the most suitable. In order to reduce the size of the crystals the forging was continued until the rods were black. The dimensions of the finished rods were about ½″ × 20″(= 1.25 cm. × 50 cm.).

With regard to the safety of reheating this pure iron in an ordinary forge without contamination the chemical analysis has shown that the carbon content of the iron was practically the same before and after forging, in this respect confirming the conclusions of Professor Burgess.

15. *Preparing the Test-pieces.*—From the forged rods the following test-pieces were made:

1. One magnetic test-piece, serving also for the electrical tests, 0.392″ (= .996 cm.) in diameter and 14″ (= 35.5 cm.) long. The earlier rods were only 12″ (= 30.5 cm.) long.

2. Two test-pieces for the mechanical tests, having a middle section 0.3″ (= .76 cm.) in diameter and 1½″ (= 3.8 cm.) long, with threaded ends ½″ by ½″ (= 1.25 cm.). (See Fig. 11.)

3. One test-piece for the metallurgical investigation ½″ by ½″ (= 1.25 cm.).

4. One test-piece for the critical temperature investigation ½″ (= 1.25 cm.) by 1″ (= 2.5 cm.), with a hole ¼″ (= .6 cm.) by ½″ (= 1.25 cm.) drilled in one end.

The shavings obtained by the process of preparing these test-pieces were collected after first removing the exposed parts of the rods and used for the chemical analysis. No oil was used during the collection of these shavings.

16. *Annealing and Quenching.*—After the test-pieces (1 and 3 above) had been tested as forged, they were sent through different heat treatments, either alone or together with other test-pieces. If they were to be annealed they were packed in pulverized sintered magnesia in an iron container, A, Fig. 62. In order to minimize the formation of oxide on the surface of the rods, it was found expedient to moisten the magnesia with alcohol. The alcohol in evaporating presumably expelled the air surrounding the rods, leaving them in a reducing atmosphere. The film of oxide formed on the rods under these conditions was very thin and could be removed with fine emery cloth, except after severe treatment, such as cooling from 900° C. to 200° C. in 48 hours. Even then no difficulty was had in removing the film. With the exception of the preliminary tests of rods No. 3-39 and No. 3-31, definite cooling curves were followed. In a few cases this was the logarithmic curve $T = 900\epsilon^{-.00105t}$, where $T$ = temperature and $t$ = time in minutes. In all other cases the cooling curve was a straight line, connecting 900° and 200° C. The time was 12, 24 or 48 hours. While it was impossible to follow the curve exactly, the deviation from it was slight. The temperature was measured by placing the pyrometer tube in direct contact with the rods about 2″ (= 5 cm.) from one end. The uniformity of temperature in the container was tested once for all by means of Seger cones placed at regular intervals along the rod with their apexes touching the rod. No marked difference could be detected. Below 200° C. the rods were allowed to cool without supervision, but they were not removed from the container until cool enough to handle with the hands.

The quenching was done by placing the test-pieces in a container, Q, Fig. 62. This was filled with magnesia and moistened with alcohol. A loose fitting iron stopper was placed at one end. At the desired temperature the container was removed from the furnace, tipped up and the contents plunged into the quenching bath by means of a sharp blow from a hammer on the top of the container. The quenching bath used consisted of iced brine at —21° C. In one case liquid air was used, but then the magnesia was omitted.

17. *Magnetic Tests.*—After each heat treatment the rods were tested magnetically and electrically. The magnetic test consisted in determining the magnetization curve up to H = 150 or 200 gilberts per cm. and the hysteresis loops for $B_{max} = 10{,}000$ and usually for $B_{max} = 15{,}000$. Points on the magnetization curve were determined by the method of reversal, the details of which are given in Appendix IV. The hysteresis loop was determined by changing from $H_{max}$ to the particular value of H wanted, and noting the deflection of the fluxmeter corresponding to this change. Thus, by using the maximum values of H and B as the basis, the magnetic viscosity does not affect the results. Unless very high values of H are used, this method appears to the author to be preferable to the one suggested by Taylor*, in which the retentivity point is used as the basis. For further details the reader is referred to Appendix IV.

18. *Metallurgical Analyses.*—As mentioned above, a small specimen for studying the microstructure was prepared from each rod as forged, and these specimens accompanied the magnetic rods wherever they went. Photomicrographs were obtained after the various treatments for nearly all the different rods. For this work an inverted metallurgical microscope was used, constructed by Ernst Leitz of Wetzlar. Great care had to be used in polishing the specimens on account of their softness. The etching was done altogether by picric acid.

The second metallurgical specimen was used for the critical temperature determinations. This was done by placing the specimen in a ½″ (1.25 cm.) "Electroquartz" pyrometer tube with the hot junction of the pyrometer inserted in the small hole made in one end of the specimen. The tube was partly filled with fused magnesia and placed in the Hoskins muffle furnace with the end at the middle of the furnace. The temperature was gradually raised to about 1,000° C., the current cut off, and the furnace allowed to cool naturally, while the time interval for each 10° C. was obtained by means of two stop-watches. From the data thus obtained three different curves were plotted for each specimen, namely:

1. The temperature-time curve, using temperature as abscissa and time as ordinate.

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*Phys. Review XXIII, p. 95, 1906.

2. The temperature-rate curve, using temperature as abscissa and number of degrees fall in temperature per minute as ordinate.

3. The inverse rate curve, using temperature as abscissa and number of seconds per degree fall in temperature as ordinate.

From these curves the critical temperatures can readily be located.

While this method is not as accurate as the differential method by means of three thermocouples, it serves as a check upon the other determinations made in the course of the investigation.

19. *Mechanical Tests.*—As only two mechanical test-pieces could be obtained from each rod it was possible to use only a few different treatments in order to secure data that could be compared. The following treatments were used:

1. As forged.
2. Annealed at 900° C. and cooled in 12 or 24 hours.
3. Quenched in brine from 1,000° C.
4. One test-piece was quenched in liquid air from 1,000° C.

The tests were made on an Olsen 10,000-pound testing machine. The load was applied very slowly and uniformly by means of an electric motor. In most cases the elastic limit was very definite, and the stress for this point was obtained whenever it was sufficiently pronounced. The elongation was also measured in a large number of cases as soon as the ultimate stress had been reached.

20. *Chemical Analyses.*—From the chemical analysis of the Swedish charcoal iron and the doubly refined electrolytic iron it is seen that the only measurable impurities in the latter are carbon and silicon. As these are the only impurities that are liable to be affected during the processes described, it seemed superfluous to analyze for anything except carbon and silicon. A few analyses were made for sulphur, but only traces could be found. A large number of carbon analyses were made. Considerable difficulty was had at first in making these analyses on account of the small quantities present. Finally a direct combustion furnace was purchased for these tests, and consistent results have since been obtained. It was found necessary with this method to clean the samples thoroughly with ether before making the analyses in order to remove any trace of oil with which they might have become contaminated in the process of preparation. Although special precautions were taken to prevent such contamination, the analyses showed that as much as 0.02 per cent carbon was removed by cleaning the shavings with ether.

## IV. Results.

In presenting the results of the investigation it has seemed desirable to divide the rods into a number of natural groups.

Group 1 includes the rods obtained from electrolytic iron melted in the resistance furnace under atmospheric pressure and is represented by rods Nos. 3-13, 3-15, 3-17, 3-18, 3-19, 3-20 and 3-25.

Group 2 includes rods obtained from electrolytic iron melted in vacuo and is represented by rods Nos. 3-34, 3-36, 3-37 and 3-38. After being forged these rods were reheated to forging temperature and cooled in air. The temperature was not measured with a pyrometer but estimated from the color to be between 1,000 and 1,100° C.

Group 3 includes rods obtained from electrolytic iron melted in vacuo and is represented by rods Nos. 3-40, 3-41, 3-43, 3-45 and 3-47. These rods were not at any time after being forged heated to a higher temperature than about 900° C. They were annealed three times at 900° C. with different cooling rates, in company with the three rods from Group 5 containing various percentages of carbon. The results show possible contamination from contact with these rods.

Group 4 includes rods obtained from electrolytic iron melted in vacuo and is represented by rods No. 3-48, 3-49 and 3-50. These rods were first annealed twice at slightly above 900° C., then annealed at 1,080° C. and cooled in about 6 hours, and finally annealed at 900° C. and cooled in 24 hours.

Group 5 includes the rods obtained from electrolytic iron melted in vacuo with different percentages of carbon added and is represented by rods Nos. 3C01, 3C02, 3C03, the amount of carbon added being 0.05 per cent, 0.10 per cent and 0.50 per cent, respectively. The rods were annealed together with the rods representing Group 3, as mentioned above.

Besides the rods included in the above five groups the following single rods were tested: Rods Nos. 3-39 and 3-31 were used separately for the preliminary test in the attempts to ascertain the most favorable annealing temperature. Rods SWI-4 and No. 1-21 were used to ascertain the effect of remelting Swedish charcoal iron in vacuo. SWI-4 was cut from a plate of Swedish iron ½″×6″×12″ (1.25×15×30 cm.) and machined into a magnetic test rod directly. The iron for No. 1-21 was cut from the same plate as SWI-4 and directly next to it and was melted in the vacuum furnace. After forging and turning, the two

## TABLE 4.

### GROUP 1: ELECTROLYTIC IRON MELTED IN RESISTANCE FURNACE UNDER ATMOSPHERIC PRESSURE. CRUCIBLES BURIED IN CRUSHED GRAPHITE.

| Rod No. | % Carbon as per Chem. Analysis | Heat Treatment | | | Values of B in Gausses for | | | | | | Spec. Elect. Resist. at 20° C. microhms |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Annealing Temp. °C. | Cooling Curve | Hours Cooling | H=.5 | H=1 | H=2 | H=8 | H=20 | H =100 | |
| 3–17 | .10 | As | Forged | | | 300 | 800 | 10,200 | 14,400 | 17,500 | 12.60 |
| 3–19 | .05 | | | | | 200 | 450 | 9,100 | 14,600 | 17,600 | 13.20 |
| 3–23 | .107 | | | | | 350 | 1,000 | 10,200 | 13,700 | 16,800 | 14.88 |
| 3–25 | | | | | | 100 | 300 | 8,200 | 14,100 | 17,500 | 14.70 |
| Avg. | | | | | | 250 | 640 | 9,425 | 14,200 | 17,350 | 13.85 |
| 3–13 | .105* | 1000 | Log. | About 6 | | 1,200 | 6,300 | 13,700 | 15,250 | 17,450 | 13.77 |
| 3–15 | | | | | | 100 | 300 | 10,800 | 14,500 | 17,600 | 14.63 |
| 3–18 | | | | | | 200 | 700 | 11,250 | 14,700 | 17,500 | 12.50 |
| 3–20 | .14* | | | | | 1,900 | 6,250 | 12,700 | 14,700 | 17,300 | 12.80 |
| Avg. | | | | | | 825 | 3,387 | 12,125 | 14,788 | 17,462 | 13.43 |
| 3–13 | .105* | 934 | Straight Line | 24 | 600 | 2,700 | 7,900 | 14,000 | 15,400 | 17,450 | 13.70 |
| 3–15 | | | | | 250 | 550 | 1,900 | 12,500 | 15,100 | 17,600 | 14.52 |
| 3–17 | .100* | | | | 500 | 1,750 | 6,450 | 13,600 | 15,200 | 17,500 | 12.55 |
| 3–18 | | | | | 50 | 100 | 400 | 11,500 | 15,000 | 17,400 | 12.40 |
| 3–19 | .050* | | | | 50 | 125 | 400 | 10,500 | 14,700 | 17,300 | 13.20 |
| 3–20 | .140* | | | | 700 | 3,150 | 8,000 | 13,650 | 15,100 | 17,300 | 12.53 |
| 3–23 | .107* | | | | 400 | 1,500 | 6,000 | 13,000 | 14,700 | 17,100 | 14.65 |
| 3–25 | | | | | 50 | 125 | 400 | 11,500 | 15,000 | 17,400 | 14.62 |
| Avg. | .100 | | | | 325 | 1,250 | 3,930 | 12,530 | 15,025 | 17,380 | 13.52 |

* Analyzed as forged

NOTE: Magnetic measurements made with L. & N. galvanometer.

## TABLE 5.

### SUMMARY OF MAGNETIC AND ELECTRIC TESTS FOR ROD NO. 3-31.

| Heat Treatment | | | | Values of B in Gausses for | | | | | Spec. Elect. Resist. at 20°C. microhms |
|---|---|---|---|---|---|---|---|---|---|
| Ref. No. | Annealing Temp. °C. | Cooling Curve | Hours Cooling from Max. to 200 °C. | H=1 | H=2 | H=8 | H=20 | H=100 | |
| A | As Forged | | | | 4,250 | 11,500 | 14,850 | 17,500 | 9.98 |
| B | 532 | Natural | ab't 3 | 1,750 | 5,100 | 12,700 | 15,300 | 17,500 | 9.96 |
| C | 700 | " | " 4 | 3,300 | 8,250 | 14,200 | 15,500 | 17,500 | 9.96 |
| D | 800 | " | " 5 | 4,900 | 10,400 | 14,600 | 15,600 | 17,600 | 9.90 |
| E | 900 | " | " 6 | 6,100 | 10,600 | 14,800 | 15,600 | 17,600 | 9.93 |
| F | 950 | " | " 6½ | 7,800 | 12,350 | 15,000 | 15,700 | 17,600 | 10.04 |
| G | 1,000 | " | " 7 | 7,000 | 10,900 | 15,000 | 15,800 | 17,600 | |
| H | 900 | " | " 6 | 7,600 | 11,500 | 15,200 | 15,750 | 17,750 | |
| J | 900 | Logarith. | 24 | 9,000 | 12,900 | 15,250 | 16,000 | 17,800 | |
| K1 | 900 | Str. Line | 48 | 10,100 | 13,000 | 15,400 | 16,000 | 17,800 | 10.03 |
| K2 | | | | 9,770* | 12,400* | 15,600* | 16,200* | 17,900* | |

* As measured with Grassot fluxmeter

## TABLE 6.

## SUMMARY OF MAGNETIC AND ELECTRIC TESTS FOR ROD NO. 3-39.

| Heat Treatment | | | | Values of B in Gausses for | | | | | Spec. Elect. Resist. at 20°C. microhms |
|---|---|---|---|---|---|---|---|---|---|
| Ref. No. | Annealing Temp. °C. | Cooling Curve | Hours Cooling from Max. to 200°C. | H=1 | H=2 | H=8 | H=20 | H=100 | |
| A | As Forged | | | 550 | 2,300 | 10,000 | 14,500 | 18,100 | 9.78 |
| B | 680 | Natural | ab't 4 | 3,700 | 9,900 | 15,600 | 16,300 | 18,100 | 9.85 |
| C | 800 | " | " 5 | 8,800 | 14,400 | 15,900 | 16,400 | 18,100 | 9.95 |
| D | 900 | " | " 6 | 9,400 | 14,500 | 15,900 | 16,400 | 18,100 | |
| E | 950 | " | " 6½ | 8,100 | 11,900 | 15,800 | 16,400 | 18,100 | |
| F | 1,000 | " | " 7 | 10,500 | 14,500 | 15,600 | 16,200 | 18,000 | |
| G | 1,060 | " | " 7 | 9,650 | 14,500 | 15,500 | 16,100 | 17,900 | 10.00 |
| H | 912 | " | " 6 | 10,650 | 14,600 | 15,500 | 16,100 | 18,000 | 9.83 |
| J | 800 | " | " 5 | 8,600 | 12,400 | 15,500 | 16,100 | 17,900 | 9.87 |
| K | 700 | " | " 4 | 9,900 | 14,200 | 15,500 | 16,100 | 17,700 | 9.87 |
| L | 616 | " | " 3½ | 11,000 | 14,500 | 15,500 | 16,100 | 17,800 | |
| M | 500 | " | " 3 | 10,800 | 14,500 | 15,500 | 16,100 | 17,800 | 9.87 |
| N | 1,000 | Cooled in brine | ab't 1 min. | 6,400 | 10,200 | 15,100 | 15,800 | 17,900 | 10.10 |
| O | 1,000 | Q'nch'd in brine | | 1,000 | 2,600 | 7,900 | 12,200 | 17,500 | 10.15 |
| P | 644 | Natural | ab't 4 | 2,000 | 7,600 | 14,700 | 15,950 | 17,900 | |
| Q | 900 | " | " 6 | 7,500 | 13,000 | 15,200 | 15,900 | 17,700 | |
| R | 900 | Logarith. | 24 | 9,900 | 13,800 | 15,400 | 16,000 | 17,800 | |
| $S_1$ | 900 | Straight Line | 48 | 10,500 | 13,600 | 15,400 | 16,000 | 17,900 | 9.95 |
| $S_2$ | | | | 12,100* | 14,400* | 15,500* | 16,200* | 17,900* | |

*As measured with Grassot fluxmeter.

## TABLE 7.

## GROUP 2: ELECTROLYTIC IRON MELTED IN VACUO.

| | Rod No. | % Carbon as per Chem. Analysis | Heat Treatment | | | Values of B in Gausses for | | | | | | Hysteresis Loss. Ergs per c.c. per cycle | | Coercive Force for B max =15,000 | Retentivity for B max =15,000 | Spec. Elect. Resist. at 20° C. microhms. | Critical Temp. $Ar_3$ °C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Annealing Temp. °C. | Cooling Curve | Hrs. Cooling from 900 to 200 °C. | H=.5 | H=1 | H=2 | H=8 | H=20 | H =100 | B max =10,000 | B max =15,000 | | | | |
| I. | 3-34 | .009 | Forging temp. | Cooled in air | | | | | | | | | | | | | 895 |
| | 3-36 | .015 | | | | | | 6,200 | 14,000 | 15,650 | 17,800 | 3,360 | | 1.40 | 9,500 | 10.33 | 875 |
| | 3-37 | .045 | | | | | 1,900 | 6,550 | 14,000 | 15,600 | 17,800 | 3,710 | | 1.70 | 10,000 | 10.39 | 885 |
| | 3-38 | .010 | | | | | | 10,000 | 15,000 | 15,900 | 17,800 | 2,000 | | .90 | 10,100 | 10.00 | 905 |
| | Avg. | .0198 | | | | | | 7,600 | 14,330 | 15,716 | 17,800 | 3,020 | | 1.33 | 9,900 | 10.24 | 890 |
| II. | 3-34 | | 902 | Log. | 24 | 3,600 | 9,400 | 12,700 | 15,200 | 16,100 | 17,800 | | | | | 10.28 | |
| | 3-36 | | | | | 2,600 | 6,900 | 10,500 | 14,800 | 15,850 | 17,700 | 1,560 | | | | 10.28 | |
| | 3-37 | | | | | 1,850 | 6,000 | 9,900 | 14,900 | 15,900 | 17,800 | 1,530 | | | | 10.38 | |
| | 3-38 | | | | | 4,000 | 9,100 | 12,300 | 15,500 | 16,050 | 17,700 | | | | | 9.99 | |
| | Avg. | | | | | 3,010 | 7,850 | 11,230 | 15,100 | 15,980 | 17,750 | | | | | 10.23 | |
| III | 3-34 | | 914 | Straight Line | 48 | 7,800 | 11,400 | 13,900 | 15,700 | 16,100 | 18,000 | 895 | 1,600 | .30 | 10,600 | 10.20 | |
| | 3-36 | | | | | 5,960 | 9,640 | 12,500 | 15,500 | 16,100 | 18,000 | 953 | 1,830 | .33 | 9,000 | 10.16 | |
| | 3-37 | | | | | 4,000 | 7,400 | 10,300 | 14,800 | 15,750 | 17,700 | 1,255 | 2,000* | .40* | 9,500* | 10.30 | |
| | 3-38 | | | | | 9,400 | 12,500 | 14,400 | 15,700 | 16,100 | 17,700 | 813 | 1,640 | .29 | 12,100 | 10.00 | |
| | Avg. | | | | | 6,800 | 10,235 | 12,750 | 15,425 | 16,010 | 17,850 | 979 | 1,770 | .33 | 10,300 | 10.16 | |

NOTE—Magnetic measurements for I and II were made with L. & N. galvanometer.
Magnetic measurements for III were made with Grassot fluxmeter.

*Estimated by comparison with 3-36.

rods were treated exactly alike. Rod No. 3-30 was made from electrolytic iron and melted in vacuo. It was given only one heat treatment, namely, annealed at 900° C. and cooled from 900 to 200° C. in 48 hours.

For the sake of comparison the results obtained with two sets of rods made from commercial iron are included in the final summary. One set of rods was made from a bar of silicon alloy steel, containing about 4 per cent silicon, while the other set of rods was made from bars such as are usually rolled into standard electrical sheets. The latter is material of commercially high permeability and is practically free from silicon. Both sets were prepared by the manufacturer and given their standard anneal.

TABLE 8.

GROUP 3: ELECTROLYTIC IRON MELTED IN VACUO.
ANNEALED WITH GROUP 5.

|  | Rod No. | % Carbon as per Chem. Analysis | Heat Treatment |  |  | Values of B in Gausses for |  |  |  |  |  | Hysteresis Loss. Ergs per c. c. per cycle |  | Coercive Force for B max =15,000 | Retentivity for B max =15,000 | Spec. Elect. Resist. at 20° C. microhms | Critical Temp. Ar 3 °C. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  | Annealing Temp. °C. | Cooling Curve | Hrs. Cooling from 900 to 200 °C. | H=.5 | H=1 | H=2 | H=8 | H=20 | H =100 | B max =10,000 | B max =15,000 |  |  |  |  |
| I. | 3-40 | .0110 | As | forg | ed |  | 900 | 4,400 | 11,000 | 14,900 | 17,900 | 4,500 |  |  |  | 9.78 | 895 |
|  | 3-41 | .0095 |  |  |  |  | 900 | 3,900 | 11,500 | 15,100 | 18,200 | 4,520 |  |  |  | 10.00 | 895 |
|  | 3-43 | .0096 |  |  |  |  | 550 | 2,800 | 9,750 | 14,200 | 17,900 | 5,620 |  |  |  | 9.90 | 895 |
|  | 3-45 | .0080 |  |  |  |  | 2,000 | 5,600 | 12,400 | 15,900 | 18,100 | 1,830 |  |  |  | 9.86 | 895 |
|  | 3-47 | .0080 |  |  |  |  | 1,900 | 5,200 | 12,300 | 15,700 | 18,200 | 1,570 |  |  |  | 10.10 | 900 |
|  | Avg. | .0092 |  |  |  |  | 1,250 | 4,380 | 11,400 | 15,150 | 18,050 | 3,690 |  |  |  | 9.93 | 896 |
| II. | 3-40 |  | 902 | Straight Line | 12 | 4,000 | 11,500 | 14,100 | 15,400 | 16,000 | 17,900 | 1,080 |  |  |  | 9.70 |  |
|  | 3-41 |  |  |  |  | 3,550 | 8,300 | 12,200 | 15,600 | 16,200 | 18,000 | 1,490 |  |  |  | 9.93 |  |
|  | 3-43 |  |  |  |  | 3,100 | 8,100 | 12,300 | 15,750 | 16,500 | 18,100 |  |  |  |  | 9.77 |  |
|  | 3-45 |  |  |  |  | 2,900 | 10,500 | 14,000 | 16,100 | 16,650 | 18,500 | 1,250 |  |  |  | 9.76 |  |
|  | 3-47 |  |  |  |  | 2,950 | 8,300 | 11,900 | 15,700 | 16,550 | 18,200 |  |  |  |  | 9.96 |  |
|  | Avg. |  |  |  |  | 3,300 | 9,350 | 12,900 | 15,710 | 16,380 | 18,100 |  |  |  |  | 9.82 |  |
| III. | 3-40 |  | 909 | Straight Line | 24 | 4,000 | 10,500 | 14,000 | 15,400 | 16,000 | 17,800 |  |  |  |  | 9.64 |  |
|  | 3-41 |  |  |  |  | 4,400 | 8,900 | 11,950 | 15,600 | 16,300 | 18,000 | 1,300 |  |  |  | 10.01 |  |
|  | 3-43 |  |  |  |  | 4,800 | 8,900 | 13,100 | 15,850 | 16,500 | 18,100 |  |  |  |  | 9.86 |  |
|  | 3-45 |  |  |  |  | 2,950 | 8,600 | 13,600 | 16,100 | 16,700 | 18,500 |  |  |  |  | 9.86 |  |
|  | 3-47 |  |  |  |  | 5,000 | 11,100 | 14,350 | 15,750 | 16,550 | 18,200 | 990 | 1,930 |  |  | 10.02 |  |
|  | Avg. |  |  |  |  | 4,030 | 9,600 | 13,400 | 15,740 | 16,410 | 18,120 |  |  |  |  | 9.88 |  |
| IV. | 3-40 |  | 903 | Straight Line | 48 | 4,800 | 8,850 | 11,700 | 15,300 | 16,000 | 17,700 | 1,240 | 2,500 | .36 | 10,100 | 9.70 |  |
|  | 3-41 |  |  |  |  | 5,560 | 8,620 | 11,300 | 15,500 | 16,200 | 18,000 | 1,155 | 2,180* | .40* | 10,000* | 10.00 |  |
|  | 3-43 |  |  |  |  | 6,330 | 9,850 | 13,000 | 16,000 | 16,500 | 18,150 | 1,165 | 2,180 | .40 | 10,200 | 9.85 |  |
|  | 3-45 |  |  |  |  | 4,300 | 8.200 | 12,000 | 16,250 | 16,800 | 18,450 | 1,470 | 2,640 | .40 | 10,600 | 9.87 |  |
|  | 3-47 |  |  |  |  | 4,600 | 8,450 | 11,900 | 15,800 | 16,500 | 18,100 | 1,190 | 2,120 | .35 | 9,000 | 10.05 |  |
|  | Avg. |  |  |  |  | 5,120 | 8,790 | 11,980 | 15,770 | 16,400 | 18,080 | 1,242 | 2,324 | .38 | 9,980 | 9.89 |  |

NOTE—Magnetic measurements for I, II and III made with L. & N. galvanometer.
Magnetic measurements for IV were made with Grassot fluxmeter.

*Estimated by comparison with 3-43.

The results of the tests are shown in the tables and curves. Tables 4, 7, 8, 9 and 10 give the results for the five groups of rods mentioned above. Table 5 gives a summary of the results for rod No. 3-31 after the various heat treatments. Table 6 gives the same for rod No. 3-39, but the heat treatments have been carried further for this rod than for No. 3-31. The results for No. 3-39 are also shown graphically in Fig. 3. Table 11 is a summary of the results for all the electrolytic iron rods melted in vacuo. Table 12 gives the final summary for all rods tested. Finally, Table 13 gives a summary of the results of the mechanical tests. Due to an oversight the rods of Groups 3 and 5 were annealed together. The results obtained were rather unusual and are, therefore, shown graphically in Fig. 4 for H=1.

TABLE 9.

GROUP 4: ELECTROLYTIC IRON MELTED IN VACUO.

| | Rod No. | % Carbon as per Chem. Analysis | Heat Treatment | | | Values of B in Gausses for | | | | | | Hysteresis Loss. Ergs per c.c. per cycle | | Coercive Force for B max =15,000 | Retentivity for B max =15,000 | Spec. Elect. Resist. at 20° C. microhms | Critical Temp. $Ar_3$ ° C. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Annealing Temp. ° C. | Cooling Curve | Hrs. Cooling from 900 to 200 °C. | H=.5 | H=1 | H=2 | H=8 | H=20 | H=100 | B max =10,000 | B max =15,000 | | | | |
| I. | 3-48 | .0080 | As | forg | ed | | 650 | 3,400 | 10,500 | 15,000 | 18,500 | 4,550 | | | | | |
| | 3-49 | .0120 | | | | | 650 | 3,050 | 10,300 | 14,950 | 18,500 | 4,550 | | | | | |
| | 3-50 | .0099 | | | | | 1,400 | 4,600 | 11,400 | 15,200 | 18,100 | 3,380 | | | | | |
| | Avg. | .0099 | | | | | 900 | 3,680 | 10,730 | 15,050 | 18,370 | 4,160 | | | | | |
| II. | 3-48 | | 925 | Str'ght Line | 24 | 6,070 | 9,750 | 12,650 | 16,100 | 16,600 | 18,300 | 970 | 1,915 | .35 | 10,400 | | |
| | 3-49 | | | | | 6,500 | 8,950 | 11,500 | 15,950 | 16,450 | 18,100 | 994 | 1,830 | .31 | 8,200 | | |
| | 3-50 | | | | | 5,100 | 9,200 | 12,500 | 15,900 | 16,400 | 18,200 | 1,195 | 2,060 | .37 | 9,300 | | |
| | Avg. | | | | | 5,890 | 9,300 | 12,220 | 15,980 | 16,480 | 18,200 | 1,053 | 1,935 | .34 | 9,300 | | |
| III. | 3-48 | | 930 | Str'ght Line | 48 | 6,400 | 9,200 | 11,800 | 15,600 | 16,300 | 18,150 | 955 | 1,850 | .29 | 9,300 | 9.70 | |
| | 3-49 | | | | | 6,100 | 8,700 | 11,200 | 15,800 | 16,500 | 18,200 | 902 | 1,710 | .31 | 8,100 | 9.90 | |
| | 3-50 | | | | | 5,920 | 8,830 | 11,500 | 15,600 | 16,250 | 18,050 | 995 | 1,940 | .30 | 9,000 | 9.70? | |
| | Avg. | | | | | 6,140 | 8,910 | 11,500 | 15,670 | 16,350 | 18,130 | 951 | 1,830 | .30 | 8,800 | 9.77 | |
| IV. | 3-48 | | 1080 | Log. | 6 | 5,200 | 8,500 | 11,500 | 15,100 | 15,850 | 17,600 | | | | | 10.20 | |
| | 3-49 | | | | | 5,500 | 8,800 | 11,700 | 15,100 | 15,850 | 17,600 | | | | | 10.05 | |
| | 3-50 | | | | | 5,250 | 8,500 | 11,700 | 15,100 | 15,700 | 17,400 | | | | | 10.20 | |
| | Avg. | | | | | 5,320 | 8,600 | 11,630 | 15,100 | 15,800 | 17,530 | | | | | 10.15 | |
| V. | 3-48 | | 918 | Str'ght Line | 24 | 5,760 | 8,620 | 11,100 | 15,200 | 15,900 | 17,600 | | | | | 9.95 | |
| | 3-49 | | | | | 5,810 | 9,200 | 11,600 | 15,150 | 15,700 | 17,600 | | | | | 10.00 | |
| | 3-50 | | | | | 5,400 | 8,830 | 11,500 | 15,150 | 15,900 | 17,600 | | | | | 10.15 | |
| | Avg. | | | | | 5,660 | 8,870 | 11,400 | 15,170 | 15,830 | 17,600 | | | | | 10.03 | |

NOTE—Magnetic measurements for I, II, III and V were made with Grassot fluxmeter.
Magnetic measurements for IV were made with L. & N. galvanometer.

## TABLE 10.

## GROUP 5: ELECTROLYTIC IRON WITH CARBON ADDED, MELTED IN VACUO. ANNEALED WITH GROUP 3.

| | Rod No. | % Carbon as per Chem. Analysis | Heat Treatment | | | Values of B in Gausses for | | | | | | Hysteresis Loss. Ergs per c. c. per cycle | | Coercive Force for B max =15,000 | Retentivity for B max =15,000 | Spec. Elect. Resist. at 20° C. microhms | Critical Temp. $Ar_3$ ° C. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Annealing Temp. ° C. | Cooling Curve | Hrs. Cooling from 900 to 200 ° C. | H=.5 | H=1 | H=2 | H=8 | H=20 | H =100 | B max =10,000 | B max =15,000 | | | | |
| I. | 3C 01 | .013 | As | forg | ed | 400 | 1,000 | 2,900 | 10,900 | 15,000 | 18,400 | 5,380 | | | | 10.2 | 895 |
| | 3C 02 | .012 | | | | 100 | 300 | 1,000 | 8,700 | 13,800 | 18,500 | 8,600 | | | | 10.66 | 895 |
| | 3C 03 | .181 | | | | 25 | 200 | 400 | 7,500 | 13,100 | 17,200 | 11,880 | | | | 12.74 | ? |
| II. | 3C 01 | | 902 | Str'ght Line | 12 | 1,450 | 5,600 | 9,600 | 15,650 | 16,450 | 18,350 | | | | | 10.25 | |
| | 3C 02 | | | | | 750 | 5,100 | 10,000 | 15,300 | 16,100 | 18,100 | | | | | 10.67 | |
| | 3C 03 | | | | | 750 | 2,000 | 6,300 | 12,900 | 14,700 | 17,100 | | | | | 12.6 | |
| III. | 3C 01 | | 909 | Str'ght Line | 24 | 3,000 | 7,000 | 10,500 | 15,900 | 16,600 | 18,500 | | | | | 10.25 | |
| | 3C 02 | | | | | 1,600 | 6,800 | 11,600 | 15,500 | 16,300 | 18,100 | | | | | 10.7 | |
| | 3C 03 | | | | | 900 | 3,150 | 7,100 | 12,900 | 14,800 | 17,100 | | | | | 12.65 | |
| IV. | 3C 01 | | 903 | Str'ght Line | 48 | 3,625 | 8,000 | 11,800 | 16,100 | 16,900 | 18,500 | 1,405 | 2,300 | .40 | 10,000 | 10.24 | |
| | 3C 02 | | | | | 1,840 | 7,750 | 11,600 | 15,500 | 16,200 | 18,000 | 1,710 | 3,190 | .50 | 10,700 | 10.64 | |
| | 3C 03 | | | | | 870 | 4,140 | 7,800 | 13,550 | 15,100 | 17,200 | 1,910 | | | | 12.4 | |

NOTE—1. Magnetic measurements for I, II, and III were made with L. & N. galvanometer.
Magnetic measurements for IV were made with Grassot fluxmeter.

2. Carbon added as follows: 3C01—.05% added.
3C02—.10% added.
3C03—.50% added.

## TABLE 11.

## ELECTROLYTIC IRON MELTED IN VACUO. SUMMARY OF GROUPS 2, 3 AND 4 AND RODS 3-30, 3-31 AND 3-39.

| No. of Rods Represented | Group or Rod No. | % Carbon as per Chem. Analysis | Heat Treatment | | | Values of B in Gausses for | | | | | | Hysteresis Loss. Ergs per c. c. per cycle | | Coercive Force for B max =15,000 | Retentivity for B max =15,000 | Spec. Elect. Resist. at 20° C. microhms | Critical Temp. $Ar_3$ ° C. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Annealing Temp. ° C. | Cooling Curve | Hrs. Cooling from 900 to 200° C. | H=.5 | H=1 | H=2 | H=8 | H=20 | H =100 | B max =10,000 | B max =15,000 | | | | |
| 4 | Gr.2 | | As | forg | ed | | | | | | | | | | | | 890 |
| 5 | Gr.3 | .0092 | | | | | 1,250 | 4,380 | 11,400 | 15,150 | 18,050 | 3,690 | | | | 9.93 | 896 |
| 3 | Gr.4 | .0099 | | | | | 900 | 3,680 | 10,730 | 15,050 | 18,370 | 4,160 | | | | 9.80 | |
| 1 | 3-31 | .0120 | | | | | 1,200? | 4,250 | 11,500 | 14,850 | 17,500 | 4,600 | | 1.80 | 7,700 | 9.98 | 885 |
| 1 | 3-39 | .0110 | | | | | 550 | 2,300 | 10,000 | 14,400 | 18,100 | 5,460 | | 1.80 | 7,000 | 9.78 | 905 |
| Avg. for 10 rds. | | .0100 | | | | | 1,070 | 3,950 | 11,070 | 15,015 | 18,100 | 4,090 | | | | 9.88 | 894 |
| 4 | Gr.2 | .0198* | 914 | Str. Line | 48 | 6,800 | 10,230 | 12,750 | 15,425 | 16,010 | 17,850 | 978 | 1,770 | .33 | 10,300 | 10.16 | |
| 5 | Gr.3 | .0092 | 903 | " | 48 | 5,120 | 8,790 | 11,900 | 15,770 | 16,400 | 18,080 | 1,242 | 2,324 | .38 | 9,980 | 9.89 | |
| 3 | Gr.4 | .0099 | 930 | " | 48 | 6,140 | 8,910 | 11,500 | 15,670 | 16,350 | 18,130 | 951 | 1,830 | .30 | 8,800 | 9.77 | |
| 1 | 3-30 | .0090 | 914 | " | 48 | 5,500 | 8,880 | 11,600 | 15,200 | 15,800 | 17,700 | | | | | 9.94 | |
| 1 | 3-31 | .0120 | 900 | " | 48 | 6,580 | 9,770 | 12,400 | 15,600 | 16,200 | 17,900 | 980 | 1,760 | .32 | 9,100 | 10.03 | |
| 1 | 3-39 | .0110 | 900 | " | 48 | 8,050 | 12,100 | 14,400 | 15,500 | 16,200 | 17,900 | 880 | 1,860 | .32 | 12,600 | 9.95 | |
| Avg. for 15 rds. | | .0125 | | | | 6,090 | 9,500 | 12,350 | 15,590 | 16,200 | 17,950 | 1,060 | 1,990 | .34 | 9,940 | 9.96 | |

*Analyzed as forged.

Fig. 5 shows the magnetization curves for the iron-carbon alloys melted in vacuo, while Fig. 6 gives the average magnetization curves for the rest of the rods. Fig. 7 shows the magnetization curves for the average and best electrolytic iron rods melted in vacuo, and Fig. 8 the hysteresis loops and permeability curve for the best rod obtained, namely, No. 3-38. Fig. 9 shows the corresponding curves for No. 3-34. While cooling curves were obtained for all the rods, only the one for No. 3-43 is included here; this is shown in Fig. 10. It was thought that it might be of interest to see some of the mechanical test-pieces after being tested; consequently, a few of these are exhibited in Fig. 11 together with a sample test-piece that has not been broken. In this figure the distance between the vertical lines is 1 in. (2.54 cm).

### TABLE 12.

### SUMMARY OF RESULTS

Rods thoroughly annealed at 900°C.

| Rod No. | Description of Iron | Furnace Used for Melting | Carbon as per Chemical Analysis % | Max. Permeability | Flux Density for Max. Permeability | Hysteresis Loss. Ergs per c.c. per cycle | | Coercive Force for B max =15,000 | Retentivity for B max =15,000 | Spec. Elect. Resist. at 20°C. microhms | Critical Temp. $Ar_3$ °C. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | B max =10,000 | B max =15,000 | | | | |
| 3–38 | Electrolytic | Vacuum | .0104 | 19,000 | 9,500 | 813 | 1,640 | .29 | 12,100 | 10.00 | 905 |
| 3–39 | " | " | .0110 | 16,500 | 8,500 | 880 | 1,860 | .32 | 12,600 | 9.95 | 905 |
| 3–34 | " | " | .0090 | 16,000 | 6,500 | 895 | 1,600 | .30 | 10,600 | 10.20 | 895 |
| 3–49 | " | " | .0120 | 15,400 | 5,000 | 902 | 1,710 | .31 | 8,100 | 9.90 | |
| 3–31 | " | " | .0120 | 13,100 | 6,200 | 980 | 1,760 | .32 | 9,100 | 10.03 | 885 |
| 3–43 | " | " | .0196 | 12,900 | 5,500 | 1,165 | 2,180 | .40 | 10,200 | 9.85 | 895 |
| 3–48 | " | " | .0080 | 12,600 | 5,500 | 955 | 1,850 | .29 | 9,300 | 9.70 | |
| 3–36 | " | " | .0150 | 12,250 | 6,000 | 953 | 1,830 | .33 | 9,000 | 10.16 | 875 |
| 3–40 | " | " | .0110 | 12,000 | 9,000 | 1,240 | 2,500 | .36 | 10,100 | 9.70 | 895 |
| 3–47 | " | " | .0080 | 11,900 | 8,000 | 1,190 | 2,120 | .35 | 9,000 | 10.05 | 900 |
| 3–50 | " | " | .0099 | 11,600 | 5,000 | 995 | 1,940 | .30 | 9,000 | 9.70 | |
| 3–41 | " | " | .0095 | 11,250 | 4,500 | 1,155 | 2,180 | .40 | 10,000 | 10.00 | 895 |
| 3–30 | " | " | .0090 | 11,050 | 5,500 | | | | | 9.94 | |
| 3–45 | " | " | .0080 | 10,500 | 9,000 | 1,470 | 2,640 | .40 | 10,600 | 9.87 | 895 |
| 3–37 | " | " | .0450 | 8,050 | 4,500 | 1,255 | 2,000 | .40 | 9,500 | 10.30 | 885 |
| Avg. | " | " | .0125 | 12,950 | 6,550 | 1,060 | 1,990 | .34 | 9,940 | 9.96 | 894 |
| Avg. for Iron melted in Resist. Furnace | | | .1000 | 1,965 | 3,930 | | | | | 13.53 | |
| 3C01 | .05% C.ad'd | Vacuum | .0130 | 8,600 | 6,000 | 1,405 | 2,300 | .40 | 10,000 | 10.24 | 895 |
| 3C02 | .10% C. " | " | .0120 | 7,600 | 7,000 | 1,710 | 3,190 | .50 | 10,700 | 10.64 | 895 |
| 3C03 | .50% C. " | " | .1810 | 4,400 | 5,500 | 1,910 | | | | 12.40 | |
| 1–21 | Sw. Charc. Iron remlt'd in Vacuo | | .0080 | 10,350 | 7,000 | 1,290 | 2,640 | .48 | 11,200 | 10.30 | |
| | *Commercial Grades* | | | | | | | | | | |
| SwI-4 | Sw. Charc. Iron cut from Plate | | .1630 | 4,870 | 6,600 | 2,490 | 4,530 | .95 | 8,000 | 10.57 | |
| G 1 & 2 | Standard Transformer Steel* | | | 3,850 | 7,000 | 3,320 | 5,910 | 1.33 | 9,900 | 11.09 | |
| H1 & 2 | 4% Silicon Steel* | | | 3,400 | 4,300 | 2,260 | 3,030 | .88 | 5,400 | 51.15 | |

* Received manufacturer's standard anneal.

## TABLE 13.

## SUMMARY OF RESULT OF MECHANICAL TESTS.

| | | A | | | | | B | | | | | C | | | | | D | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Specimen No. | Carbon Content. % | AS FORGED | | | | | Annealed at 900° C. Cooled in 12 hrs. accd. to Straight Line. | | | | | Annealed at 900° C. Cooled in 24 hrs. accd. to log. curve exc. as noted. | | | | | Quenched in Brine from 1000° C. except as noted. | | | | | REMARKS |
| | | Stress at Yield Pt. lb. per sq. in. | Ultimate Strength lb. per sq. in. | Elong. % 1 | Elong. % 2 | Reduc'n of Area % | Stress at Yield Pt. lb. per sq. in | Ultimate Strength lb. per sq. in. | Elong. % 1 | Elong. % 2 | Reduc'n of Area % | Stress at Yield Pt. lb. per sq. in. | Ultimate Strength lb. per sq. in. | Elong. % 1 | Elong. % 2 | Reduc'n of Area % | Stress of Yield Pt. lb. per sq. in. | Ultimate Strength lb. per sq. in. | Elong. % 1 | Elong. % 2 | Reduc'n of Area % | |
| 3-30 | .0090 | 36,600 | 44,800 | | 41 | 87.7 | | | | | | | | | | | | | | | | |
| 3-31 | .0120 | 69,100 | 70,500 | | 20 | 82.8 | 20,700 | 35,500 | 30 | 60 | 89.5 | | | | | | | | | | | |
| 3-33 | | 64,000 | 68,000 | | 22 | 81.5 | | | | | | | | | | | | 40,500 | 24 | 53 | 84.5 | |
| 3-34 | .0090 | | | | | | | | | | | | 34,300 | | 47 | 61.1 | | 55,500 | 14 | 32 | 83.1 | |
| 3-35 | | | | | | | | | | | | 14,400 | 34,500 | | 54 | 87.3 | | 44,900 | 22 | 44 | 86.0 | |
| 3-36 | .0150 | | | | | | | | | | | 20,100 | 37,500 | | 52 | 77.0 | | | | | | |
| 3-37 | .0450 | | | | | | | | | | | | | | | | 41,250 | 64,600 | 15 | 37 | 73.7 | |
| 3-38 | .0104 | | | | | | | | | | | | 35,150 | | 56 | 85.5 | 37,400 | 45,500 | 17 | 42 | 86.1 | |
| 3-39 | .0110 | 35,800 | 44,700 | | 39 | 80.4 | | | | | | 16,400 | 36,100 | | 61 | 80.9 | | | | | | Column C: cooled in 48 hrs. accd. to straight line. |
| 3-40 | .0110 | 39,600 | 43,500 | | 38 | 82.3 | | 41,600 | | 42 | 85.7 | | | | | | | | | | | |
| 3-41 | .0095 | 62,200 | 65,500 | | 26 | 83.8 | | 40,200 | 18 | 53 | 85.1 | | | | | | | | | | | |
| 3-43 | .0096 | | | | | | 18,750 | 35,800 | 31 | 59 | 84.5 | | | | | | | | | | | |
| 3-45 | .0080 | 51,400 | 55,500 | | 32 | 85.8 | 15,000 | 39,400 | 27 | 40 | 88.0 | | | | | | | | | | | |
| 3-47 | .0080 | | | | | | 17,400 | 36,100 | 31 | 57 | 91.3 | | | | | | 30,200 | 40,000 | 20 | 47 | 80.8 | |
| | | | | | | | | | | | | | | | | | 15,500 | 35,400 | 27 | 45 | 36.6 | ← Quenched in liq. air. |
| 3-48 | .0080 | 45,600 | 51,800 | 12.0 | 36 | 86.4 | | | | | | 15,300 | 35,800 | 23 | 42 | 73.2 | | | | | | Column C: cooled 900° to 200° C. in 24 hrs. accd. to str. line. |
| 3-49 | .0120 | 41,400 | 48,700 | 12.0 | 44 | 76.2 | | | | | | 11,100 | 34,900 | 23 | 32 | 75.7 | | | | | | |
| 3-50 | .0099 | 38,200 | | 12.5 | 35 | | | | | | | 19,300 | 35,800 | 28 | 45 | 86.5 | | | | | | |
| Avg. | .0125 | 48,390 | 54,800 | 12.2 | 33.3 | 83.0 | 17,960 | 38,100 | 27.4 | 51.8 | 87.4 | 16,100 | 35,500 | 24.7 | 48.6 | 78.4 | 36,280 | 50,170 | 18.6 | 42.5 | 82.4 | |
| 3 CO1 | .0130 | | | | | | 21,000 | 51,100 | 27 | 41 | 72.5 | | | | | | 33,200 | 53,000 | 20 | 65 | 86.2 | %Carbon added=.05 |
| 3 CO2 | .0120 | | | | | | 19,400 | 38,900 | 29 | 60 | 73.5 | | | | | | 67,700 | 76,900 | 7 | 25 | 79.7 | %Carbon added=.10 |
| 3 CO3 | .1810 | | | | | | 22,100 | 51,000 | 24 | 50 | 64.5 | | | | | | | | | | | %Carbon added=.50 |
| Sw.I-4 | .1630 | | 40,000 | | 36 | 57.0 | | | | | | | | | | | | | | | | As cut from plate |

1. Elongation before specimen has commenced to "neck." 2. Ultimate elongation.

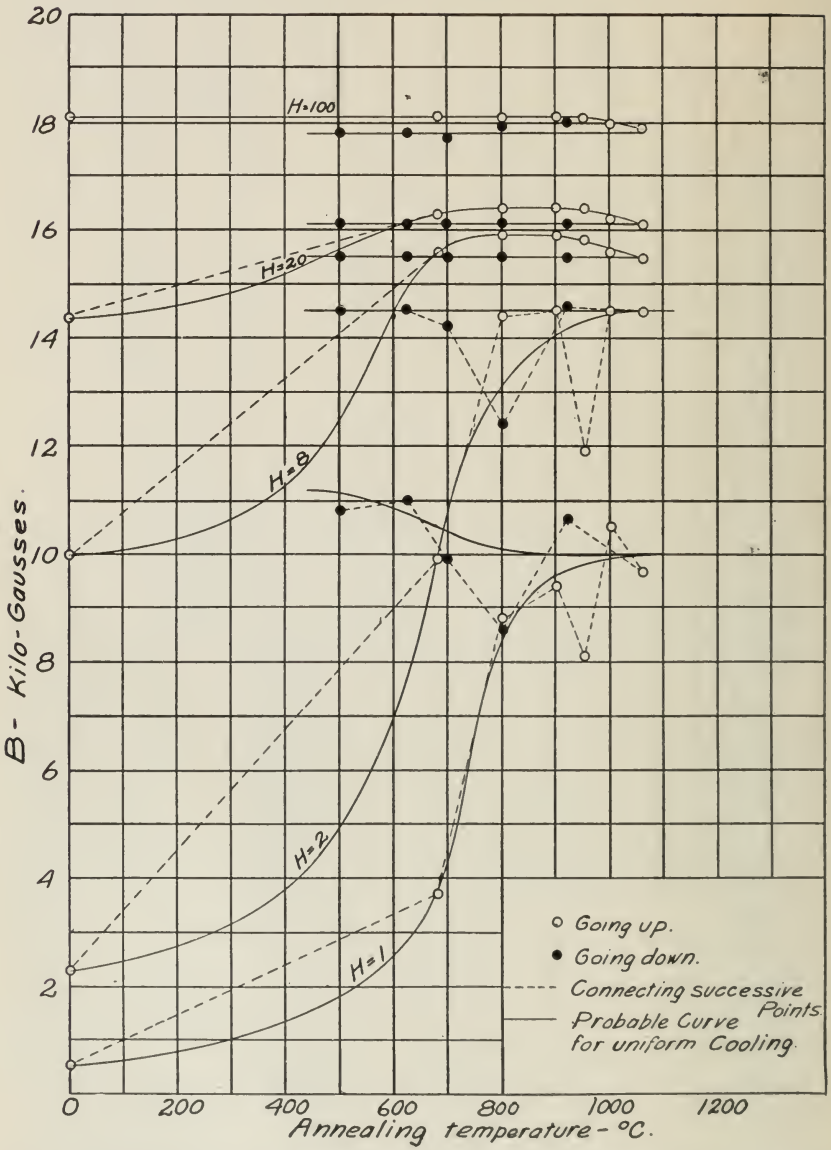

Fig. 3. Electrolytic Iron Melted in Vacuo. Effect of Annealing Temperature. Rod No. 3-39.

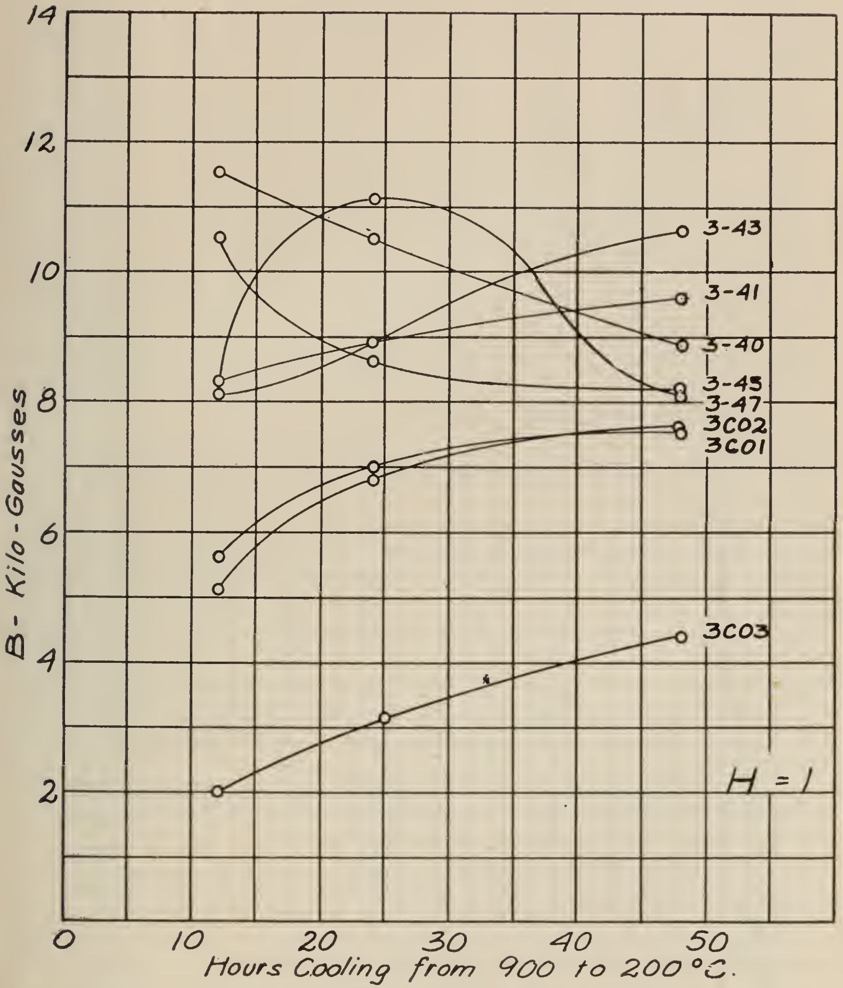

Fig. 4. Electrolytic Iron Annealed at 900° C. in Contact with Rods Containing Carbon.

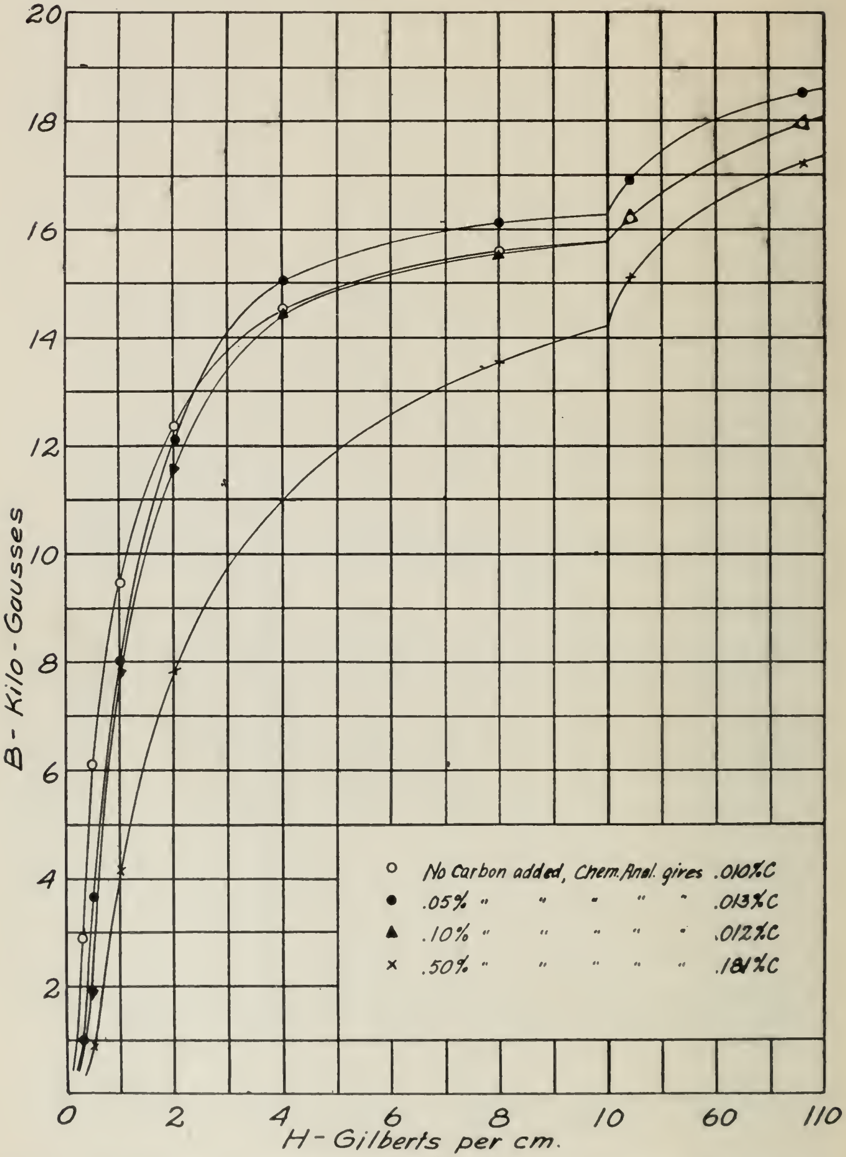

Fig. 5. Electrolytic Iron Melted in Vacuo. Effect of Carbon. Annealed at 900° C.

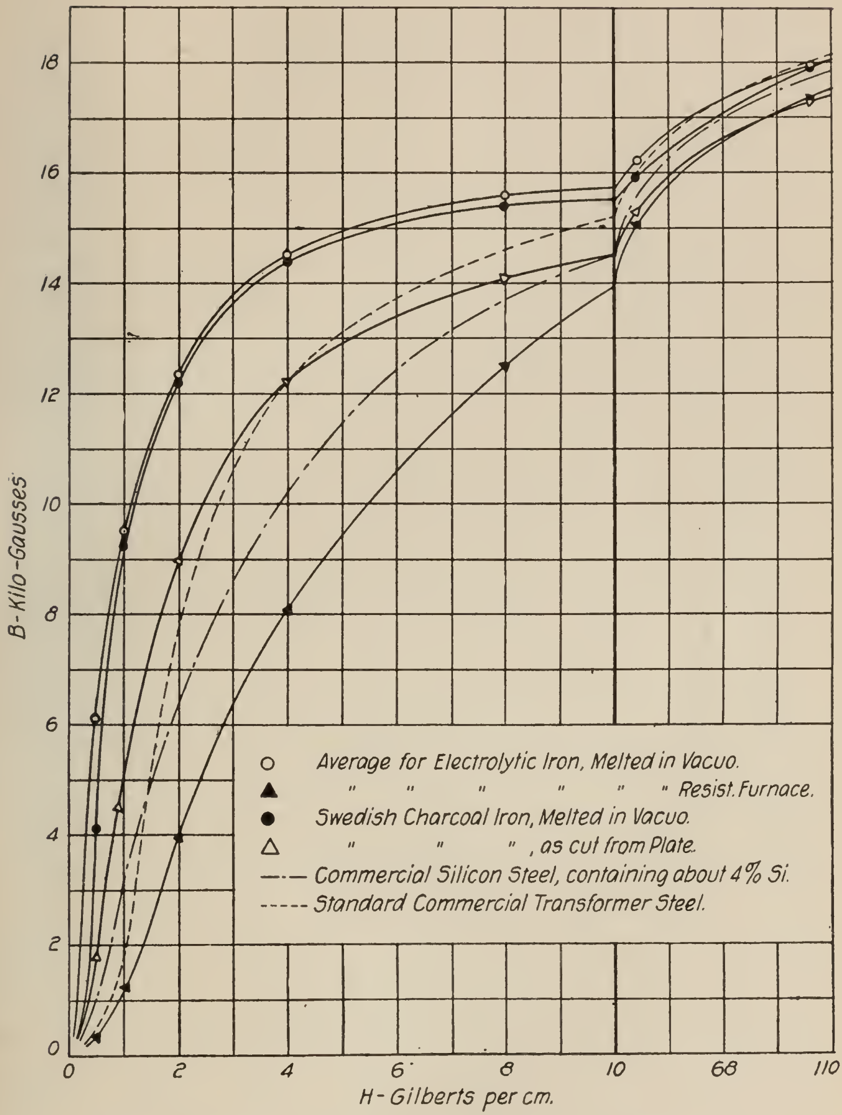

FIG. 6. MAGNETIZATION CURVES FOR DIFFERENT GRADES OF IRON.

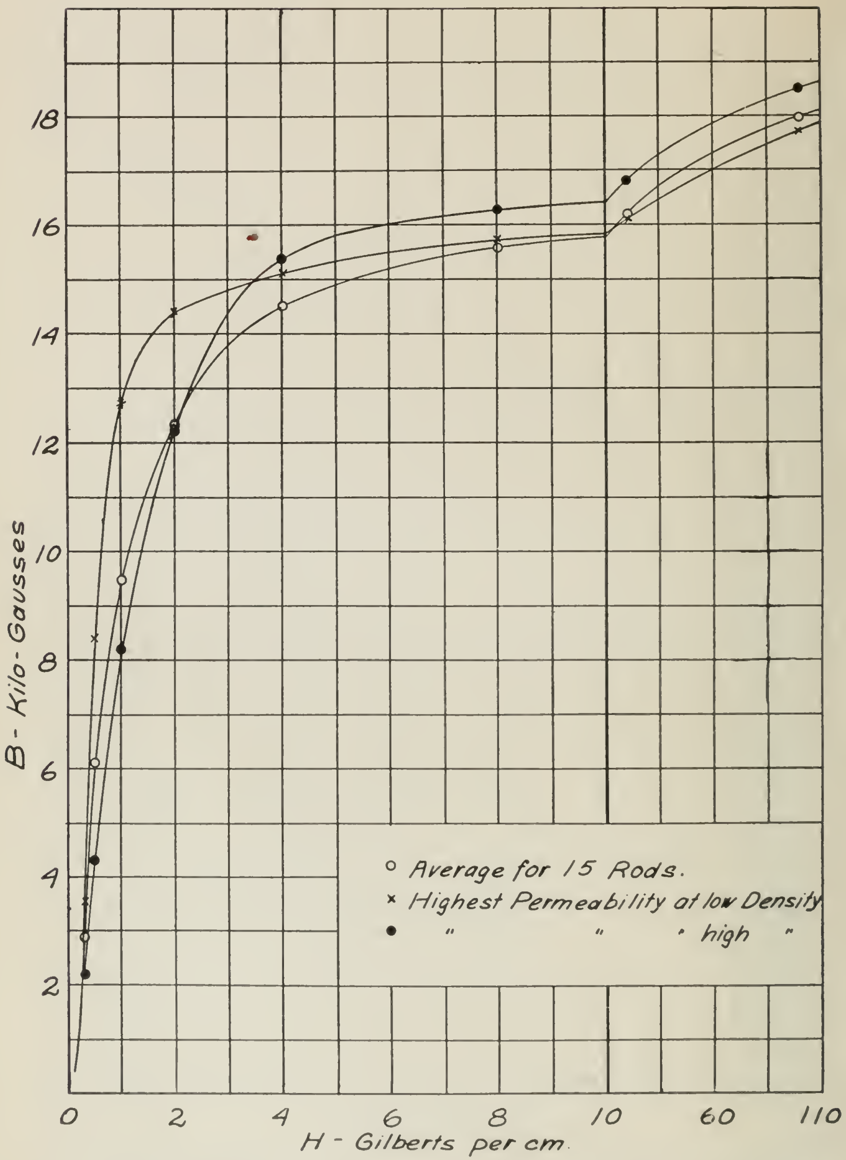

FIG. 7. ELECTROLYTIC IRON MELTED IN VACUO. ANNEALED AT 900° C. COOLED IN 48 HOURS.

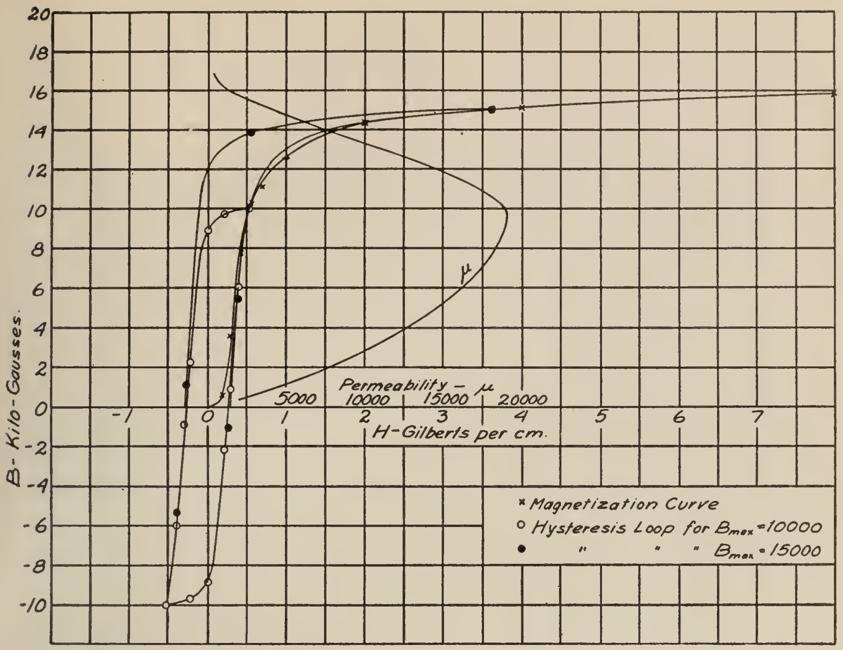

FIG. 8. ELECTROLYTIC IRON MELTED IN VACUO. ANNEALED AT 900° C. COOLED IN 48 HOURS. ROD NO. 3-38.

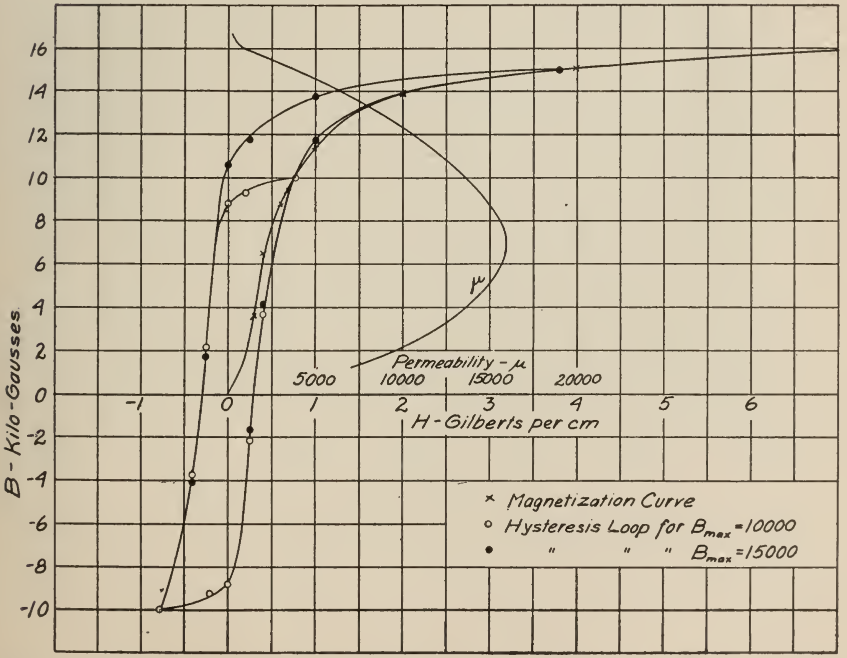

FIG. 9. ELECTROLYTIC IRON MELTED IN VACUO. ANNEALED AT 900° C. COOLED IN 48 HOURS. ROD NO. 3-34.

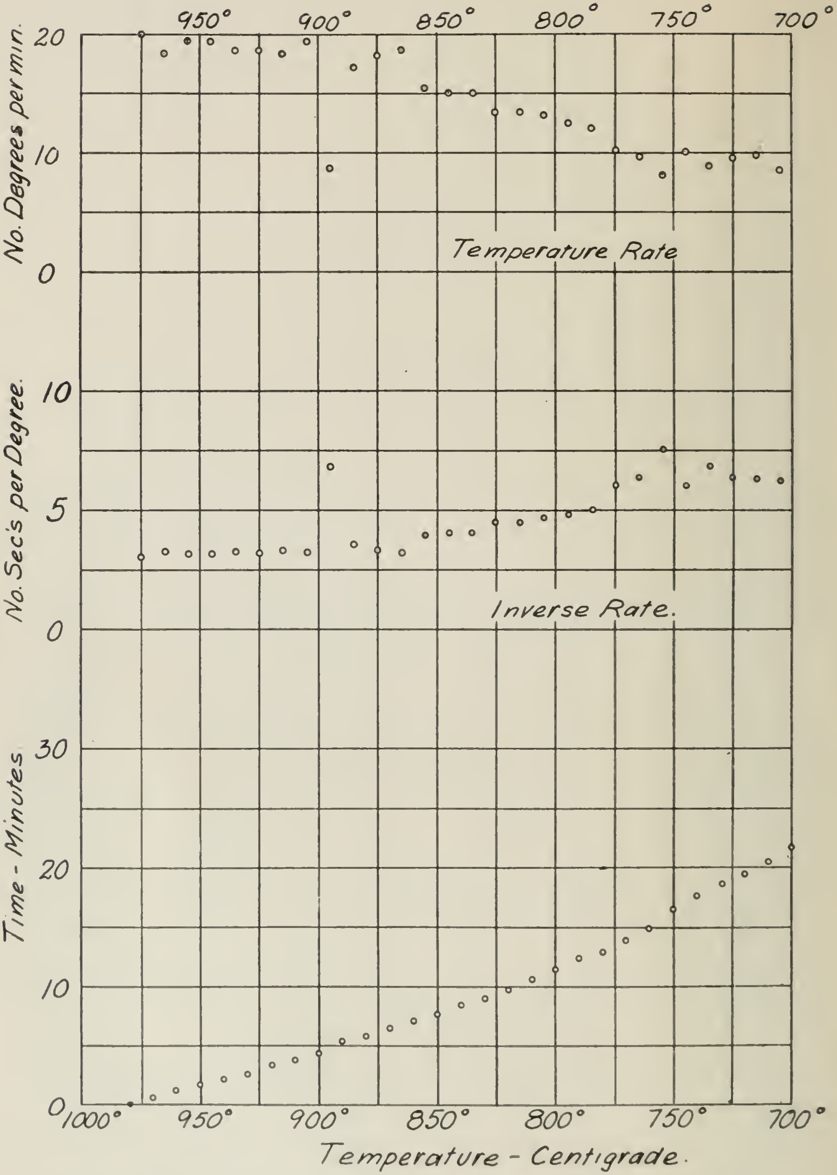

FIG. 10. COOLING CURVES FOR ELECTROLYTIC IRON MELTED IN VACUO. ROD NO. 3-43.

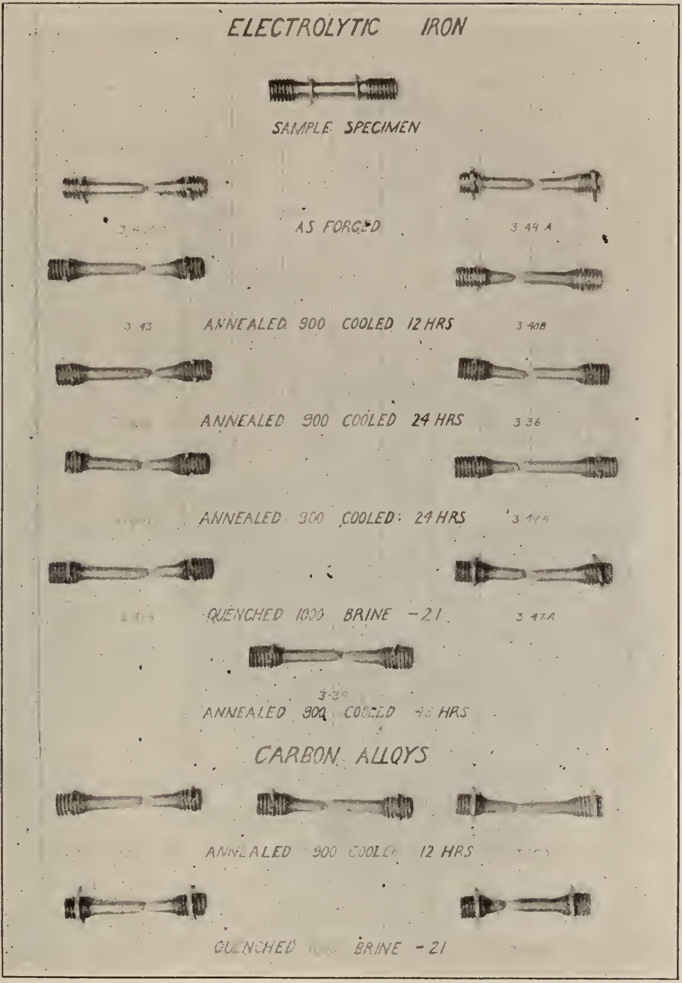
ELECTROLYTIC IRON
SAMPLE SPECIMEN
AS FORGED
3 44 A
3 43
ANNEALED 900 COOLED 12 HRS
3 40B
ANNEALED 900 COOLED 24 HRS
3 36
ANNEALED 900 COOLED 24 HRS
QUENCHED 1000 BRINE −21
3 47A
3-39
ANNEALED 900 COOLED 48 HRS
CARBON ALLOYS
ANNEALED 900 COOLED 12 HRS
QUENCHED BRINE −21

Fig. 11. A Few Mechanical Test Pieces.

As already mentioned, photomicrographs were obtained for nearly all rods after each heat treatment. Only a few of these will be exhibited here. Figs. 19-30 show the complete series for rod No. 3-39, as this rod was given a larger variety of heat treatments than any other rod. This series may be studied to advantage together with Table 6 and Fig. 3. Besides these photomicrographs for No. 3-39 a few for some of the other electrolytic iron rods melted in vacuo are shown. To show the effect of carbon there has been included an exhibit of the three iron-carbon alloys No. 3C01, No. 3C02, and No. 3C03. The difference between SWI-4 and No. 1-21, one being Swedish iron cut from the plate and the other after being melted in vacuo, is clearly shown in Figs. 43-45. Fig. 46 shows the structure of the electrolytic iron melted in the resistance furnace under atmospheric pressure.

## V. Discussion of Results.

The results obtained may perhaps best be discussed by considering first the equilibrium diagram for iron-carbon alloys. Fig. 12 shows this diagram as represented by Rozenhain. This diagram should probably be modified somewhat in view of the recent researches upon the critical temperatures for pure iron. It has been shown for instance by Burgess and Crowe[501] in a bulletin just issued by the Bureau of Standards that the point F, usually called $Ar_2$, for pure iron is 768° C. Furthermore, it has been shown conclusively that the temperature at which pure iron changes from a ferromagnetic to a paramagnetic substance, or vice versa, is 785° C., thus showing that these two transformation points do not coincide.

Only that part of the diagram lying to the left of "I" will be considered here. From this diagram it is seen that after the iron has passed from the liquid state it exists in the form of a solid solution of iron and carbon, called austenite. When this solution cools it eventually reaches the line E-G-I, where ferrite crystals begin to be precipitated. Upon further cooling the solution, now enriched in carbon, passes downwards towards the right following E-G-I, more and more ferrite being precipitated. Finally, upon reaching the point I, the part of the solution still remaining, containing now 0.9 per cent carbon, is decomposed into cementite ($Fe_3C$) and ferrite, and the resulting mixture is called pearlite. At this temperature the iron consequently consists of pure ferrite crystals with the spaces between the crystals filled up with a mixture of ferrite and cementite. One generally accepted theory states that iron has three different allotropic modifications, Alpha, Beta, and Gamma. Gamma iron is stable above the line E-G-I, Beta iron between E-G and G-F, and Alpha iron below G-F. It has been shown that iron in the Beta and Gamma modifications is weakly paramagnetic,* while Alpha iron is ferromagnetic. Under certain conditions Beta and Gamma iron may exist to some extent below the boundaries given above. It is supposed, for instance, that if iron be cooled rapidly from a high temperature, Gamma and Beta iron may not have sufficient time to change into Alpha

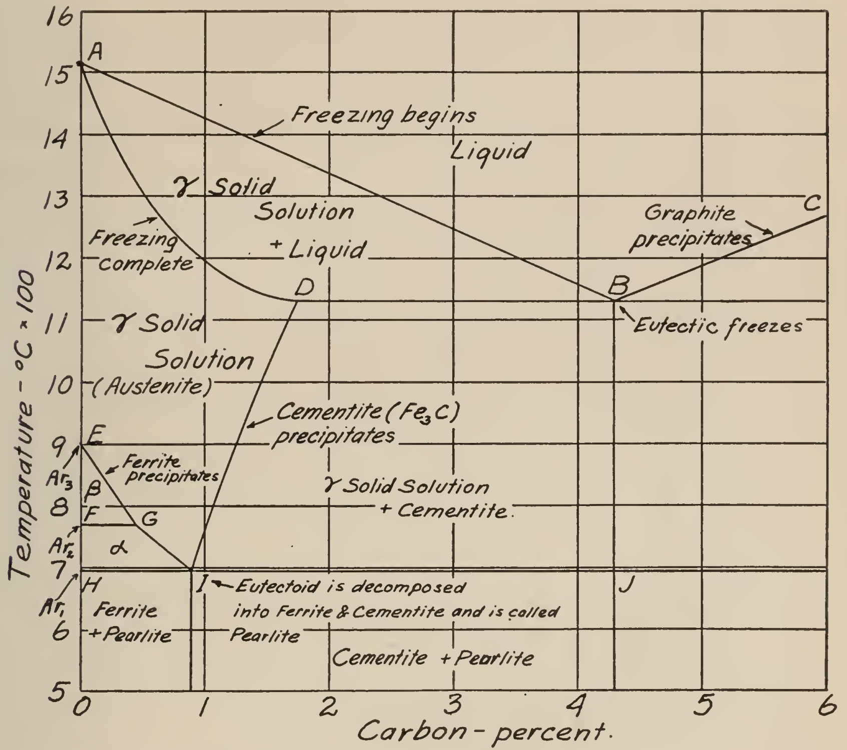

FIG. 12. IRON-CARBON EQUILIBRIUM DIAGRAM ACCORDING TO ROZENHAIN.

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*A paramagnetic substance is a substance capable of being magnetized in the same direction as that of the magnetizing force applied to it, in contradistinction to a diamagnetic substance, which, when placed in a magnetic field, is magnetized in the opposite direction to that of the field. A ferromagnetic substance is a paramagnetic substance whose permeability is of the same order as that of iron under ordinary conditions.

iron. If, however, such iron be reheated to above the line E-G-I and slowly cooled, the iron may ultimately all be obtained in the ferromagnetic Alpha modification. This theory is confirmed by the results obtained by Hadfield and Hopkinson[105], who found that the "specific magnetism" of iron-carbon alloys is decreased to a large but uncertain extent by quenching from a high temperature. Certain elements, carbon and manganese for example, when added to the iron assist in retaining the iron in the Gamma form. Thus Hadfield obtained an iron-manganese alloy whose permeability is nearly unity at ordinary temperatures.

Another factor which may influence the magnetic properties of iron is mechanical strain, due either to outside influence or to rapid cooling. It is not definitely known which is the more important, removing mechanical strain, or changing all the iron into Alpha iron, but it is certain that both are important and that both may be accomplished by proper annealing. It is well known that if subsequent to annealing the iron be mechanically strained, its magnetic quality is depreciated. The author noticed this while measuring a rod that had been slightly bent during the annealing process. Its permeability seemed, compared with previous measurements, to be unusually low. It was noticed that in clamping it in the permeameter it had been strained to some extent. It was then adjusted, so as to lessen the strain as far as possible, and the permeability was found to have increased materially. This fact helped to explain some rather puzzling results obtained in the course of the investigation. It also showed that, if the iron has not been strained beyond a certain point the effect of the strain is only temporary.

With the above explanations and facts in mind the results given in the previous part of the bulletin will now be considered.

It is at once apparent from Table 12 and Fig. 6 that melting electrolytic iron, or low carbon iron in general, in an atmosphere of carbon monoxide gases under atmospheric pressure gives the iron an opportunity to absorb carbon, the amount absorbed varying from 0.05 to 0.15 per cent. This result is in agreement with the experiments of Burgess and Aston referred to in the introduction. It is furthermore seen that carbon is not the only factor that affects the magnetic properties of iron. Swedish charcoal iron SWI-4 with a carbon content of 0.163 per cent and No. 3C03, an electrolytic iron-carbon alloy melted in vacuo, with a carbon content of 0.181 per cent, both have a decidedly higher permeability than the electrolytic iron melted in the resistance furnace, although the latter has an average carbon content of only 0.10 per cent. Again, the electrical resistance of the resistance furnace iron is 13.52 microhms, while that of SWI-4 and No. 3C03 is 10.57 and 12.40 microhms respectively. The carbon content of 0.1 per cent does not by itself account for a resistance of 13.52 microhms. According to Barrett[19] this resistance corresponds to a carbon content of 0.6 per cent. As carbon is probably absorbed from CO by the reversible reaction

$$\mathrm{Fe} + \mathrm{CO} \rightleftarrows \mathrm{C} + \mathrm{FeO}$$

it seems natural to assume that not only carbon, but also iron oxide has been absorbed by the iron. In that case the result would be an alloy of ferrite, cementite ($Fe_3C$) and iron oxide ($FeO$). For a carbon content of 0.1 per cent, the alloy would contain 1.5 per cent $Fe_3C$, and 0.59 per cent FeO. While cementite is slightly ferromagnetic as shown by Hadfield and Hopkinson[105], FeO is weakly paramagnetic, and a very poor conductor of electricity. With these two substances interspersed among the ferrite crystals the low permeability and high resistance can be satisfactorily explained. From a further study of Fig. 6 and Table 12 it is seen that the Swedish charcoal iron has lost nearly all of its carbon by being melted in vacuo, the carbon content of the final iron being only .008 per cent. At the same time its electrical resistance has been lowered and its magnetic permeability considerably increased. Similarly, the three iron-carbon alloys melted in vacuo have lost the larger portion of the carbon added to them. While the additions amounted to .05, .10 and .50 per cent, respectively, the chemical analysis of the final iron shows a carbon content of .013, .012 and .181 per cent. If to the former figures is added the carbon content of the electrolytic iron as deposited, namely, .006 per cent, the loss of carbon is found to be 76.8, 88.5 and 64.4 per cent respectively. The corresponding loss for the Swedish iron was 95 per cent. It has been demonstrated by the Bureau of Standards[501] that such a loss occurs by melting electrolytic iron in vacuo, the loss found being in one case 67 per cent, in another case 87.5 per cent.

In the present case the electrolytic iron melted in vacuo not only did not lose any carbon, but actually gained a slight amount. In order to explain this apparent discrepancy it should be mentioned that the pressure used by the Bureau of Standards in their vacuum furnace was .01 mm. of mercury, while the pressure used by the author was from 2 to 5 mm. Based upon the facts above enumerated the following explanation is now offered: Any oxygen left in the furnace combines with the carbon of the heating element and forms carbon monoxide (CO). This gas, as already stated in connection with the results obtained with the resistance furnace, reacts with the iron according to the reversible equation

$$\mathrm{Fe} + \mathrm{CO} \rightleftarrows \mathrm{FeO} + \mathrm{C},$$

until equilibrium is established. If now the pressure in the furnace be lowered the reaction will take place from right to left; that is, any FeO that may be present will be reduced by the carbon in the iron with the result that the carbon content will be decreased, until equilibrium is restored. Conversely, with an increase of pressure due to an admission of oxygen, the reaction will take place from left to right, and the carbon content, as well as the FeO, of the iron will be increased. Upon this hypothesis the difference between the carbon content obtained by the Bureau of Standards and that obtained by the author is immediately explained. The final carbon content in any particular case, besides depending upon the pressure, will, according to this hypothesis, depend upon the relative amounts of carbon and iron oxide present; it will probably also depend upon the length of time the iron is kept in the molten state.

The changes in the structure of the Swedish iron are very well illustrated by the photomicrographs shown in Figs. 43-45. Fig. 43 shows the nonhomogeneous Swedish iron as rolled. The dark spots are probably partly slag and partly pearlite, shown to a higher magnification in Fig. 44. Fig. 45 shows the iron after being melted in vacuo, being perfectly homogeneous and without any sign of pearlite. Figs. 37 to 42 show the structures of the iron-carbon alloys No. 3C01, No. 3C02 and No. 3C03, verifying in general the chemical analysis as to carbon content. However, Fig. 5 and Table 10 show that the electrical resistance and magnetic permeability for No. 3C01 and No. 3C02 are not strictly in accordance with the chemical analysis. From the latter, No. 3C01 and No. 3C02 are approximately alike, while the electrical and magnetic tests indicate that No. 3C02 should have a higher carbon content than No. 3C01. It is difficult to explain why this discrepancy should exist, as the analysis for No. 3C02 was repeated four times, each giving results that agree quite closely.

The effect of annealing at different temperatures is shown in Tables 5 and 6 and graphically in Fig. 3. From the figure it is seen that the permeability increases as the annealing temperature is raised, at first slowly, and then very rapidly as the temperatures are raised from 700° to 800° C. Above 900° C. some uncertainty appears. The permeability is decreased for 950° and again increased for 1,000° C. Whether this result is due to mechanical strain in clamping the rod in the permeameter or whether it is due to different rates of cooling or to some other cause, the author is not prepared to say. From the experience previously related it seems most probable that the drop for 950° is due to mechanical straining, as it occurs for low values of H only. The experiments of Terry[226] show that 1,100° is the most favorable annealing temperature for pure iron. While the results shown in Fig. 3 are not conclusive in themselves, they help to confirm the results obtained by Terry. Another interesting point is shown in Fig. 3. For values of $H = 8$ or above, the permeability is decreased by annealing at temperatures above 900° C. and does not return to the higher value by further annealing at lower temperatures. Furthermore, upon annealing successively at decreasingly lower temperatures the permeability is increased for low values of H. Such a process is, of course, equivalent to annealing at the highest temperature and cooling at a slower rate than was used for each of the successive annealings. This assumption was verified by annealing at 900° C. and cooling, first in 24 hours, according to a logarithmic time-temperature curve, and finally in 48 hours according to a straight line connecting 900° and 200° C. As seen from Table 6 these treatments increased the permeability considerably for low values of H, but did not alter it for high values of H. Quenching from 1,000° C. in iced brine produced a decided magnetic hardening, but this hardening was again removed by reheating to 900° C. followed by slow cooling.

The results obtained with rod No. 3-39 are confirmed by those obtained with the three groups of rods made from electrolytic iron melted in vacuo. Thus, the rods of Group 2 (see Table 7) that were reheated to forging temperature, between 1,000° C. and 1,100° C., gave a decidedly lower permeability for high values of H than rods that had not been heated above 900° C. The same is shown by the results obtained for the rods of Group 4 after these rods had been annealed at 1,080° C., as seen in Table 9. While this table shows that the permeabiltiy for low values of H were not altered materially by such annealing, the rods of Group 2 as well as rod No. 3-39, that had been annealed above 1,000° C., include the best rods, magnetically, of the entire series.*

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*Rod No. 3-37 belonging to Group 2 has an exceptionally low permeability, probably due to an accident somewhere in the process, as the chemical analysis shows that it contains .045 per cent carbon.

The results shown in Fig. 4, taken from Tables 8 and 10, appear somewhat puzzling. The rods represented in Fig. 4 were all annealed together in such a way as to come in intimate contact with each other, but no record was kept as to the relative location of each rod. However, it is not probable that they occupied the same relative position each time. It will be seen that after each heat-treatment the permeability of two or three rods is decidedly lower than it was previous to that heat-treatment, and while the permeability for two of the pure iron rods is constantly increasing and that for two others is constantly decreasing, the permeability for No. 3-47 is at first increased and then decreased. No such variation occurs for any other group of rods, and the only explanation that can be offered is that the only rod containing any appreciable amount of carbon, namely No. 3C03, which contains .181 per cent, has contaminated the two or three rods with which it came in contact during each heat-treatment.

Various other inconsistencies appear upon a close scrutiny of the tables and figures, but they are of less importance and do not affect the general results. These inconsistencies show, however, how difficult it is, even by using great care, to obtain uniform results with pure iron.

The summary for all the rods made from electrolytic iron melted in vacuo, as given in Table 11, includes the results as forged and after the 48-hour cooling from 900° C., whether this last heat-treatment gave the best result with regard to magnetic quality or not. The average values obtained from this table for all the 15 rods, including rod No. 3-37, which perhaps should have been thrown out, are therefore somewhat lower than may be expected for 15 rods treated under more favorable conditions. However, the magnetization curve plotted from these average values, as shown in Fig. 6, serves to indicate the place occupied by electrolytic vacuum iron in relation to other grades of iron. While the average curve for the electrolytic vacuum iron is the highest of those shown in Fig. 6, the curve for the Swedish charcoal iron remelted in vacuo is a close second, and these two curves are far above the curves for commercial iron used at present for magnetic purposes.

The maximum permeability for the average magnetization curve as obtained from Table 12 is 12,950 occurring at a flux density of 6,550 gausses, while the maximum obtained for the best rod of the series, No. 3-38, is 19,000 occurring at a flux density of 9,500 gausses. Terry gives 11,000 as the maximum permeability obtained for electrolytic iron as deposited, but it is not perfectly clear whether this permeability occurred at ordinary temperatures or in the neighborhood of 760° C., as he compares this maximum value obtained with those obtained by Morris and Wells, which occurred near the critical temperature, 785° C. However, it is believed that Terry's maximum occurred at room temperature after annealing at 1,100° C.

While high permeability is of interest for certain purposes, the characteristics of iron that are of special interest to the manufacturer and user of electrical machinery are hysteresis and eddy current losses, the latter depending, to a large extent, upon the electrical resistance of the iron. (See ref. 32 in Bibliography.)

From Table 12 the hysteresis loss for $B_{max} = 10,000$ is seen to be less than one-half as high for the average electrolytic vacuum iron as it is for the commercial silicon steel, while for $B_{max} = 15,000$, the corresponding figure is about two-thirds. The reason why the hysteresis loss for the vacuum iron is not even still lower compared with commercial iron is readily seen by looking at the figures for retentivity. (See also Figs. 8 and 9.) While the retentivity for $B_{max} = 15,000$ is 9,940 for the average electrolytic vacuum iron, it is only 5,400 for silicon steel. The hysteresis loss for Hadfield's best magnetic steels, the 2½ per cent silicon alloy and the 2¼ per cent aluminum alloy, as reported by Barrett, Brown and Hadfield[15-18], is 1,550 and 1,440 ergs per c.c. per cycle for $B_{max} = 9,000$. Comparing these results, it is seen that in spite of the high retentivity, the hysteresis loss for the electrolytic vacuum iron is much lower than for any material thus far produced of which the literature gives information. This is due to the low coercive force, namely, .34 gilberts per cm. being the average value for the 15 rods.

The specific electrical resistance for the average vacuum iron is 9.96 microhms. The resistance for the standard transformer steel is 11.09 microhms, while that for the silicon steel is 51.15 microhms. Thus the eddy current losses per unit volume, for the same thickness of sheet and for the same maximum flux density, would be much in favor of the silicon steel. However, it is definitely known that the resistance of the electrolytic vacuum iron can be raised by the addition of silicon or aluminum. What the effect of such additions upon the magnetic quality of the iron will be remains to be seen, but, judging from the effect they have upon commercial grades of iron, it seems probable that it will not be harmful.

While the photomicrographs are of interest primarily to show the structure of the electrolytic vacuum iron in general, and in what respect it differs from other grades of iron, certain conclusions as to the relation between the microstructures and the magnetic quality of the iron may be drawn. Considering the series for rod No. 3-39 it appears that there is no general growth in the size of the crystals. Annealing at 900° C. or above with subsequent slow cooling breaks the crystals up into smaller parts. Quenching from 1,000° C., this fine structure again gives way for larger crystals with indefinite boundaries, but the fine structure reappears upon subsequent annealing and slow cooling. The structure that seems to give the best magnetic quality is that shown in Figs. 27 and 30 for rod No. 3-39. This same structure is obtained for the best rod of the series, No. 3-38, and appears in Fig. 18, also for rod No. 3-43 in Fig. 35 and for No. 3-31 in Fig. 15.

The photomicrographs of the quenched specimens, Figs. 47 to 56, show that the structure of pure iron is not changed materially by quenching either in iced brine or in liquid air. The only specimen that has undergone a decided change is No. 3C03 that contains .181 per cent C. As quenched in iced brine, Figs. 50 and 51, this specimen exhibits a structure suggesting that some of the iron has been retained in the Beta and Gamma modifications. The dark crystals are probably martensite, the first transition stage in the decomposition of austenite. Fig. 55 shows the structure of No. 3C03 after quenching in liquid air. This structure is quite different from the one shown in Figs. 50 and 51, and the substance represented is probably austenite.

These results show that the changes in the properties of pure iron resulting from rapid cooling are not so much due to changes in the structure of the iron as to mechanical strains caused by such cooling. With a small carbon content, however, the changes in the properties of the iron due to rapid cooling may be partly attributed to the retention of the iron in the Beta and Gamma modifications, as shown by Figs. 50, 51 and 55.

The results of the critical temperature determinations show that the point $Ar_3$ (see Fig. 12) for the average electrolytic vacuum iron is 894°C, agreeing with the value found by the Bureau of Standards for pure iron.

Table 13 shows that magnetic and mechanical hardness go together and confirms further the results obtained by previous writers. It emphasizes the extreme mechanical softness of the electrolytic iron melted in vacuo, particularly after annealing at 900° C. In comparing No. 3C01, No. 3C02, and No. 3C03 with the rest of the rods, it should be remembered that the actual carbon content of the former is .013, .012 and .181 per cent respectively.

While the results show some inconsistencies that cannot at present be satisfactorily explained, it should be borne in mind that the experiments have necessarily been made upon relatively small samples, that the processes involved in order to obtain the test pieces in the final condition are numerous, and that the magnetic properties are very readily affected by outside influences.

It is hoped that employing a vacuum furnace for annealing as well as for melting the iron will help to remove some of these inconsistencies.

## VI. Summary and Conclusion.

The results recorded in the previous pages may be summarized as follows:

1. Pure iron melted in an atmosphere of carbon monoxide under atmospheric pressure will absorb both carbon and oxygen with the result that the iron thus produced is of an inferior magnetic quality.

2. Low carbon iron melted in vacuo will lose 50 to 90 per cent of its original carbon content.

3. The magnetic quality of electrolytic iron melted in vacuo is decidedly superior to any grade of iron thus far produced, the maximum permeability obtained being 19,000 at a flux density of 9,500 gausses. The average hysteresis loss obtained is less than 50 per cent of that found in the best grades of commercial transformer iron.

4. The specific electrical resistance of pure iron melted in vacuo is 9.96 microhms.

5. Swedish charcoal iron melted in vacuo has a magnetic quality approximating that of electrolytic iron melted in vacuo, chiefly due to the reduction of the carbon content.

From these facts it appears that a superior quality of iron for magnetic purposes may be obtained by melting electrolytic iron in vacuo. While the electrical resistance of the iron thus obtained is very low, this defect may be remedied by the addition of such alloying elements as silicon or aluminum, elements that are known to increase the electrical resistance very materially without affecting the magnetic quality to any large extent. Experiments are now under way for determining the effect of such alloying elements, and the results will be published at some later date.

Whether iron melted in vacuo will ever become a commercial product depends, of course, upon whether any apparatus can be devised for producing such iron on a commercial scale at a cost that will not be prohibitive.

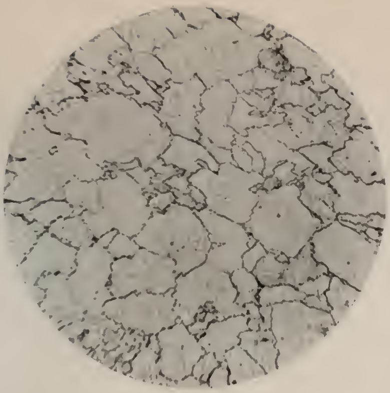

Fig. 13. 3-31A x 40.
As forged.

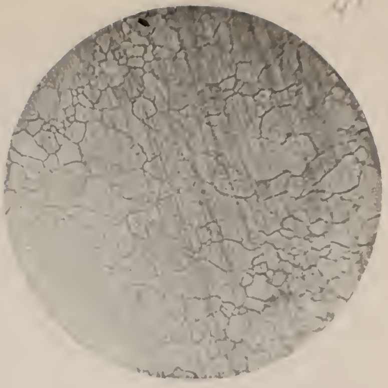

Fig. 14. 3-31F x 40.
Annealed at 950° C.

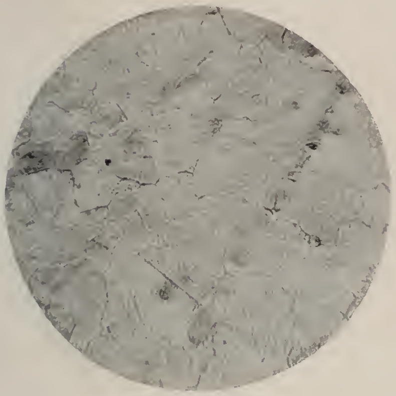

Fig. 15. 3-31K x 40.
Annealed at 900° C. Cooled in 48 hours.

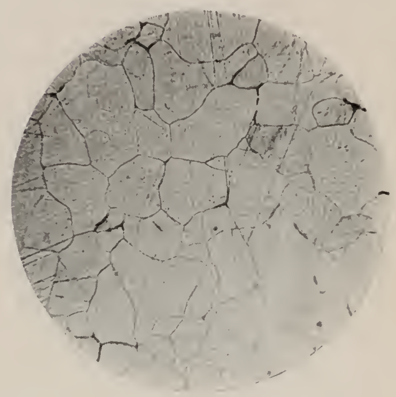

Fig. 16. 3-36 x 40.
Annealed at 900° C.

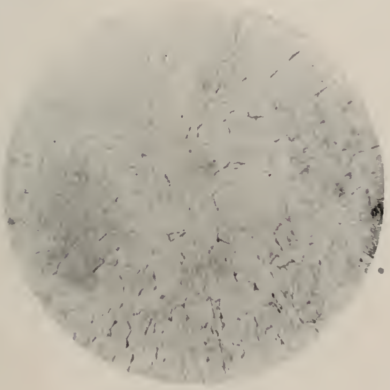

Fig. 17. 3-37 x 40.
Forged and reheated to 1,050° C.

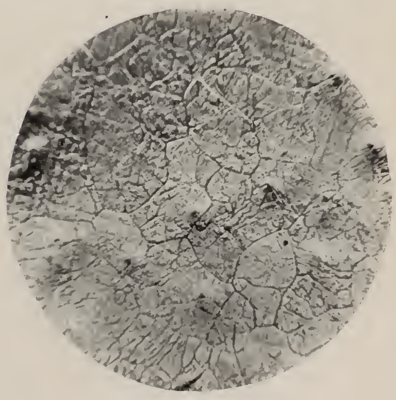

Fig. 18. 3-38 x 40.
Annealed at 900° C.

Electrolytic Iron, Melted in Vacuo. See Tables 5 and 7.

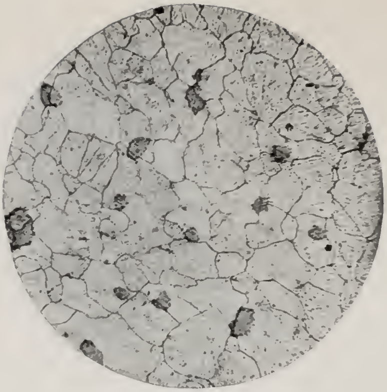

Fig. 19. 3-39A x 40.
As forged.

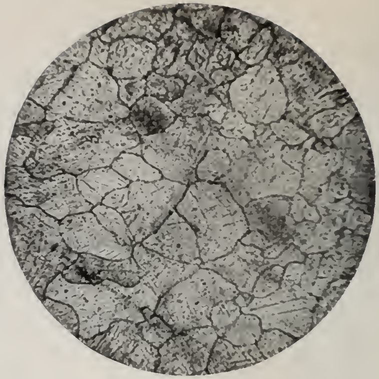

Fig. 20. 3-39B x 40.
Annealed at 680° C. Going up.

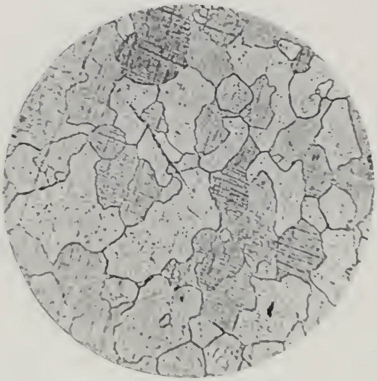

Fig. 21. 3-39D and E x 40.
Annealed at 900 and 950° C. Going up.

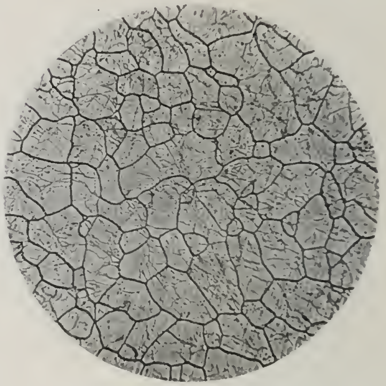

Fig. 22. 3-39F x 40.
Annealed at 1,000° C. Going up.

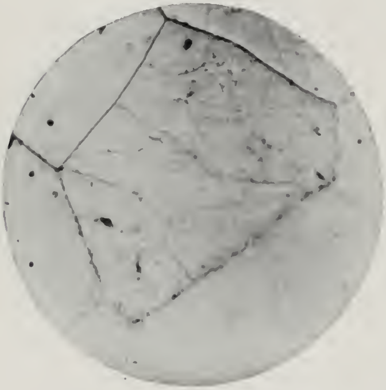

Fig. 23. 3-39F x 240.
Same as Fig. 22.

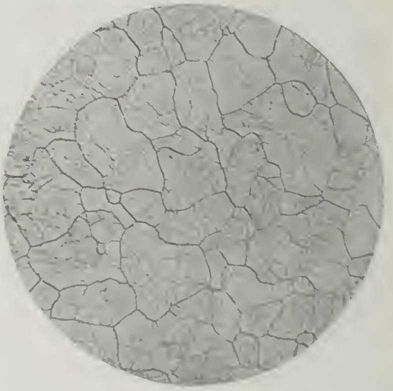

Fig. 24. 3-39G x 40.
Annealed at 1,060° C.

Electrolytic Iron, Melted in Vacuo. Series for Rod No. 3-39. See Table 6.

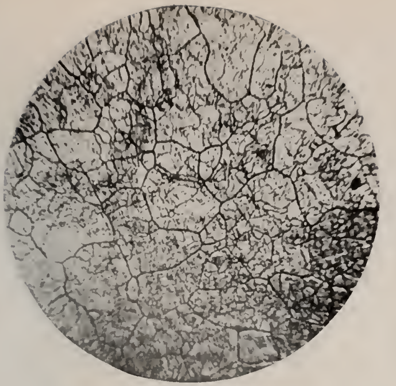

Fig. 25. 3-39H x 40.
Annealed at 912° C. Going down.

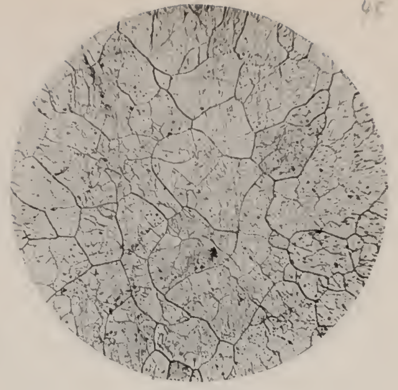

Fig. 26. 3-39J x 40.
Annealed at 800° C. Going down.

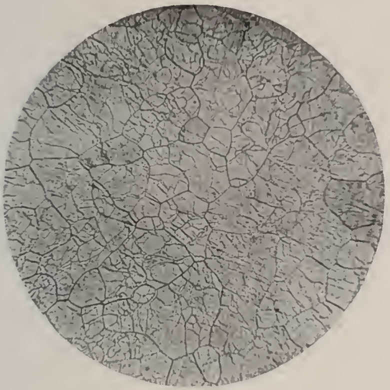

Fig. 27. 3-39L x 40.
Annealed at 616° C. Going down.

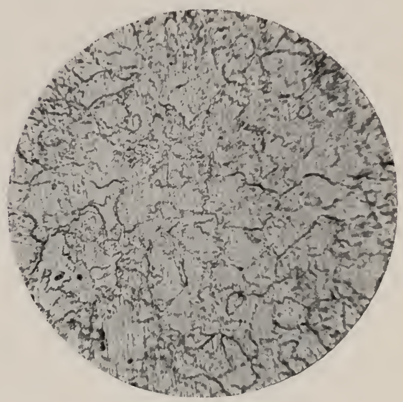

Fig. 28. 3-39O x 40.
Quenched in brine from 1,000° C.

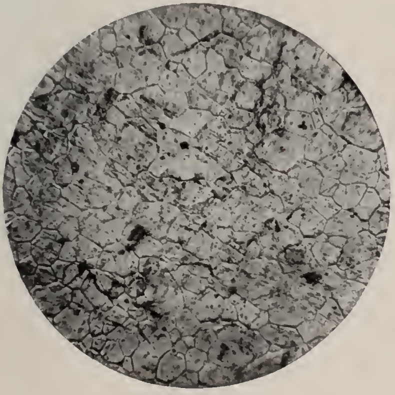

Fig. 29. 3-39Q x 40.
Annealed at 99° C. Cooled in 24 hours.

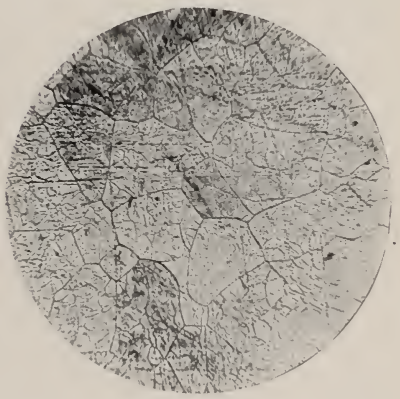

Fig. 30. 3-39S x 40.
Annealed at 900° C. Cooled in 48 hours.

Electrolytic Iron, Melted in Vacuo. Series for Rod No. 3-39. See Table 6.

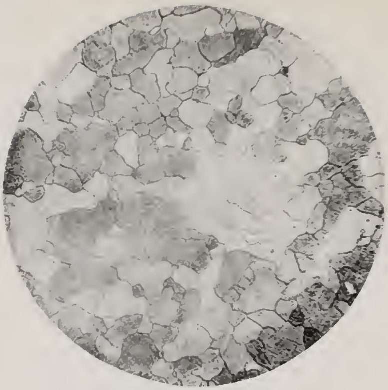

Fig. 31. 3-41 x 40.
Annealed at 900° C. Cooled in 12 hours.

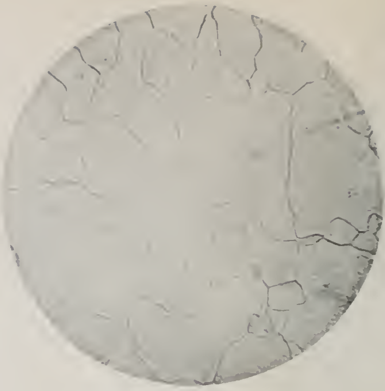

Fig. 32. 3-43 x 40.
Same treatment as Fig. 31.

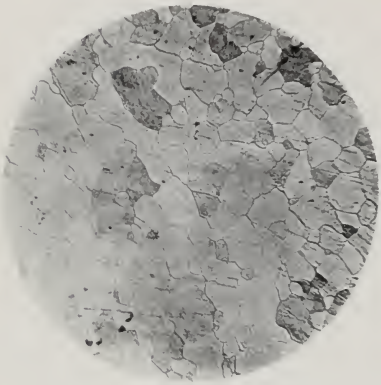

Fig. 33. 3-45 x 40.
Same treatment as Fig. 31.

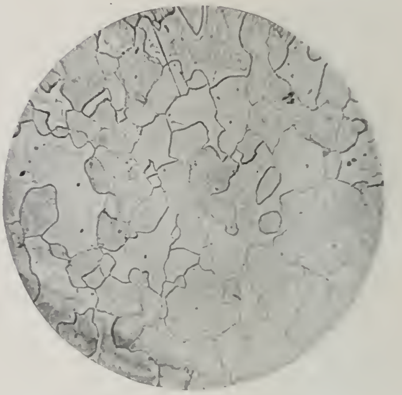

Fig. 34. 3-47 x 40.
Same treatment as Fig. 31.

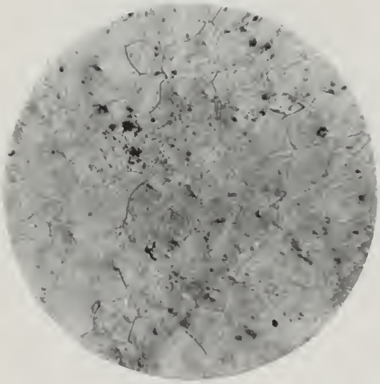

Fig. 35. 3-43 x 40.
Annealed at 900° C. Cooled in 48 hours.

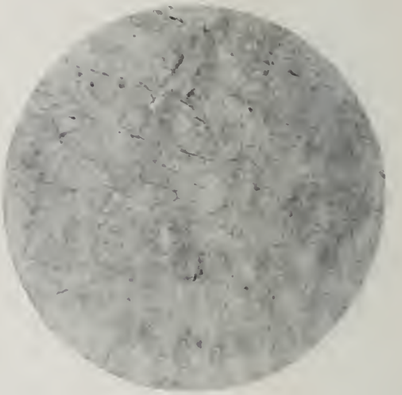

Fig. 36. 3-45 x 40.
Same treatment as Fig. 35.

Electrolytic Iron Melted in Vacuo. See Table 8.

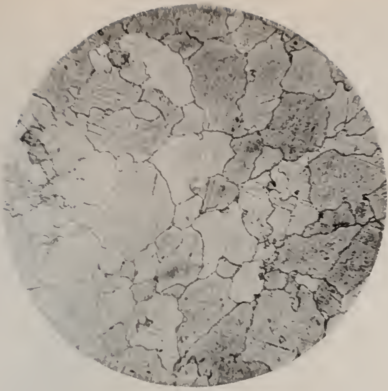

Fig. 37. 3C01 x 40.
As forged.

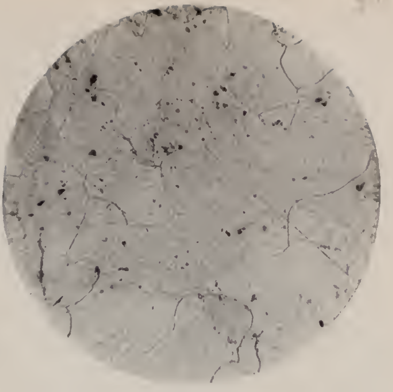

Fig. 38. 3C01 x 40.
Annealed at 900° C. Cooled in 48 hours.

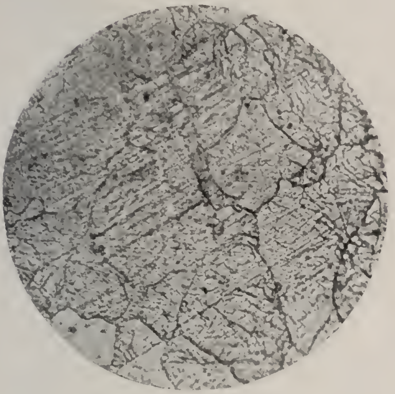

Fig. 39. 3C02 x 240.
As forged.

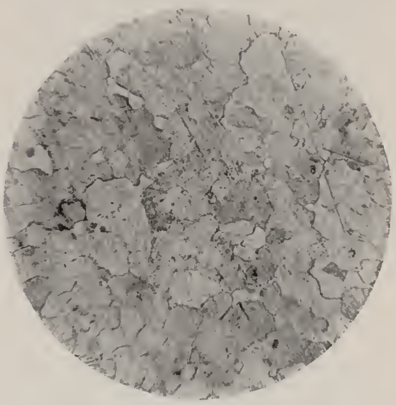

Fig. 40. 3C02 x 40.
Same treatment as Fig. 38.

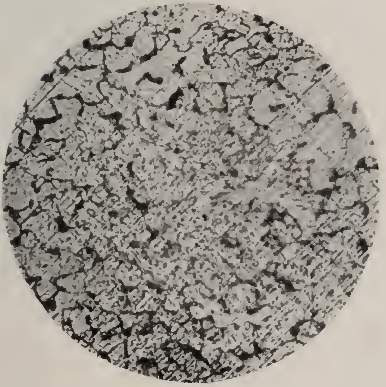

Fig. 41. 3C03 x 240.
As forged.

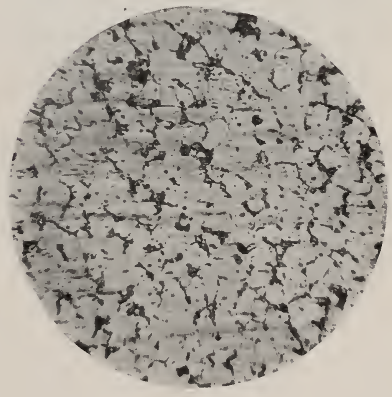

Fig. 42. 3C03 x 40.
Same treatment as Fig. 38.

Electrolytic Iron with Carbon Added, Melted in Vacuo. See Table 10.
Carbon added: 3C01—.05%, 3C02—.10%, 3C03—.50%.
Carbon as per chemical analysis: 3C01—.013%, 3C02—.012%, 3C03—.181%.

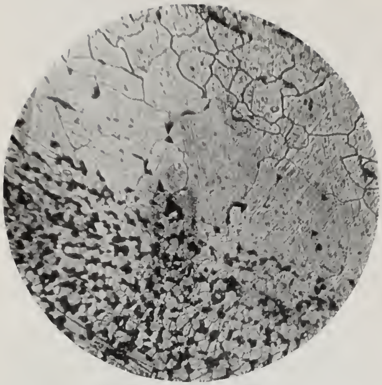

Fig. 43. SwI-4 x 40.
Annealed at 945° C.

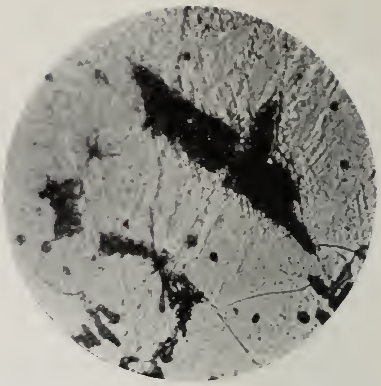

Fig. 44. SwI-4 x 240.
Same treatment as Fig. 43.

Swedish charcoal iron as cut from plate. Carbon content .163%.

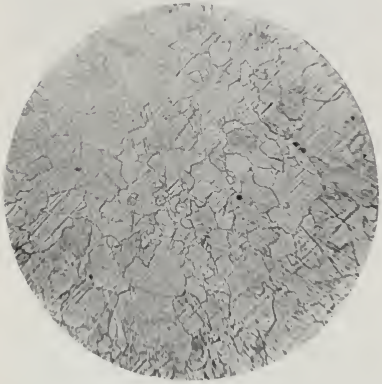

Fig. 45. 1-21 x 40.
As forged.

Swedish charcoal iron remelted in vacuo. Carbon content .008%.

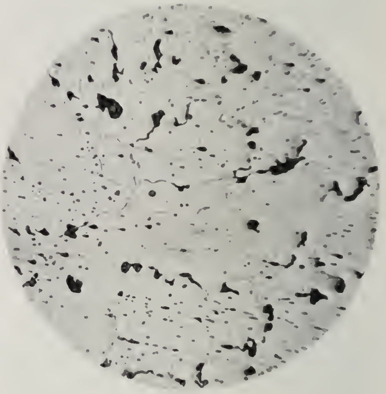

Fig. 46. 3-13 x 40.
Annealed at 900° C.

Electrolytic Iron melted in resistance furnace under atmospheric pressure. Carbon content .105%.

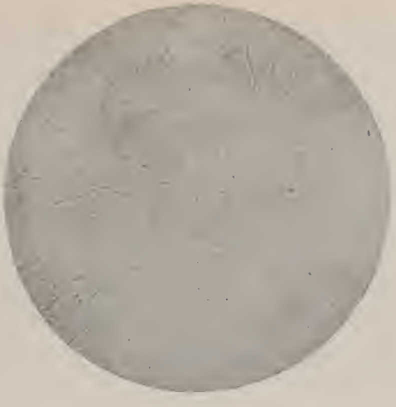

Fig. 47. 3-34 x 40.
Carbon content .009%.

Fig. 48. 3C01 x 40.
Carbon content .013%.

Fig. 49. 3C02 x 240.
Carbon content .012%.

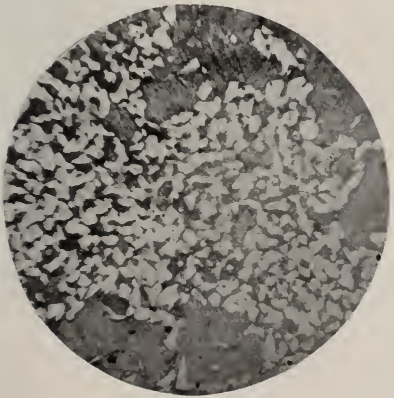

Fig. 50. 3C03 x 40.
Carbon content .181%.

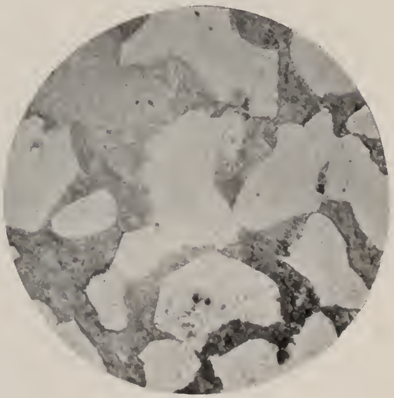

Fig. 51. 3C03 x 240.
Same as Fig. 49.

Electrolytic Iron with and without Carbon Added, Melted in Vacuo.
Quenched from 1000° C. in Brine at —21° C.

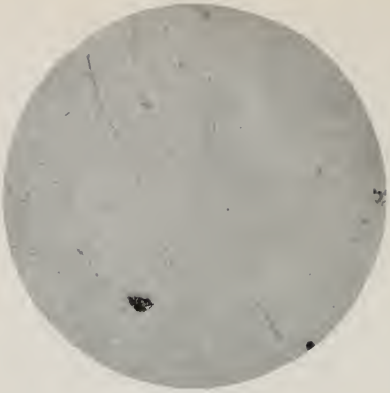

Fig. 52. 3-36 x 40.
Carbon content .015%.

Fig. 53. 3-47 x 40.
Carbon content .008%.

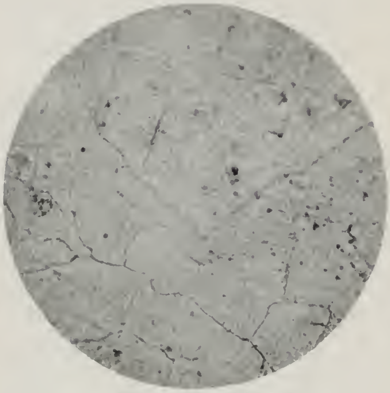

Fig. 54. 3C02 x 40.
Carbon content .012%.

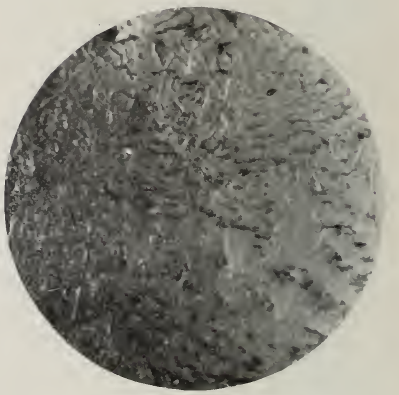

Fig. 55. 3-C03 x 240.
Carbon content .181%.

Fig. 56. 1-21 x 40.
Swedish charcoal iron, remelted in vacuo.
Carbon content .008%.

ALL SPECIMENS QUENCHED FROM 1000° C. IN LIQUID AIR.

## APPENDIX I.

### CRUCIBLES AND ARC FURNACE.

The crucibles in which the electrolytic iron was melted were made from electrically fused magnesia, made in the arc furnace shown in Fig. 57. This furnace was the outcome of a series of experiments with different furnaces. It consists essentially of an iron shell 28″ (71 cm.) in diameter and 48″ (122 cm.) in height, placed upon an iron tray 60″ (152 cm.) in diameter. The top is provided with four 6″ (15 cm.) openings with iron covers. The lower electrode consists of an 8″ (20 cm.) round carbon block, hollowed out on the top and placed upon a layer of magnesia bricks. Into this block is screwed a 3″ (7.6 cm.) carbon rod that extends through the iron shell. The upper electrode consists or a 3″ (7.6 cm.) carbon rod capable of being raised or lowered by means of a rope and counterweight. The guides for the electrodes consist of iron pipes fastened to the iron shell by locknuts and insulated from the shell by asbestos washers.

The raw material used at first consisted of a product known as "native powdered magnesite," calcined at about 1000° C. Chemical analysis showed that it contains 4 per cent $SiO_2$ and 2 per cent Fe. The fused material contains 2 per cent $SiO_2$. Later, a purer product was used, obtained by precipitation. This product had not been calcined and was found upon analysis to contain .4 per cent $SiO_2$, 3 per cent $Fe_2O_3$ and $Al_2O_3$, and .179 per cent CaO. The fused magnesia from this product contains 1.0 per cent $SiO_2$.

The operation of the furnace was as follows: The shell was completely filled with the raw material. On account of its consistency it served as an excellent heat insulator, so that no other lining of the shell was needed. The arc was started by lowering the upper electrode until it touched the carbon block, the current being limited by means of a water rheostat. The electrode was then raised until the voltage across the arc was about 50 volts. As the magnesia fused it filled the hollow in the carbon block, and, being a good conductor of electricity in the fused state, the arc formed between it and the upper electrode. The latter consequently had to be raised gradually as more and more magnesia fused in order to maintain the voltage. As the arc was raised the magnesia gradually solidified but continued to act as a conductor.

During this operation fresh magnesite had to be supplied on account of the shrinkage. The "native powdered magnesite" shrank to about one-fourth, while the precipitated magnesite shrank to about one-tenth of the original volume. At the end of 10 to 12 hours the operation was completed, and the result was a core of fused magnesia, weighing 75 to 100 pounds (35 to 45 kg.), surrounded by sintered magnesia weighing 100 pounds or more.

Attending this furnace was by no means a pleasure. As fresh magnesite fell down into the arc an explosion would take place, forcing steam, $CO_2$, and other gases out of the furnace, accompanied by a cloud

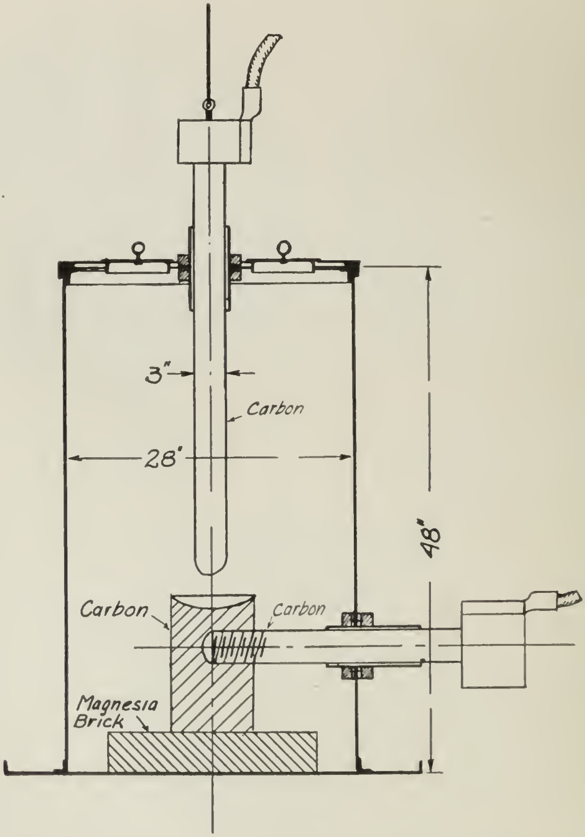

FIG. 57. ARC FURNACE FOR PRODUCING FUSED MAGNESIA.

of fine particles of magnesite. This was particularly true of the precipitated magnesite, and it would probably have been cheaper and easier to have calcined this material in some other way previous to fusing it in the arc furnace.

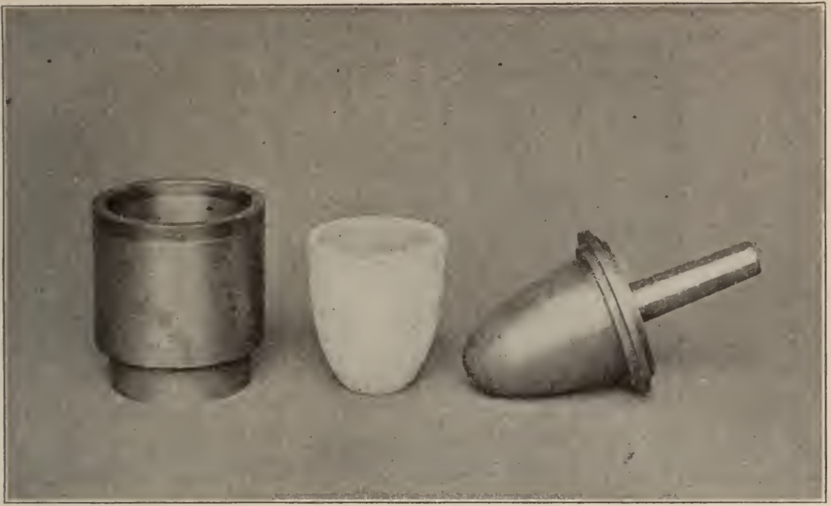

Fig. 58. Form for Making Crucible and Finished Product.

For making the crucibles the fused magnesia thus obtained was ground to go through a 40 mesh sieve. The binder used consisted of hydrated magnesia, made by mixing 600 grams "native powdered magnesite" with 1,000 c.c. water. This was placed in a stone mill that was kept going for twenty-four hours. Just a sufficient amount of this binder was used with the fused magnesia to make the mass moist. The crucibles were pressed in a steel form, shown in Fig 58, fitting into a hand operated press capable of exerting a pressure of 4,000 pounds. After being formed the crucibles were left to dry over steam pipes for a couple of days and were then baked in the electric furnace, shown in Fig. 62, at a temperature of 1,600 to 1,800° C. The finished product is shown in Fig. 58. If burned in a slightly oxidizing atmosphere the crucibles will come out perfectly white. They are strong mechanically and do not begin to show signs of softening below 1,800° C.

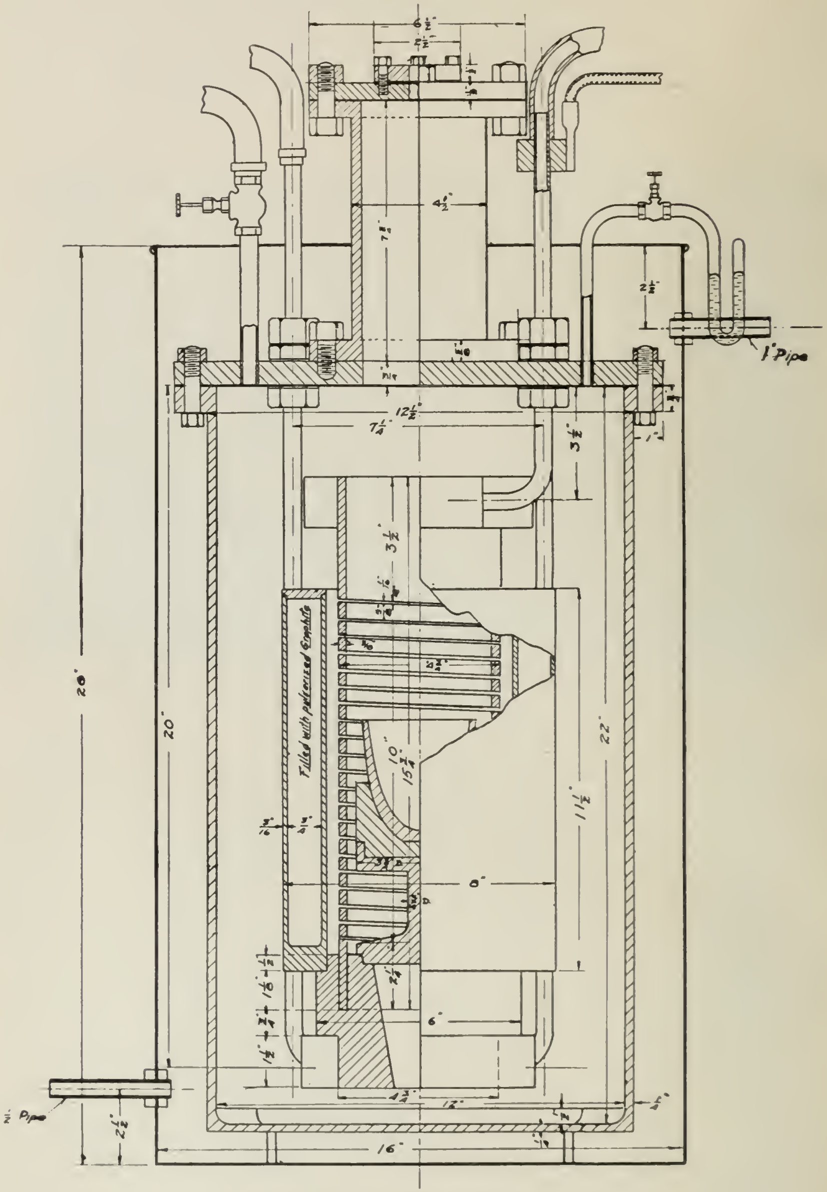

FIG. 59. VACUUM FURNACE.

# APPENDIX II.

## VACUUM FURNACE.

The furnace described here was modeled after the vacuum furnace developed by W. C. Arsem, who presented a paper descriptive of this type of furnace before the American Electrochemical Society in 1906. The furnace as constructed by the author is illustrated in Figs. 59, 60 and 61. The vacuum tank was made from a 12″ (30 cm.) wrought iron pipe, with a ¼″ (.64 cm.) steel plate welded on at one end. A steel ring was shrunk around the top and the joint made airtight by means of a groove filled with molten lead. The cover was made from a ¾″ (1.9 cm.) steel plate and the dome from a 4″ (10 cm.) wrought iron pipe provided with flanges at each end. A mica window was placed in the top of the dome for the inspection of the charge during operation. As the furnace is submerged in water, all the joints were made airtight by means of rubber gaskets.

The heating element was cut from a solid 4¾″ (12 cm.) carbon electrode by first cutting a helical groove of the desired pitch and depth and then cutting out the core by means of a pipe provided with teeth on one end. Care had to be exercised to obtain a perfectly uniform cross-section. The radiation screen surrounding the helix was made from an 8″ (20 cm.) carbon electrode and filled with crushed graphite. Both the helix and the screen are supported at the lower end by a carbon base which in turn is supported by a water cooled copper tubing. This tubing is connected to the base by means of a cast iron clamp, and it serves both as a support for the heating part of the furnace and as a conductor for the current. It is clamped in place by two airtight insulating bushings in the cover. The upper end of the helix is similarly supported by another water cooled copper tubing connected to the helix through a cast iron clamp.

An ordinary mercury manometer was used to measure the pressure, as it was deemed unnecessary to reduce the pressure below .5 cm.

At first the furnace was submerged in ordinary water, but as direct current was used it was found that under these conditions the bushings were very badly attacked by electrolysis. Consequently, the furnace was placed in a tank filled with distilled water and this was kept cool by circulating water in the outside tank and also by means of cooling coils placed on the top of the cover. With this arrangement the temperature of the distilled water seldom rose above 60° C.

The furnace was exhausted down to a pressure of 2 to 3 cm. by means of a water aspirator. The Geryck pump was then started and this reduced the pressure to .5 cm. or below.

With an input of 15 kw. the temperature in the crucible was raised to 1,600° C. in half an hour and could be maintained at that temperature with 12 kw. No instrument was used for measuring the temperature, as the melting of the iron could be observed through the window in the top of the dome.

The life of the helix heating element has been about 50 melts, each melt lasting approximately one hour with a temperature of 1,500° to 1,600° C. in the crucible. The temperature of the helix itself was, of course, much higher, possibly 2,000° C.

It was found that the helix gradually sagged and after a certain length of time the lower turns came close enough together to allow an arc to form. This arc increased the current so as to raise the drop between the next two turns sufficiently to cause an arc, and so on. In this way a dead short circuit took place. As after this length of time the helix had become partly disintegrated, any attempt to increase the distance between the turns resulted in a broken helix, and a new one had to be substituted. For this reason a different shaped heating element might be preferable, although the helix can be made quite cheaply for this size furnace. For larger sizes the helix form would probably be out of the question.

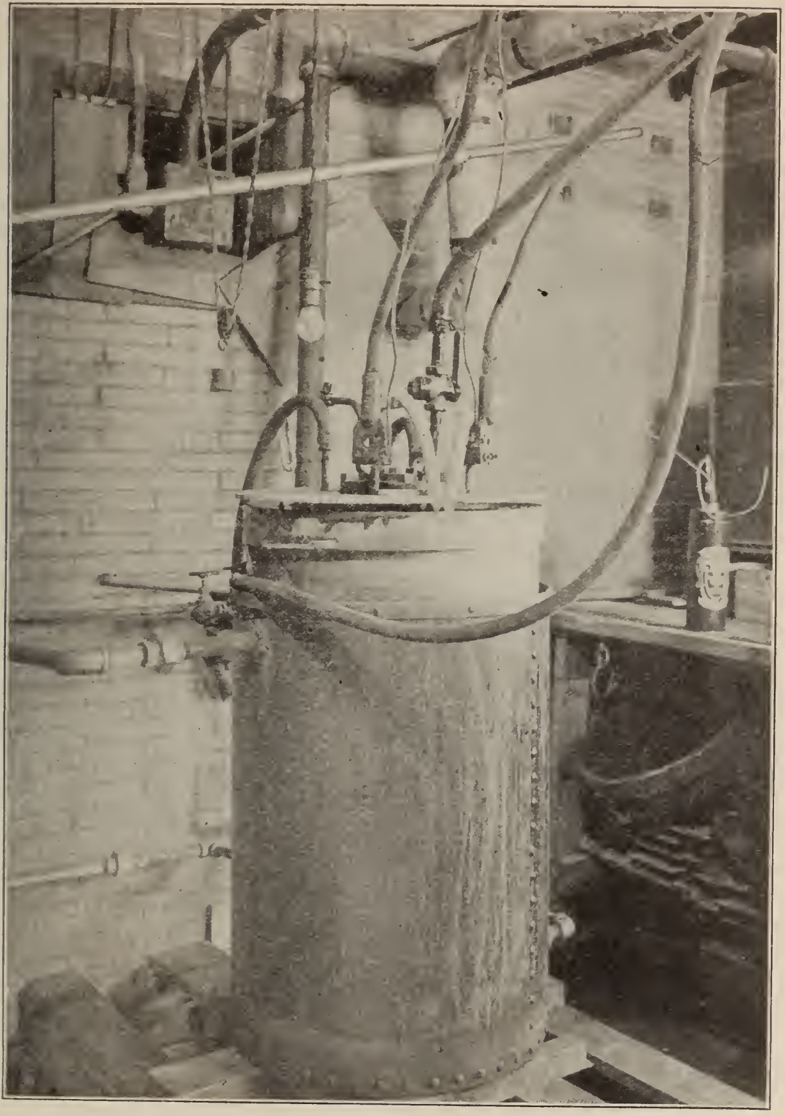

Fig. 60. Vacuum Furnace.

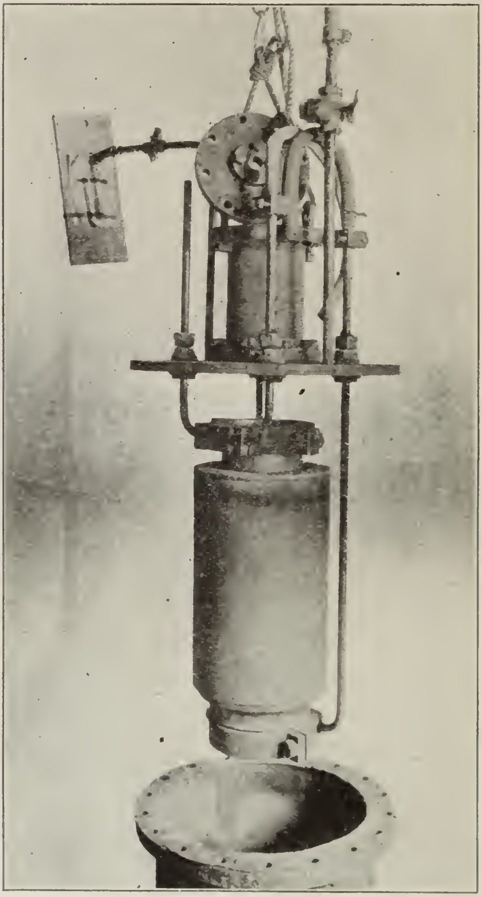

FIG. 61. CONSTRUCTION OF VACUUM FURNACE.

## APPENDIX III.

### REHEATING FURNACE.

For the annealing and quenching of the iron rods a resistance type furnace was used, modeled after the Hoskins carbon plate furnace. A photograph of the furnace is shown in Fig. 62. The lining of the furnace consists of one layer of magnesia brick next to the heating chamber and another layer of firebrick next to the iron shell. The heating element consists of carbon plates, ¼″ (.63 cm.) × 1⅝″ (4.13 cm.) × 8″ (20 cm.) arranged along the sides of the heating chamber. The two rows of plates are connected in parallel by means of 2″ (5 cm.) carbon plates, and 3″ (7.6 cm.) carbon electrodes are screwed into these end plates. The connections to the transformer consist of water cooled cast iron clamps and heavy copper cables. The pressure on the carbon plates can be varied by means of screws at each end. The furnace requires for its operation about 1,500 amperes at 25 volts. With this input the temperature will rise to 1,000° C. in 1½ hours. If the current is cut off at this temperature, the furnace will cool along a logarithmic temperature-time curve to 200° C. in about seven hours, depending somewhat upon the previous rate of heating. In most cases, however, slower cooling along definite temperature-time curves was necessary, and for this purpose the energy input was regulated either by varying the primary voltage of the transformer, or by varying the pressure on the carbon plates, or by varying both.

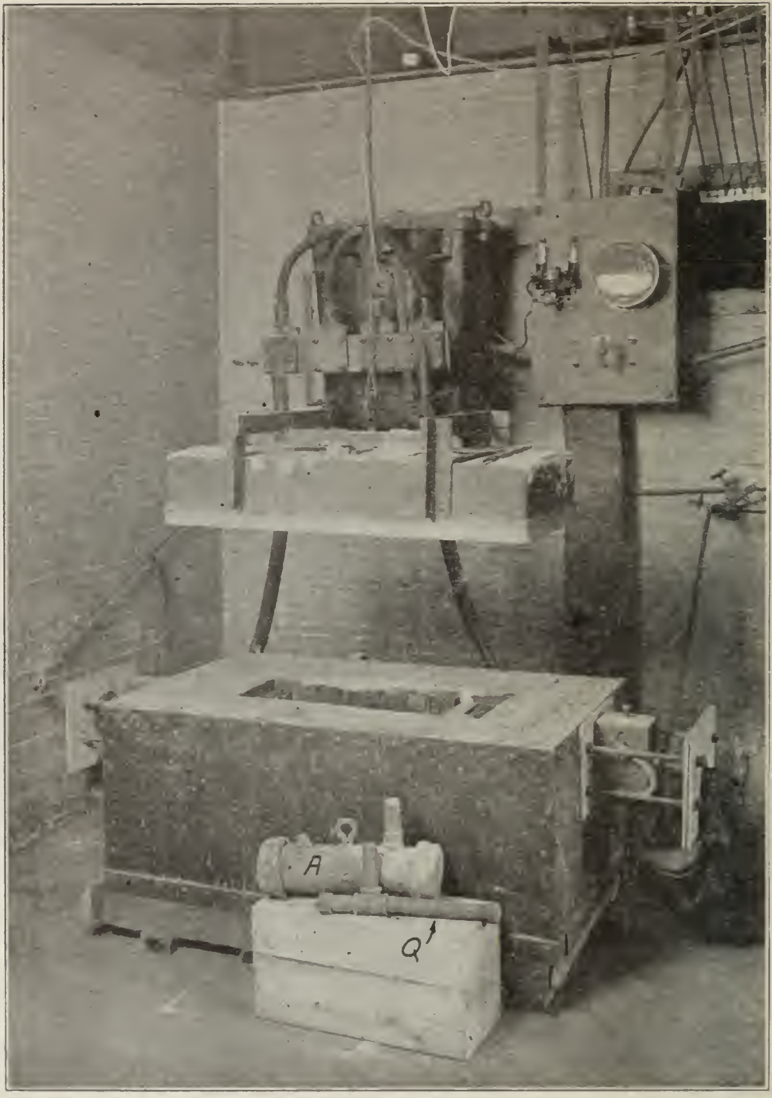

FIG. 62. ANNEALING FURNACE.

## APPENDIX IV.

### PERMEAMETER.

As the apparatus for measuring the magnetic properties is the one upon which the numerical accuracy of the results depend, this apparatus will be described in detail and the probable error in each measurement calculated. The method used is known as the compensated double bar and yoke method and is based upon the results obtained by the Bureau of Standards and described by Charles W. Burrows in Reprint No. 117. The apparatus as constructed by the author is shown in Fig. 63 drawn to scale. Two rods are used, of approximately the same magnetic and electrical quality, one of which is the rod to be measured and the other to be referred to as the auxiliary rod. The two rods are clamped together by means of split yokes made from Swedish charcoal iron, thoroughly annealed. Thus a complete magnetic circuit is established. The magnetizing coils T and A are wound on red fibre tubing and consist of No. 18 B. & S. double cotton covered copper wire, wound 20 turns per inch (7.875 turns per cm.) in 10 layers. The first layer was wound on a screw thread cut in the tube so as to make exactly 20 turns per inch, the next layer wound in the grooves of the first layer, and so on for all 10 layers. After completion the separate layers were connected in series. Four compensating coils, C, serve to overcome the demagnetizing effect of the yokes and the joints, and these coils were wound in the same manner as the two long coils. The secondary coils, *t*, *a* and *c* are also wound on fibre tubes, and each consists of 126 turns of No. 30 B. & S. double silk covered copper wire; *t* is the secondary of the main magnetizing coil T, *a* that of the auxiliary coil A, and *c* is divided into two halves, one at each end of T, directly under the compensating coils. After being wound these secondary coils were boiled in paraffin so as to insure perfect insulation.

For calibration purposes a mutual inductance M was constructed as shown in Fig. 67. It is wound on a cylinder of seasoned wood. The primary consists of No. 18 B. & S. double cotton covered copper wire and in order to insure uniformity the wire was laid in a screwthread of 20 threads per inch (7.875 threads per cm.). The secondary, *m*, consists of 1,260 turns of No. 30 double silk covered copper wire wound in 10 layers of 126 turns each, the layers being separated by layers of paraffin poured on in a molten state and turned down smooth.

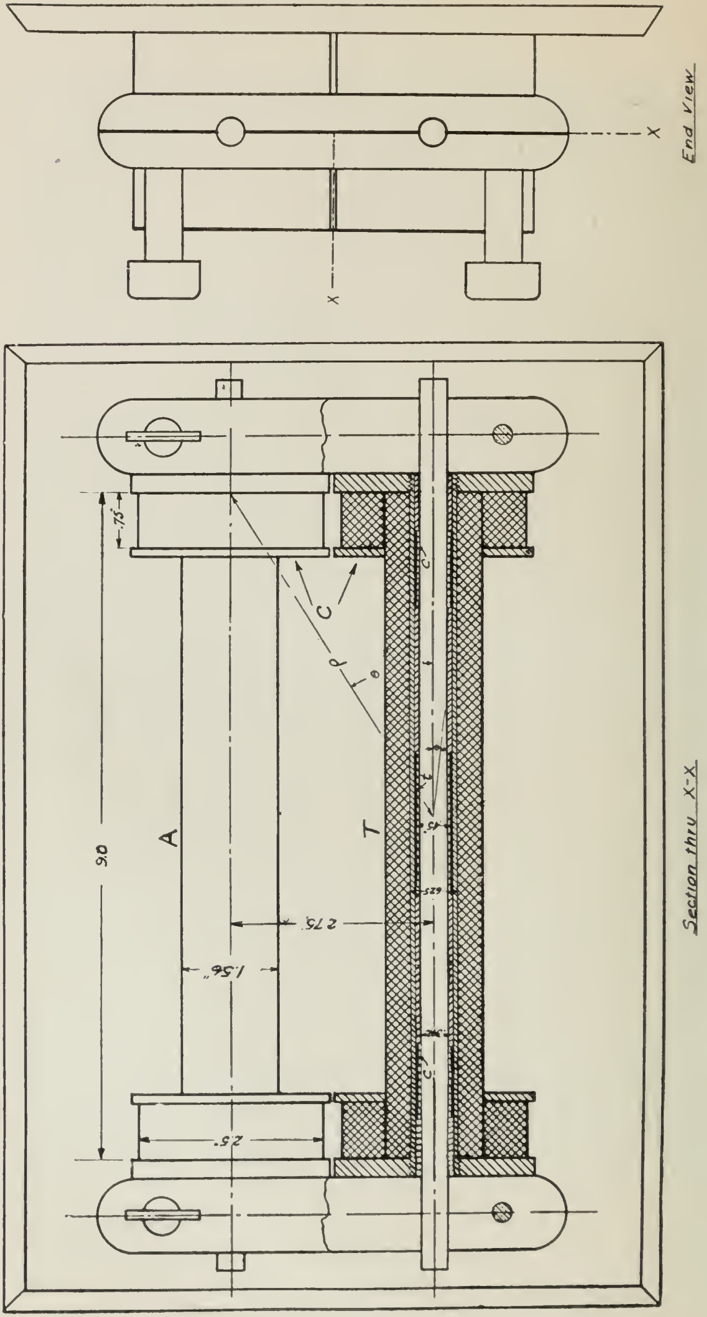

FIG. 63. PERMEAMETER.

The following notation will be used.

$I_T$, $H_T$ = magnetizing current and magnetizing force for coil T.
$I_A$, $H_A$ = magnetizing current and magnetizing force for coil A.
$I_C$, $H_C$ = magnetizing current and magnetizing force for coil C.
$I_M$, $H_M$ = magnetizing current and magnetizing force for coil M.
$n_T$ = number of turns per cm, of coil T.
$n_M$ = number of turns per cm. of coil M.
$n_t$ = number of turns of coil *t*.
$n_m$ = number of turns of coil *m*.
$\Phi_t$ = total flux threading coil *t*.
$\Phi_m$ = total flux threading coil *m*.

The principle upon which this method depends is that the true value of H for a certain section of the magnetic path can be obtained from the value of the magnetizing current in the coil for that section, provided, first, that this coil is wound with absolute uniformity, second, that the flux is absolutely uniform throughout that section, and third, that the end effects of the magnetizing coils are small. In that case

$$H = .4\pi \, n \, I.$$

The uniformity of the flux can be investigated by means of the three secondary coils, *t, a,* and *c,* see Fig. 64. With a fixed magnetizing current in T and the galvanometer switch connected at *ta,* the current in A can be adjusted until the galvanometer shows no deflection upon reversal of the currents. Since *t* and *a* are connected in opposition and contain the same number of turns, they must be threaded by the same flux. The sensitivity of the galvanometer, for this purpose, may be made a maximum by short-circuiting the resistence $\omega$. In the same way the flux may be adjusted to equality at *t* and *c* by means of the current in the compensating coils C.

With the flux the same at *t, a,* and *c,* it is justifiable to assume that, for all practical purposes, it is the same throughout the magnetic path and in particular that it is the same throughout the rod to be measured. Then, with a magnetizing current of $I_T$ amperes in T, the magnetizing force is

$$H_T = .4\pi n_T \, I_T \text{ gilberts per cm.}$$

provided the end effects of the coils may be neglected. These end effects will now be investigated with reference to the particular apparatus used.

The correction due to one end of the solenoid T, to be applied at the middle of the same, is*

$$\frac{1}{2} k_T = -\frac{1}{2} (1 - \cos \alpha) H_o \qquad (1)$$

---

*Burrows: Bull. Bur. of Stand., Vol. 6, No. 1, reprint 117.

where $H_0$ is the magnetizing force due to an infinite solenoid and $\tan a = \frac{2r}{l}$, $r$ being the mean radius of the coil and $l$ the length of the coil. Expanded (1) becomes

$$\frac{1}{2}\,k_T = -\frac{H_0}{2}\left[\frac{1}{2}\left(\frac{2r}{l}\right)^2 - \frac{3}{8}\left(\frac{2r}{l}\right)^4 + \frac{5}{16}\left(\frac{2r}{l}\right)^6 - \ldots\ldots\right] \quad (2)$$

Similarly the correction due to one end of solenoid A, to be applied along the axis of T at its center, for the same magnetizing force $H_0$, but opposite in direction, is

$$\frac{1}{2}\,k_A = +\frac{H_0}{2}\left[\frac{1}{2}\left(\frac{r}{\rho}\right)^2 P_1 - \frac{3}{8}\left(\frac{r}{\rho}\right)^4 P_3 + \frac{5}{16}\left(\frac{r}{\rho}\right)^6 P_5 \ldots\right] \quad (3)$$

where d = distance between the axis of the two coils, and $P_1$, $P_3$, $P_5$, etc., are the zonal harmonics of $\cos\theta$ (see Fig. 63). The corrections to be applied at the center of T due to the compensating coils may be obtained from (2) and (3) by substituting the appropriate constants.

For the particular case under consideration the following constants apply:

| For solenoids T and A. | For the compensating solenoids C. For the outer end | For the compensating solenoids C. For the inner end |
|---|---|---|
| $r = 1.39$ cm. | $r_c = 2.62$ cm. | $r_c = 2.62$ cm. |
| $l = 22.8$ cm. | $l = 22.8$ cm. | $l_c = 18.6$ cm. |
| $d = 7.0$ cm. | $d = 7.0$ cm. | $d = 7.0$ cm. |
| $\theta = 31^\circ.5$ | $\theta = 31^\circ.5$ | $\theta_c = 37^\circ.0$ |
| $\rho = 13.4$ cm. | $\rho = 13.4$ cm. | $\rho_c = 11.65$ cm. |

Substituting in (2) and (3), the correction due to the four ends of solenoids T and A is

$$2\left(\frac{1}{2}k_T + \frac{1}{2}k_A\right) = -H_0\left\{\left[\frac{1}{2}\left(\frac{1.39}{11.4}\right)^2 - \frac{3}{8}\left(\frac{1.39}{11.4}\right)^4 + \frac{5}{16}\left(\frac{1.39}{11.4}\right)^6 - ..\right]\right.$$
$$\left. - \left[\frac{1}{2}\left(\frac{1.39}{13.4}\right)^2 \cos\theta - \frac{3}{8}\left(\frac{1.39}{13.4}\right)^4\left(\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta\right) + \ldots\right]\right\}$$
$$= -H_0\left[.00365 - .0023\right]$$
$$= -.0027\,H_0$$

Substituting in (2) and (3), the correction due to the eight ends of the compensating coils, C, for the same current as in the main coil, T, is

$$2\left(\frac{1}{2}k_{c_1}+\frac{1}{2}k_{c_2}+\frac{1}{2}k_{c_3}+\frac{1}{2}k_{c_4}\right)=$$

$$+H_o\left\{\left[\frac{1}{2}\left(\frac{2.62}{9.3}\right)^2-\frac{3}{8}\left(\frac{2.62}{9.3}\right)^4+\dots\right]-\left[\frac{1}{2}\left(\frac{2.62}{11.4}\right)^2-\frac{3}{8}\left(\frac{2.62}{11.4}\right)^4+\dots\right]\right.$$

$$-\left[\frac{1}{2}\left(\frac{2.62}{11.65}\right)^2\cos\theta_c-\frac{3}{8}\left(\frac{2.62}{11.65}\right)^4\left(\frac{5}{2}\cos^3\theta_c-\frac{3}{2}\cos\theta_c\right)+\dots\right]$$

$$\left.+\left[\frac{1}{2}\left(\frac{2.62}{13.4}\right)^2\cos\theta-\frac{3}{8}\left(\frac{2.62}{13.4}\right)^4\left(\frac{5}{2}\cos^3\theta-\frac{3}{2}\cos\theta\right)+\dots\right]\right\}$$

$$=+H_o\left[.03754-.02550+.01614-.02014\right]$$

$$=+\ .00804\ H_o$$

Consequently, the total correction due to the ends of all the coils, for the same current in all of them, is

$$k=(-.00270+.00804)\ H_o=+.00534\ H_o.$$

As it has been found by experience that for the highest permeabilities the compensating current may be 5 times as large as the main magnetizing current, the maximum correction to be applied is

$$k_{max}=(-.0027+.0402)\ H_o+.0377\ H_o.$$

From this result it is seen that for ordinary iron the values of H as obtained by the expression $H=.4\pi n_T I_T$ are correct within 1 per cent for all values of H, while for the highest permeability iron the errors in the values thus obtained vary from 4 per cent for low values of H to less than 1 per cent for values of H above 20 gilberts per cm. These corrections have not been applied to the values of H as given in the preceding account. From the data given for the magnetizing coils,

$$\begin{aligned}H&=.4\pi\,n\,I\\&=.4\pi\times78.75\ I\\&=99\ I\end{aligned}$$

Thus H might be measured by means of an ordinary milliammeter by dividing the readings by 10 and deducting 1 per cent from the result. In order to avoid this correction and at the same time to yield greater accuracy for the main solenoid the following scheme was adopted:

The magnetizing force $H_T$ for the rod to be tested was measured by the drop of potential across the shunt shown in Fig. 64. This shunt has three contacts so adjusted that if the first switch is closed the millivoltmeter reads 10 $H_T$, if switch No. 2 is closed it reads $H_T$ directly, and if switch No. 3 is closed it reads ¼ $H_T$. Thus the error in reading H for $H_T=.5$ is ± .005 or ± 1 per cent. The magnetizing force in coils A and

C are measured by milliammeters, as the accuracy of these readings are of less importance. All these meters have been calibrated against a Weston standard and corrections applied.

The flux density in the rod under test was at first measured by means of a Leeds and Northrup ballistic galvanometer whose period was adjusted to 15 seconds. This galvanometer was calibrated before every test by the mutual inductance previously referred to and shown in Fig. 67.

The inside diameter of the primary is 3.9″ (9.9 cm.) and the mean diameter $D_{av.} = 4.0''$ (10.025 cm.). In order to include the total flux inside of the secondary coil, the equivalent value of the diameter at which the magnetizing current may be considered concentrated is found to be

$$D_{eq.} = \sqrt{D^2_{av.} - \left(\frac{d}{2}\right)^2}$$

where $d =$ radius of the primary wire $\doteq .127$ cm. Substituting numerical values, $D_{eq.} = \sqrt{\overline{10.025}^2 - \overline{.064}^2} = 10.025$ —. Thus the error in using $D_{av.}$ as the effective diameter is negligible. The effective area is then

$$A = \pi \frac{D_{av.}{}^2}{4} = 78.6\ \overline{\text{cm.}}^2$$

With a current of $I_M$ flowing in the primary, the total flux threading the secondary for an infinitely long solenoid is

$$\begin{aligned} \Phi'_m &= .4\pi n_M I_M \times A \\ &= 782\ I_M. \end{aligned}$$

The correction to be applied at the center due to the effect of the ends is

$$\begin{aligned} k_M &= -H_o\left[\frac{1}{2}\left(\frac{10}{90}\right)^2 - \frac{3}{8}\left(\frac{10}{90}\right)^4 + \frac{5}{16}\left(\frac{10}{90}\right)^6 - \cdots\right] \\ &= -\ .0061\ H_o. \end{aligned}$$

Thus the flux as corrected is

$$\begin{aligned} \Phi_m &= (1 - .006)\ 782\ I_m \\ &= 777\ I_M \end{aligned}$$

With 1260 turns on the secondary, the total flux turns are

$$\Phi_m n_m = 1260 \times 777\ I_M.$$

The test rods have a diameter of .392″ (.995 cm) and the area is thus .779 $\overline{\text{cm.}}^2$ With an induction of B gausses the total flux in the rod is

$$\Phi_t = B_t \times .779,$$

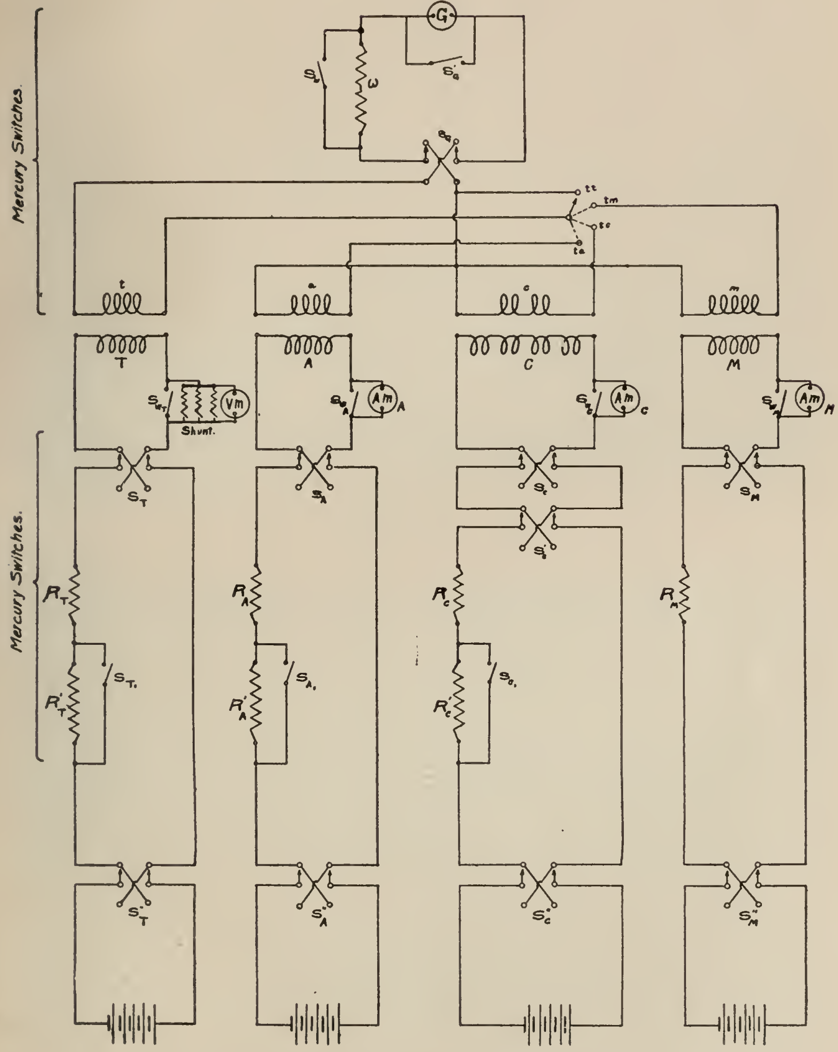

Fig. 64. Electrical Connections.

and as the number of secondary turns are 126, the flux turns are

$$\Phi_t\, n_t = 126 \times .779 \times B_t.$$

If now the secondary coil, *t*, be connected in opposition to the coil, *m*, in series with the galvanometer and the latter shows no deflection upon reversing all the magnetizing currents, including $I_M$, then

$$\Phi_m\, n_m = \Phi_t\, n_t,$$

or

$$1260 \times 777\, I_M = 126 \times .779 \times B_t,$$

$$B_t = .997 \times 10{,}000\, I_M.$$

As it has been assumed that $B_t = 10{,}000\, I_M$, the correction to be applied is — .3 per cent.

On account of the difference in the time required to establish the flux in M and T it was found impracticable to apply this zero method. Instead, the resistance, $\omega$, was so adjusted that the deflection of the galvanometer upon reversal of $I_M$ was

$$D = 20\, I_M.$$

Thus with the coil *t* connected to the galvanometer, the value of $B_t$ corresponding to a deflection D, upon reversal of the magnetizing currents, is

$$B_{t\,\max} = \frac{1{,}000\ D}{2}$$

and the total change of $B_t$ is

$$\triangle B_t = 1{,}000\ D,$$

as $B_t$ passes from $+ B_{t_{\max}}$ to $- B_{t_{\max}}$ upon reversal.

All the different switching operations are accomplished by means of rocking mercury switches, as shown in Figs. 65 and 66. With this arrangement any number of switches may be operated simultaneously. As a matter of fact, the three switches, $S_T$, $S_A$, and $S_C$, and also $S_{T_1}$, $S_{A_1}$, and $S_{C_1}$, are connected together so that they operate in unison.

The magnetization curves are obtained by the method of reversals. Preliminary to each measurement the flux is adjusted to equality around the magnetic path as already described.

Hysteresis loops are traced by a similar method. All values are referred to the maximum. After having made the adjustments for equality for the maximum value to be used, the resistances $R_T$, $R_A$, and $R_C$ are not changed. If $H_2$ be the desired value of $H_T$ on the loop, this is obtained by opening the switches $S_{T_1}$, $S_{A_1}$, and $S_{C_1}$, and adjusting the resistances $R'_T$, $R'_A$, and $R'_C$. The latter are then short-circuited and the flux reversed a few times so as to follow the loop. These operations have taken place with the galvanometer short-circuited. Adjustments for

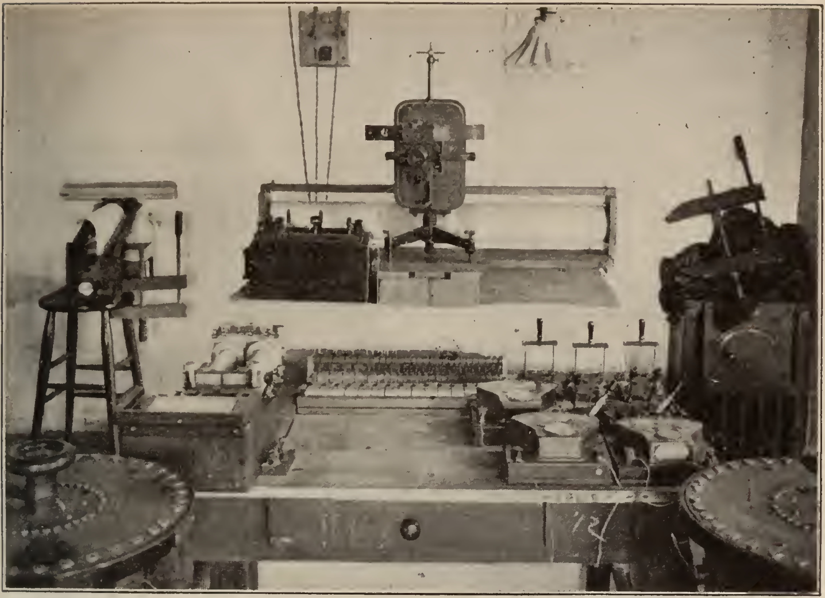

FIG. 65. PERMEAMETER. OPERATING TABLE.

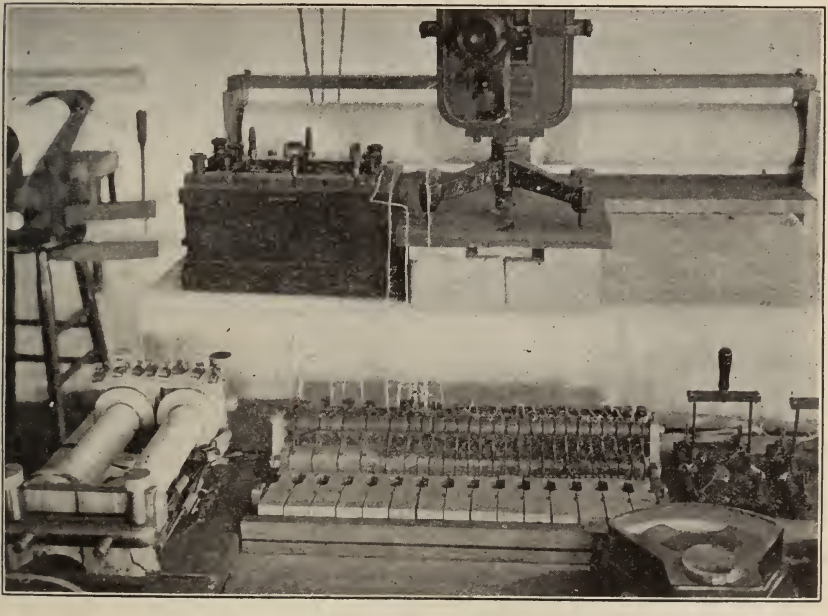

FIG. 66. PERMEAMETER, OPERATING TABLE.

equality of flux for $H_2$ are now made by connecting *a* in opposition to *t* through the galvanometer and noting the deflection upon opening $S_{T1}$, $S_{A1}$, and $S_{C1}$. If there is a deflection, $R'_A$ is adjusted so that upon opening these switches no deflection takes place. A few reversals have to be made after each such trial. In a similar manner $R'_C$ is adjusted until no deflection takes place with *c* connected in opposition to *t*. When equality has been obtained all along the line, the coil *t* is connected directly to the galvanometer and the deflection noted upon opening $S_{T1}$, etc. This operation gives a point between $B_{max}$ and the retentivity point $B_r$, or it may give $B_r$ if $R'_T$ in the above case is infinity, i. e., open-circuited. For values of $B_2$ between $B_r$ and $-B_{max}$ the reversing switches $S_T$, $S_A$ and $S_C$ are operated at the same time as $S'_T$, etc. Otherwise the procedure is exactly the same.

The Leeds and Northrup galvanometer was sufficiently accurate for iron of low permeability, but when attempts were made to use it for vacuum iron the errors introduced on account of the viscosity of the iron were too large. This was particularly true in measuring hysteresis. It was found, for instance, that the time required for a complete change of flux in a particular case was 5 to 6 seconds. As the period of the galvanometer was 15 seconds for a complete swing, or about 4 seconds for ¼ swing, the change of flux had not been completed by the time the galvanometer coil had reached its maximum deflection. As the accuracy of a ballistic galvanometer depends upon the completion of the impulse before the coil has moved appreciably, it was very evident that the ballistic galvanometer had reached its limit of usefulness, even though its period had been doubled.

For all the final measurements of the vacuum iron a Grassot fluxmeter was therefore used. In this meter the suspension effect has been practically eliminated, so that the same result is obtained whether the deflection is made from maximum towards zero or vice versa. The needle does creep towards a zero point, but the travel is very slow and hardly noticeable. Consequently with this instrument it makes no difference how quickly or how slowly the change of flux takes place. It can be shown that in any case

$$\Delta\Phi = K\Theta$$

where K is a constant and $\Theta$ is the deflection. The time element does not enter. The meter, besides being provided with a needle, is also equipped with a mirror, so that measurements may be taken in connection with a lamp and scale. By this means the sensitivity may be increased 20 times. For the present purpose the lamp and scale were used

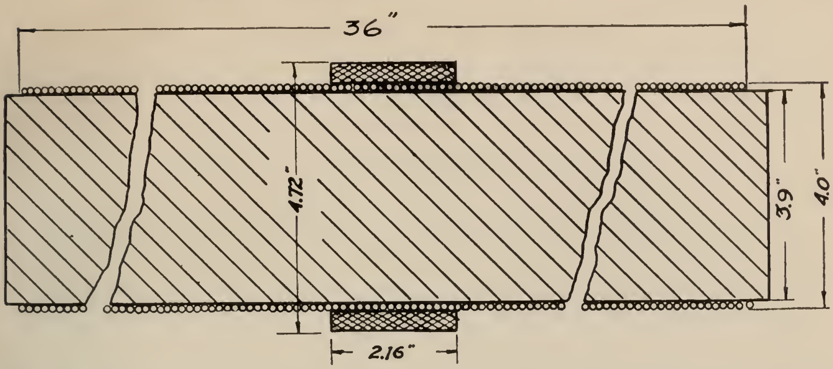

FIG. 67. MUTUAL INDUCTANCE FOR CALIBRATION OF GALVANOMETER OR FLUXMETER.

for the preliminary adjustments for equality of flux, while the final measurement was obtained from the deflection of the needle. The meter was calibrated to read in 10,000 maxwells per scale division per turn of the search coil. As the secondary coil, *t*, contains 126 turns and the cross-section of the rod is .779 sq. cm. the deflections of the fluxmeter were multiplied by

$$\frac{10{,}000}{126 \times .779} = 102$$

in order to give $\Delta B_t$.

By comparing the results obtained with the fluxmeter with those obtained with the galvanometer, it was found that the results are identical for permeabilities up to 4,000 or 5,000. For permeabilities of 10,000 the results obtained by the fluxmeter are 10 to 15 per cent higher, while for values of $\frac{\Delta B}{\Delta H}$ of 20,000 or above, such as may occur on the steepest part of the hysteresis loop, the fluxmeter gives results 50 to 100 per cent higher than the galvanometer. The values of the standard rod as obtained by the two meters are shown in Fig. 2.

The preliminary adjustments for equality of flux are made very readily and accurately with the fluxmeter. If the two rods are not just alike the change of flux may occur at a different rate in one than in the other with the result that, if the initial and final flux values are nearly the same, the needle will deflect first in one direction and then in the other. While these two kicks caused considerable trouble with the galvanometer, no uncertainty exists with the fluxmeter. If the spot of light on the scale returns to the same spot from which it started it makes no difference what happens to it in the meantime. In that case it is definitely known that the fluxes threading the two opposing coils are equal.

# APPENDIX V.

## BIBLIOGRAPHY.

This bibliography is not complete. However, it covers the field of researches upon the magnetic properties of ferromagnetic substances during the last half century quite thoroughly, so that anyone who wishes to make a study of this subject may find in these references a fairly continuous history of the progress in the field. The papers in each group are arranged chronologically.

### MAGNETIC AND ALLIED PROPERTIES IN GENERAL.

Ref.
Number.
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# ACOUSTICS OF AUDITORIUMS

BY

F. R. WATSON

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## BULLETIN No. 73

## ENGINEERING EXPERIMENT STATION

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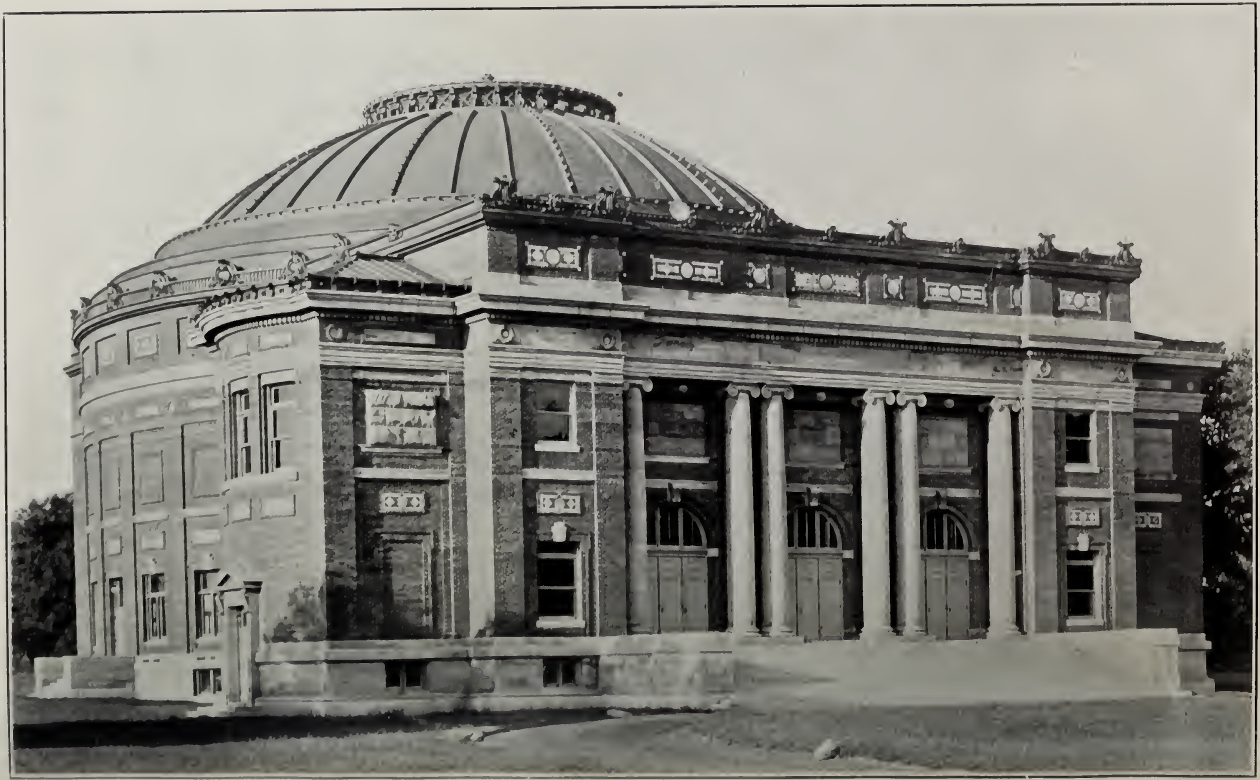

Fig. 1. Auditorium, University of Illinois.

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 73 MARCH, 1914

# ACOUSTICS OF AUDITORIUMS.

BY F. R. WATSON, ASSISTANT PROFESSOR OF PHYSICS.

## CONTENTS.

Page
I. Introduction ........ 3
Acknowledgment ........ 5
II. Behavior of Sound Waves in a Room........ 5
III. Methods of Improving Faulty Acoustics........ 8
  A. Reverberation and Its Cure........ 8
    Experimental Work on Cure of Reverberation........ 9
    Formulae for Reverberation of Sound in a Room...... 10
  B. Echoes and Their Remedy........ 11
  C. Popular Conception of Cures. Use of Wires and Sounding Boards ........ 11
    Sounding Boards ........ 12
    Modeling New Auditorium after Old Ones with Good Acoustics ........ 12
  D. The Effect of the Ventilation System on the Acoustics... 13
IV. The Investigation in the Auditorium at the University of Illinois ........ 13
  A. Preliminary Work ........ 13
  B. Details of the Acoustical Survey in the Auditorium..... 17
  C. Conclusion Drawn from the Acoustical Survey........ 26
  D. Methods Employed to Improve the Acoustics........ 26
    Reflecting Boards ........ 26
    Sabine's Method ........ 26
    Method of Eliminating Echoes........ 29
    Proposed Final Cure........ 29
V. Bibliography of Publications on Acoustics of Auditoriums... 31

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# ACOUSTICS OF AUDITORIUMS

## An Investigation of the Acoustical Properties of the Auditorium at the University of Illinois.

### I. Introduction.

Much concern has arisen in late years in the minds of architects because of the faulty acoustics that exist in many auditoriums. The prevalence of echoes and reverberations with the consequent difficulty in hearing and understanding on the part of the auditor defeats the purpose of the auditorium and diminishes its value.

The Auditorium at the University of Illinois presents such a case. The building is shaped nearly like a hemisphere, with several large arches and recesses to break up the regularity of its inner surface. The original plans of the architect were curtailed because of insufficient money appropriated for the construction. The interior of the hall, therefore, was built absolutely plain with almost no breaking up of the large, smooth wall surfaces; and, at first, there were no furnishings except the seats and the cocoa matting in the aisles. The acoustical properties proved to be very unsatisfactory. A reverberation or undue prolongation of the sound existed, and in addition, because of the large size of the room and the form and position of the walls, echoes were set up.

If an observer stood on the platform and clapped his hands, a veritable chaos of sound resulted. Echoes were heard from every direction and reverberations continued for a number of seconds before all was still again. Speakers found their utterances thrown back at them, and auditors all over the house experienced difficulty in understanding what was said. On one occasion the University band played a piece which featured a xylophone solo with accompaniment by the other instruments. It so happened that the leader heard the echo more strongly than the direct sound and beat time with it. Players near the xylophone kept time to the direct sound, while those farther away followed the echo. The confusion may well be imagined.

Thus it seemed that the Auditorium was doomed to be an acoustical horror; that speakers and singers would avoid it, and that auditors would attend entertainments in it only under protest. But the apparent misfortune was in one way a benefit since it provided an opportunity to

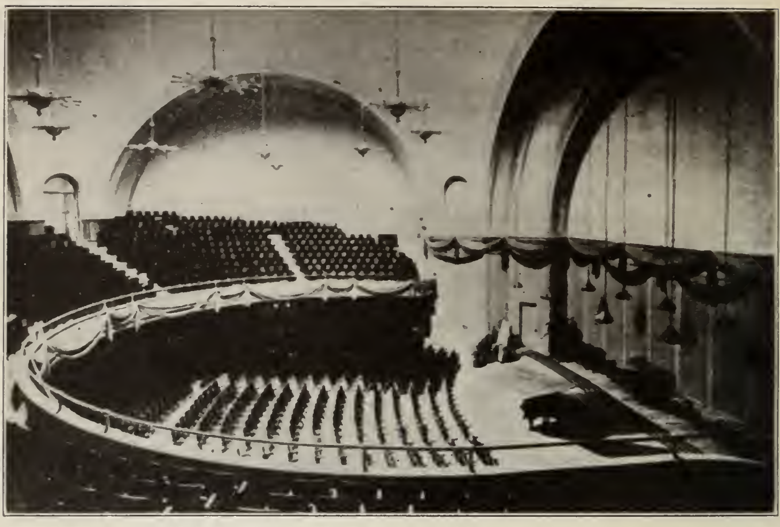

FIG. 2. PHOTOGRAPH OF INTERIOR. VIEW OF STAGE.

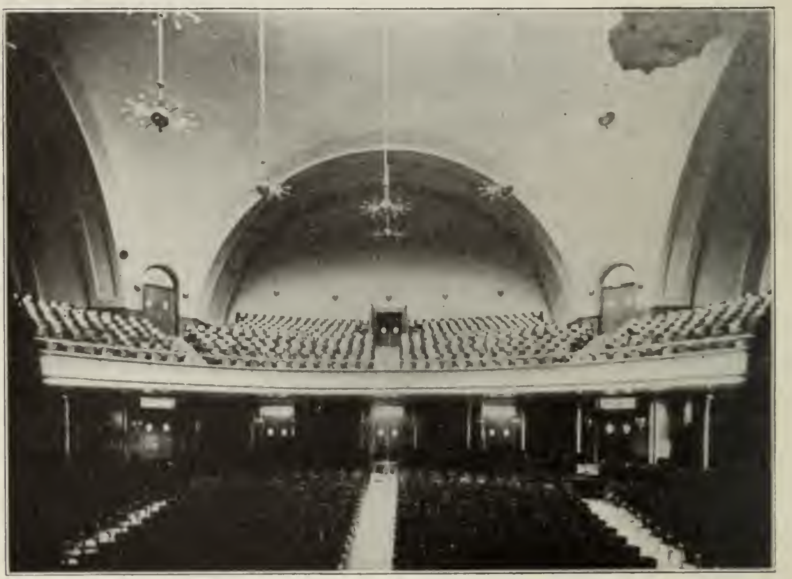

FIG. 3. PHOTOGRAPH OF INTERIOR. VIEW TOWARD BALCONY.

study defective acoustics under exceptionally good conditions and led to conclusions that not only allowed the Auditorium to be improved but also indicate some of the pitfalls to be avoided in future construction of other halls.

An investigation of the acoustical properties of the Auditorium was begun in 1908 and has continued for six years. It was decided at the outset not to use "cut and try" methods of cure, but to attack the problem systematically so that general principles could be found, if possible, that would apply not only to the case being investigated but to auditoriums in general. This plan of procedure delayed the solution of the problem, since it became necessary to study the theory of sound and carry out laboratory investigations at the same time that the complex conditions in the Auditorium were being considered. The author spent one year of the six abroad studying the theory of acoustics and inspecting various auditoriums.

The main echoes in the Auditorium were located by means of a new method for tracing the path of sound, the time of reverberation was determined by Sabine's method, and a general diagnosis of the acoustical defects was made. Hangings and curtains were installed in accordance with the results of the study so that finally the acoustical properties were improved.

*Acknowledgment.*—The author desires to express his great appreciation of the advice and encouragement given by President E. J. James, Supervising Architect J. M. White, and Professor A. P. Carman of the Physics Department. He desires also to acknowledge the material assistance cheerfully rendered by the workmen at the University, which contributed in no small degree to the successful solution of the problem.

## II. BEHAVIOR OF SOUND WAVES IN A ROOM.

When a speaker addresses an audience, the sounds he utters proceed in ever widening spherical waves until they strike the boundaries of the room. Here the sound is partly reflected, partly transmitted, and the rest absorbed. The amounts of reflection, absorption and transmission depend on the character of the walls. A hard, smooth wall reflects most of the sound so that but little is transmitted or absorbed. In the case of a porous wall or a yielding wall, the absorption and transmission are greater, and the reflection is less. After striking a number of reflecting surfaces, the energy is used up and the sound dies out.

The reflection of sound produces certain advantages and disadvantages for the acoustics. When it is considered that sound travels about 1100 feet a second it may be seen that a room of ordinary size is almost immediately filled with sound because of the many reflections. In a room 40 feet square, for instance, the number of reflections per second between opposite walls is 1100 ÷ 40, or approximately 27. The number is really greater than this, since the sound that goes into the corners is reflected much more frequently than out in the middle where the distances between walls are greater. The result is that the sound mixes thoroughly in all parts of the room so as to give the same average intensity; that is, the sound is of the same average *loudness* for all auditors, even for those in the remotest corners.

Though the reflection of sound has the advantage of fulfilling the conditions for loudness, it introduces at the same time possibilities for setting up defective acoustics. For instance, when the walls of the room are hard and smooth very little energy is lost at each impact of the sound and many reflections take place before it finally dies out. This slow decadence of the sound, or *reverberation* as it is called, is the most common defect in auditoriums.

If a speaker talks in such a hall the auditors have difficulty in understanding. Each sound, instead of dying out quickly, persists for some time so that the succeeding words blend with their predecessors and set up a mixture of sounds which produces confusion. The cure for the trouble is brought about by the introduction of materials such as carpets, tapestries, and the like, which act as absorbers of sound and reduce the time of reverberation.

When music is played in an auditorium with a prolonged reverberation, the tones following one another blend and produce the same effect as that of a piano when played with the loud pedal in use. A reverberation is more advantageous for music than for speech, since the prolongation and blending of the musical tones is desired, but the mixing of the words in a speech is a distinct disadvantage. When curing this defect for halls used for both music and speaking, a middle course must be steered, so that the reverberation is made somewhat long for speaking and somewhat short for music, yet fairly satisfactory for both.

Going back to the consideration of the reflection of sound, it is found that another defect may be produced, namely, an *echo*. This is the case when a wall at some distance reflects the sound to the position of the auditor. He hears the sound first from the speaker, then later by reflection from the wall. The time interval between the direct and reflected sound must be great enough to allow two distinct impressions to be made. This time is about 1/15 of a second, but varies with the acuteness of the observer. The farther off the wall is, the greater is the time interval and the more pronounced is the echo. If the wall is not very distant, the time interval is too short to allow two distinct impressions to be made, and the effect on the auditor is then much the same as if his neighbor at his side speaks the words of the discourse in his ear at the same time that he gets them directly from the speaker. In case the reflecting wall is curved so as to focus the sound the echoes are much more pronounced. A curved wall wherever it may be placed in an auditorum is thus always a menace to good acoustics.

There are other actions of the sound that may result in acoustical defects. The phenomena of *resonance,* for instance, may cause trouble. Suppose that the waves of sound impinge on an elastic wall, not too rigid. If these waves are timed right they set the wall in vibration in the same way that the bell ringer causes a bell to ring by a succession of properly timed pulls on the bell rope. The wall of the room will then vibrate under the action of this sound with which it is in tune and will reinforce it. Now suppose a band is playing in a room. Certain tones are reinforced, while the others are not affected. The original sound is then distorted. The action is the same on the voice of the speaker. The sounds he utters are complex and as they reach the walls certain components are reinforced and the quality of the sound is changed. This action of resonance may also be caused by the air in a room. Each room has a definite pitch to which it responds, the smaller the volume of the room the higher being the pitch. A large auditorium would respond to the very low pitch of the bass drum. In small rooms and alcoves the response is made to higher pitched tones, as may be observed by singing the different notes of the scale until a resonance is obtained.

Another action of sound causes the *interference* of waves. Thus the reflected waves may meet the oncoming ones and set up concentrations of sound in certain positions and a dearth of sound in others.

Summing up, it is seen that the effects of sound which may exist in a room are *loudness, reverberation, echoes, resonance,* and *interference,* and that the most common defects are reverberation and echoes. We now turn to the discussion of the methods of cure.

## III. Methods of Improving Faulty Acoustics.

### A. REVERBERATION AND ITS CURE.

Everyone has doubtless observed that the hollow reverberations in an empty house disappear when the house is furnished. So, in an auditorium, the reverberation is lessened when curtains, tapestries, and the like are installed in sufficient numbers. The reason for this action is found when we inquire what ultimately becomes of the sound.

Sound is a form of energy and energy can not be destroyed. When it finally dies out, the sound must be changed to some other form of energy. In the case of the walls of a room, for instance, it has been shown in a preceding paragraph that the sound may be changed into mechanical energy in setting these walls in vibration. Again, some of the sound may pass out through open windows and thus disappear. The rest of the sound, according to Lord Rayleigh, is transformed by friction into heat. Thus[1] a high pitched sound, such as a hiss, before it travels any great distance is killed out by the friction of the air. Lower pitched sounds, on reaching a wall, set up a friction in the process of reflection between the air particles and the wall so that some of the energy is converted into heat.[2] The amount of sound energy thus lost is small if the walls are hard and smooth. The case is much different, however, if the walls are rough and porous, since it appears that the friction in the pores dissipates the sound energy into heat. In this connection, Lamb[3] writes: "In a sufficiently narrow tube the waves are rapidly stifled, the mechanical energy lost being of course converted into heat. * * * * When a sound wave impinges on a slab which is permeated by a large number of very minute channels, part of the energy is lost, so far as the sound is concerned, by dissipation within these channels in the way just explained. The interstices in hangings and carpets act in a similar manner, and it is to this cause that the effect of such appliances in deadening echoes in a room is to be ascribed, a certain proportion of the energy being lost at each reflection. It is to be observed that it is only through the action of true dissipative forces, such as viscosity and thermal conduction, that sound can die out in an enclosed space, no mere modifications of the waves by irregularities being of any avail."

It should be pointed out in this connection that any mechanical breaking up of the sound by relief work on the walls or by obstacles in the room will not primarily diminish the energy of the sound. These may break up the regular reflection and eliminate echoes, but the sound energy as such disappears only when friction is set up.

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[1] "Theory of Sound," Vol. II, p. 316.
[2] "Theory of Sound," Vol. II, § 351.
[3] "Dynamical Theory of Sound," p. 196.

The following quotation from Rayleigh[1] emphasizes these conclusions: "In large spaces, bounded by non-porous walls, roof, and floor, and with few windows, a prolonged resonance seems inevitable. The mitigating influence of thick carpets in such cases is well known. The application of similar material to the walls and roof appears to offer the best chance of further improvement."

*Experimental Work on Cure of Reverberation.*—The most important experimental work in applying this principle of the absorbing power of carpets, curtains, etc., has been done by Professor Wallace C. Sabine of Harvard University.[2] In a set of interesting experiments lasting over a period of four years, he was able to deduce a general relation between $t$, the time of reverberation, $V$, the volume of the room, and $a$, the absorbing power of the different materials present. Thus:

$$t = 0.164\, V \div a \tag{1}$$

For good acoustical conditions, that is, for a short time of reverberation, the volume $V$ should be small and the absorbing materials, represented by $a$, large. This is the case in a small room with plenty of curtains and rugs and furniture. If, however, the volume of the room is great, as in the case of an auditorium, and the amount of absorbing materials small, a troublesome reverberation will result.

Professor Sabine determined the absorbing powers of a number of different materials. Calling an open window a perfect absorber of sound, the results obtained may be written approximately as follows:

| | |
|---|---|
| One square meter of open window space | 1.000 |
| One square meter of glass, plaster, or brick | .025 |
| One square meter of heavy rugs, curtains, etc. | .25 |
| One square meter of hair felt, 1 inch thick | .75 |
| One square meter of audience | .96 |

These values, together with the formula, allow a calculation to be made in advance of construction for the time of reverberation. This pioneer work cleared the subject of architectural acoustics from the fog of mystery that hung over it and allowed the essential principles to be seen in the light of scientific experiment.

In a later investigation[3] Sabine showed that the reverberation depended also on the pitch of sound. As a concrete example, the high notes of a violin might be less reverberant with a large audience than the lower tones of the bass viol, although both might have the same reverberation in the room with no audience. Again, the voice of a man with notes of low pitch might give satisfactory results in an auditorium while the voice of a woman with higher pitched notes would be unsatisfactory.

---

[1] "Theory of Sound," p. 333.

[2] "Architectural Acoustics." A series of articles in the Engineering Record, 1900; also the American Architect, 1900.

[3] "Architectural Acoustics," Proc. of Amer. Acad. of Arts and Sciences. Vol. 42, pp. 49-84, 1906.

These considerations show that the acoustics in an auditorium vary with other factors than the volume of the room and the amount of absorbing material present. The audience may be large or small, the speaker's voice high or low, the entertainment a musical number or an address. The best arrangement for good acoustics is then a compromise where the average conditions are satisfied. The solution offered by Professor Sabine is such an average one, and has proved satisfactory in practice.

The problem of architectural acoustics has been attacked experimentally by other workers. Stewart[1] proposed a cure for the poor acoustical conditions in the Sibley Auditorium at Cornell University. His experiments confirmed the work of Sabine. Marage[2], after investigating the properties of six halls in Paris, approved Sabine's results and advocated a time of reverberation of from ½ to 1 second for the case of speech.

*Formulae for Reverberation of Sound in a Room.*—On the theoretical side, Sabine's formula has been developed by Franklin,[3] who obtained the relation $t = 0.1625\, V \div a$, an interesting confirmation, since Sabine's experimental value for the constant was 0.164.

A later development has been given by Jäger,[4] who assumes for a room whose dimensions are not greater than about 60 feet, that the sound, after filling the room, passes equally in all directions through any point, and that the average energy is the same in different parts of the room. By using the theory of probability and considering that a beam of sound in any direction may be likened to a particle with a definite velocity, he was able to deduce Sabine's formula and write down the factors that enter into the constants. Applying his results to the case of reflection of sound from a wall, he showed that sound would be reflected in greater volume when the mass of the wall was increased and the pitch of the sound made higher. He showed also that when sound impinges on a porous wall, more energy is absorbed when the pitch of the sound is high than when it is low, since the vibrations of the air are more frequent, and more friction is introduced in the interstices of the material.

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[1] G. W. Stewart. "Architectural Acoustics," Sibley Journal of Engineering, May, 1903. Published by Cornell University, Ithaca, N. Y.

[2] "Qualités acoustiques de certaines salles pour la voix parlée." Comptes Rendus, 142, 878, 1906.

[3] W. S. Franklin. "Derivation of Equation of Decaying Sound in a Room and Definition of Open Window Equivalent of Absorbing Power." Physical Review, Vol. 16, pp. 372-374, 1903.

[4] G. Jäger. "Zur Theorie des Nachhalls." Sitzungsberichte der Kaiserliche Akad. der Wissenschaften in Wien, Math-naturw. Klasse; Bd. CXX. Abt. 11 a. Mai, 1911.

### B. ECHOES AND THEIR REMEDY.

An echo is set up by a reflecting wall. If an observer stands some distance from the front of a cliff and claps his hands, or shouts, he finds that the sound is returned to him from the cliff as an echo. So, in an auditorium, an auditor near the speaker gets the sound first directly from the speaker, then, an instant later, a strong repetition of the sound by reflection from a distant wall. This echo is more pronounced if the wall is curved and the auditor is at the point where the sound is focused.

To cure such an echo, two methods may be considered. One method consists in changing the form of the wall so that the reflected sound no longer sets up the echo. That is, either change the angle of the wall, so that the reflected sound is sent in a new direction where it may be absorbed or where it may reinforce the direct sound without producing any echoes, or else modify the surface of the wall by relief work or by panels of absorbing material, so that the strong reflected wave is broken up and the sound is scattered. The second method is to make the reflecting wall a "perfect" absorber, so that the incident sound is swallowed up and little or none reflected. These methods have been designated as "surgical" and "medicinal" respectively. Each method has its disadvantages. Changing the form of the walls in an auditorium is likely to do violence to the architectural design. On the other hand, there are no perfect absorbers, except open windows, and these can seldom be applied. The cure in each case is, then, a matter of study of the special conditions of the auditorium. Usually a combination of the surgical and the medicinal cures is adopted. For instance, coffering a wall so that panels of absorbing material may be introduced has been found to work well in bettering the acoustics, and also, in may cases, it fits in with the architectural features.

### C. POPULAR CONCEPTION OF CURES.—USE OF WIRES AND SOUNDING BOARDS.

A few words should be written concerning the popular notion that wires and sounding boards are effective in curing faulty acoustics. Experiments and observations show that wires are of practically no benefit, and sounding boards can be used only in special cases. Wires stretched in a room scarcely affect the sound, since they present too small a surface to disturb the waves. They have much the same effect on sound waves that a fish line in the water has on water waves. The idea has, perhaps, grown into prominence because of the action of a piano in responding to the notes of a singer. The piano has every advantage over a wire in an auditorium. It has a large number of strings tuned to different pitches so that it responds to any note sung. It also has a sounding board that reinforces strongly the sound of the strings. Finally, the singer is usually near the piano. The wire in the auditorium responds to only one tone of the many likely to be present, it has no sounding board, and the singer is some distance away. But little effect, therefore, is to be expected.

The author has visited a number of halls where wires have been installed, and has yet to find a case where pronounced improvement has resulted.[1] Sabine[2] cites a case where five miles of wire were stretched in a hall without helping the acoustical conditions. It is curious that so erroneous a conception has grown up in the public mind with so little experimental basis to support it.

*Sounding Boards.*—Sounding boards or, more properly, reflecting boards, have value in special cases. Some experiments are described later where pronounced effects were obtained. The sounding board should be of special design to fit the conditions under which it is to be used.

*Modeling New Auditoriums after Old Ones with Good Acoustics.*—Another suggestion often made is for achitects to model auditoriums after those already built that have good acoustical properties. It does not follow that halls so modeled will be successful, since the materials used in construction are not the same year after year. For instance, a few years ago it was the usual custom to put lime plaster on wooden lath; now it is frequently the practice to put gypsum plaster on metal lath, which forms an entirely different kind of a surface. This latter arrangement makes hard, non-porous walls which absorb but little sound, and thus aggravate the reverberation. Further, a new hall usually is changed somewhat in form from the old one, to suit the ideas of the architect, and it is very likely that the changes will affect the acoustics.

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[1] Science Vol. 35, p. 833, 1912.

[2] Arch. Quarterly of Harvard University, March, 1912.

### D. THE EFFECT OF THE VENTILATION SYSTEM ON THE ACOUSTICS.

At first thought it might seem that the ventilation system in a room would affect the acoustical properties. The air is the medium that transmits the sound. It has been shown that the wind has an action in changing the direction of propagation of sound.[1] Sound is also reflected and refracted at the boundary of gases that differ in density and temperature.[2] It is found, however, that the effect of the usual ventilation currents on the acoustics in an auditorium is small. The temperature difference between the heated current and the air in the room is not great enough to affect the sound appreciably, and the motion of the current is too slow and over too short a distance to change the action of the sound to any marked extent.[3]

Under special circumstances, the heating and ventilating systems may prove disadvantageous.[4] A hot stove or a current of hot air in the center of the room will seriously disturb the action of sound. Any irregularity in the air currents so that sheets of cold and heated air fluctuate about the room will also modify the regular action of the sound and produce confusion. The object to be striven for is to keep the air in the room as homogeneous and steady as possible. Hot stoves, radiators, and currents of heated air should be kept near the walls and out of the center of the room. It is of some small advantage to have the ventilation current go in the same direction that the sound is to go, since a wind tends to carry the sound with it.

## IV. THE INVESTIGATION IN THE AUDITORIUM AT THE UNIVERSITY OF ILLINOIS.

### A. PRELIMINARY WORK.

As already stated, a chaos of sound was set up when an observer in the Auditorium spoke or shouted or clapped his hands. Both echoes and reverberations were present and could be heard in all parts of the room, though the echoes seemed to be strongest on the stage and in the balcony. The prospects for bettering the acoustics were not very encouraging. Luckily, the cure for the reverberation was fairly simple, since Sabine's method gave a definite procedure that could be applied to this case. The cure for the echo, however, was yet to be found. It was first necessary to find out which walls set up the defect.

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[1] Osborne Reynolds. Proc. of Royal Soc., Vol. XXII, p. 531, 1874.

[2] Joseph Henry, "Report of the Lighthouse Board of the United States for the year 1874."
J. Tyndall, Phil. Trans., 1874.

[3] Sabine, Engineering Record, Vol. 61, p. 779, 1910.
Watson, Engineering Record, Vol. 67, p. 265, 1913.

[4] Sabine and Watson. Ibid.

The attempt to locate echoes by generating a sound and listening with the ear met with only partial success. The ear is sensitive enough but becomes confused when many echoes are present, coming apparently from every direction, so that the evidence thus obtained is not altogether conclusive. It became apparent that the successful solution lay in fixing the attention on the sound going in a particular direction and finding out where it went after reflection; then tracing out the path in another particular direction, and so on until the evidence obtained gave some hint of the general action of the sound.

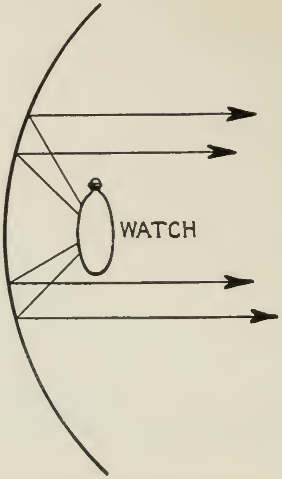

Fig. 4. Watch as Source of Sound, Backed by a Concave Reflector.

The first step in the application of this principle was to use a faint sound which could not be heard at any great distance unless reinforced in some way. The ticks of a watch were directed, by means of a reflector (Fig. 4) to certain walls suspected of giving echoes. Using the relation that the angle of incidence equals the angle of reflection, the reflected sound was readily located, and the watch ticks heard distinctly after they had traveled a total distance as great as 70 to 80 feet from the source.

In a later experiment, a metronome was used which gave a louder sound. It was enclosed in a sound-proof structure (Fig. 5) with only one opening, so that the sound could be directed by means of a horn. This method was suggested by the work of Gustav Lyon in the Hall of the Trocadero at Paris,* where a somewhat similar arrangement was used. The method was successful and verified the observations taken previously.

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*La Nature (Paris), April 24, 1909.

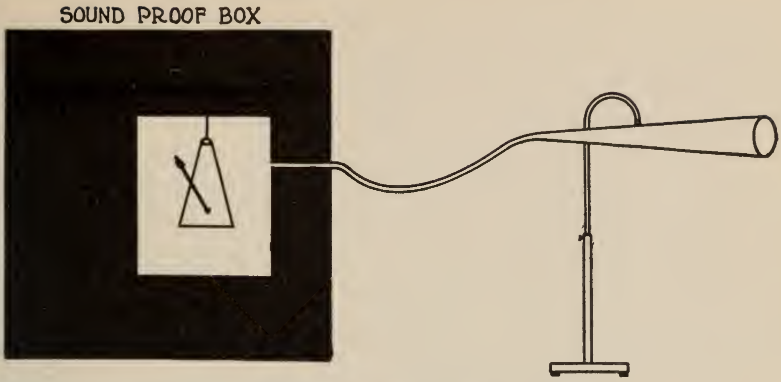

FIG. 5. METRONOME AS SOURCE OF SOUND.

Though the results obtained with the watch and metronome seemed conclusive, yet the observer was not always confident of the results. A further method was sought, and a more satisfactory one found by using an alternating current arc-light at the focus of a parabolic reflector (Fig. 6). In addition to the light, the arc gave forth a hissing sound, which was of short wave length and therefore experienced but little diffraction. The bundle of light rays was, therefore, accompanied by a bundle of sound, both coming from the same source and subject to the same law of reflection. The path of the sound was easily found by noting the position of the spot of light on the wall. The reflected sound was located by applying the relation that the angles of incidence and reflection are equal. The arc-light sound was intense and gave the observer confidence in results that was lacking in the other

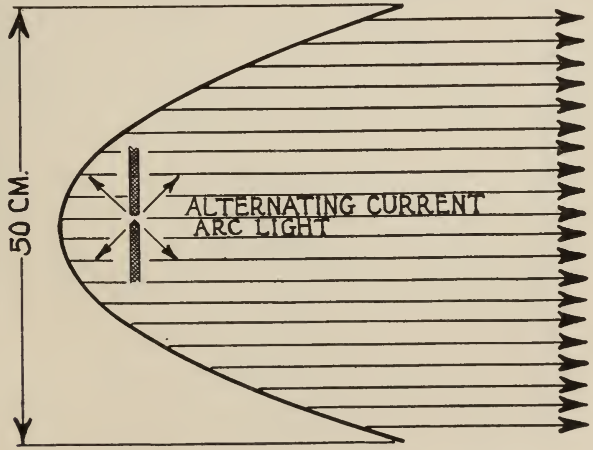

FIG. 6. ARC-LIGHT AS SOURCE OF SOUND.

methods. To trace successive reflections, small mirrors were fastened to the reflecting walls so that the path of the reflected sound was indicated by the reflected light. A "diagnosis" of the acoustical troubles of the Auditorium was then made by this method.

It should be noted here that the arc-light sound is not the same as the sounds of music or speech, these latter ones being of lower pitch and of longer wave length. It was, therefore, a matter of doubt whether the results obtained would hold also for the case of speech or music. Tests made by observers stationed in the Auditorium when musical numbers and speeches were rendered, however, verified the general conclusions obtained with the arc-light.

It should be pointed out in this connection that there is an objection to applying the "ray" method of geometrical optics to the case of sound. It is much more difficult to get a ray of sound than it is to get a ray of light.* This is due to the difference in the wave lengths in the two cases. It appears that the waves are diffracted, or spread out, in proportion to their length, the longer waves being spread out to a greater extent. The short waves of light from the sun, for instance, as they come through a window mark out a sharp pattern on the floor, which shows that the waves proceed in straight lines with but little diffraction or spreading. Far different is it with the longer waves of sound. If the window is open, we are able to hear practically all the sounds from outdoors, even that of a wagon around the corner, although we may be at the other end of the room away from the window. The longer sound waves spread out and bend at right angles around corners, so that it is almost impossible to get a sound shadow with them. Furthermore, in the matter of reflection, it appears that the area of the reflecting wall must be comparable with the length of the waves being reflected. In the case of light, the waves are very minute, hence a mirror can be very small and yet be able to set up a reflection; but sound waves are of greater length, the average wave length of speech (45 cm.) being about 700 000 times longer than the wave length of yellow light (.00006 cm.), hence the reflecting surface must be correspondingly larger. An illustration will perhaps make this clearer. Suppose a post one foot square projects through a water surface. The small ripples on the water will be reflected easily from the post, but the large water waves pass by almost as if the post were not there. The reflecting surface must have an area comparable with the size of the wave if it is to cause an effective reflection. Relief work in auditoriums, if of small dimensions, will affect only the high pitched sounds, i. e., those of short wave length, while the low pitched sounds of long wave length are reflected much the same as from a rather rough wall. It is also shown that the area of the reflecting surface is dependent on its distance from the source of sound and from the observer; the greater these distances are the larger must be the reflecting surface.*

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*Rayleigh "Theory of Sound," Vol. II, § 283.

These considerations all show that the reflection of sound is a complicated matter. The dimensions of a wall to reflect sound, or of relief work to scatter it, are determined by the wave length and by the various other factors mentioned. It should be said with caution that a "ray" of sound is reflected in a definite way from a small bit of relief work. We must deal with *bundles* of sound, not too sharply bounded, and have them strike surfaces of considerable area in order to produce reflections with any completeness.

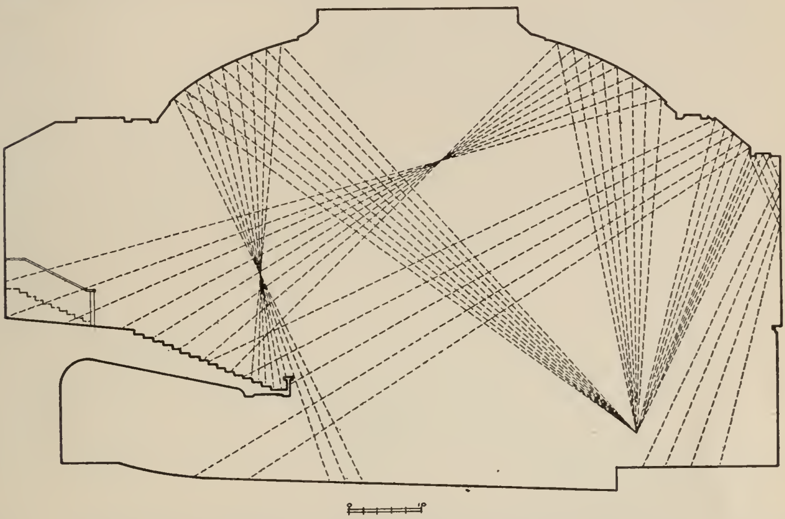

FIG. 7. LONGITUDINAL SECTION SHOWING THE CHIEF CONCENTRATIONS OF SOUND, THE DIFFRACTION EFFECTS BEING DISREGARDED.

### B. DETAILS OF THE ACOUSTICAL SURVEY IN THE AUDITORIUM.

The general effect of the walls of the Auditorium on the sound may be anticipated by considering analogous cases in geometrical optics, but with the restrictions on "rays" described in the preceding paragraph. The sound does not actually confine itself to the sharp boundaries shown. The diagrams are intended to indicate the main effect of the sound in the region so bounded. Fig. 7 gives such an idea for the concentration of sound in the longitudinal section of the Auditorium.

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*Rayleigh, ibid, 283.

The plan followed in the experimental work was to anticipate the path of the sound as indicated in Fig. 7, then to verify the results with the arc-light reflector. Figs. 8 and 9 show the effect of the rear wall in the balcony in forming echoes on the stage. The speaker was particularly unfortunate, being afflicted with no less than ten echoes.

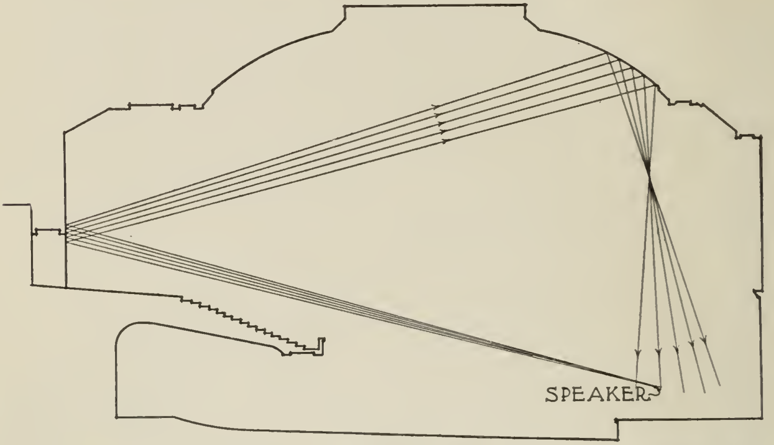

Fig. 8. Longitudinal Section Showing how Sound Is Returned to the Stage to Form an Echo.

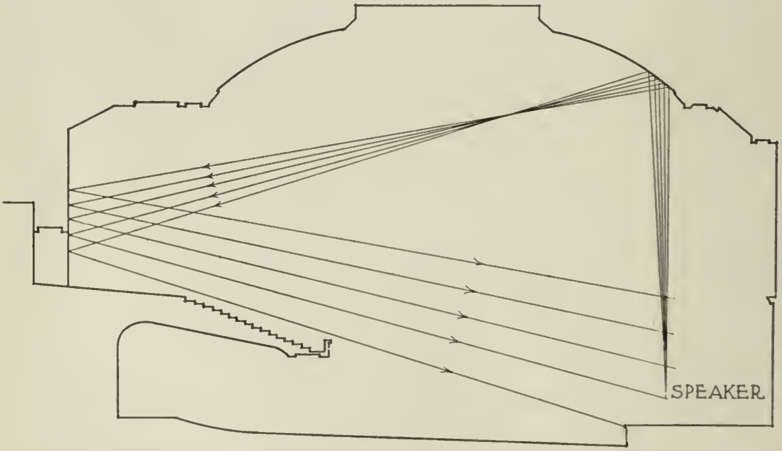

Fig. 9. Longitudinal Section Showing Formation of Echo on the Stage.

The hard, smooth, circular wall bounding the main floor under the balcony gave echoes as shown in Fig. 10, the sound going also in the reverse direction of the arrows.

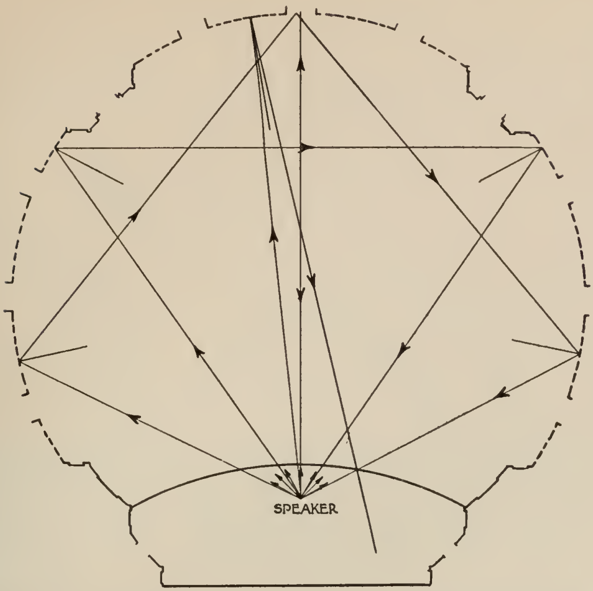

Fig. 10. Plan of Auditorium Showing Action of Rear Wall on the Sound.

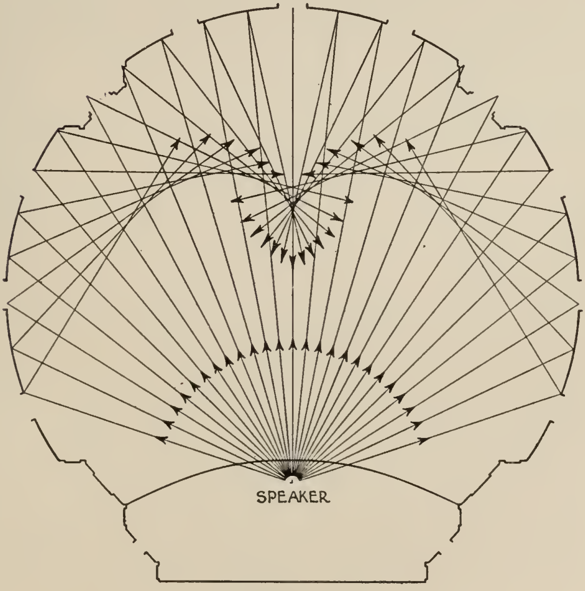

Fig. 11. Plan of Auditorium Showing Concentration of Sound by the Rear Wall.

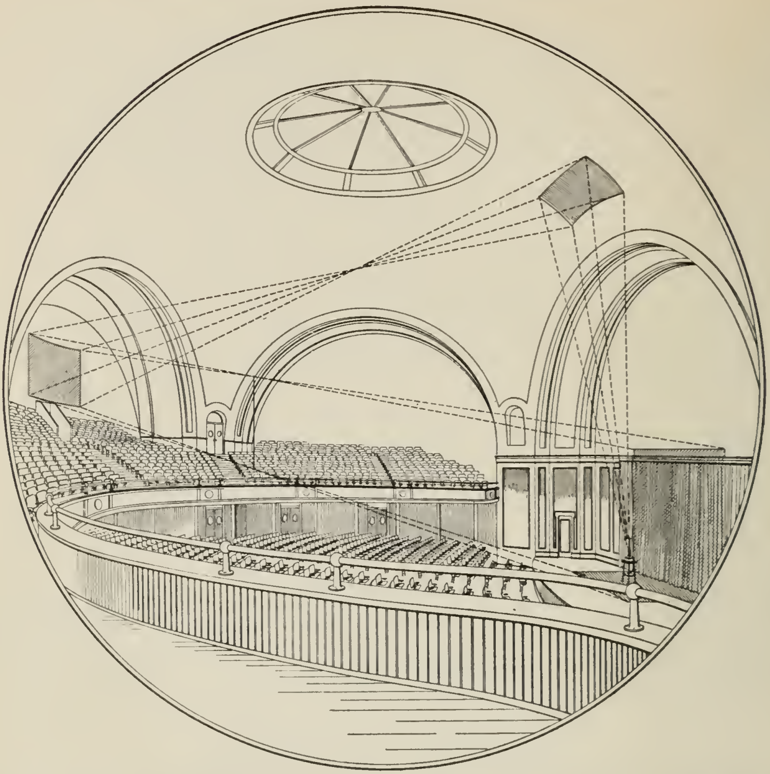

Fig. 12. This Figure Taken with Fig. 9 Shows How an Echo Is Set Up on the Stage.

A more comprehensive idea of the action of this wall is shown in Fig. 11. This reflected sound was small in amount and therefore not a serious disadvantage.

The cases cited were fairly easy to determine since the bundles of sound considered were confined closely to either a vertical or a horizontal plane for which the plans of the building gave some idea of the probable path of the sound. For other planes, the paths followed could be anticipated by analogy from the results already found. Fig. 12 shows in perspective the development of the result expressed in Fig. 9.

A square bundle of sound starts from the stage and strikes the spherical surface of the dome. After reflection, it is brought to a point focus, as shown, and spreads out until it strikes the vertical cylindrical wall in the rear of the balcony. This wall reflects it to a line focus, after which it proceeds to the stage. Auditors on all parts of the stage complained of hearing echoes.

Referring to Fig. 7, it is seen that the arch over the stage reflects sound back to the stage. Fig. 13 shows in perspective the focusing action of this overhead arch. Fig. 14 shows the effect of the second arch.

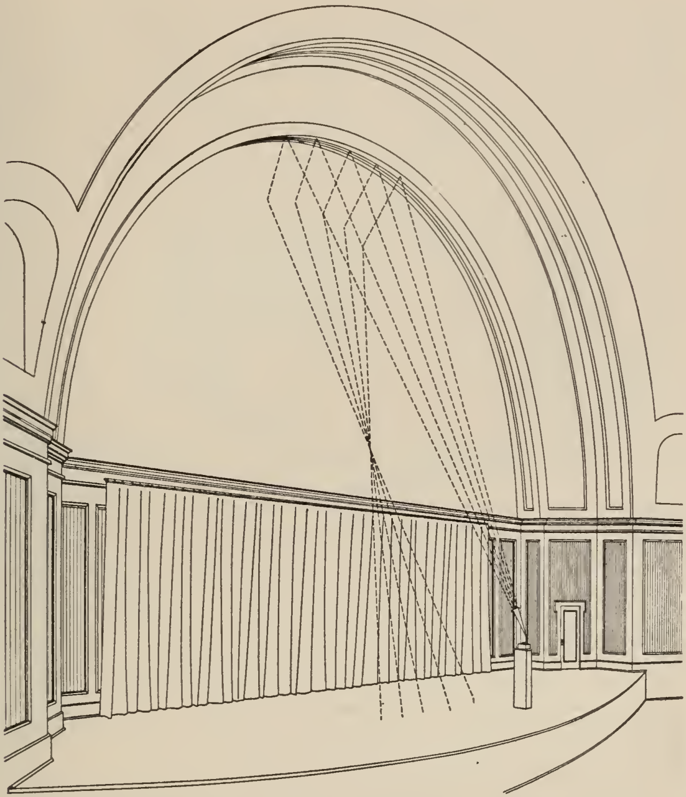

FIG. 13. PERSPECTIVE OF STAGE SHOWING FOCUSING ACTION OF ARCH ON SOUND.

Some of this sound is reflected to the stage and to the seats in front of the stage; other portions, striking more nearly horizontally, are reflected to the side balconies. The echoes are not strong except for high pitched notes with short wave lengths, since the width of the arch is small.

Passing now to the transverse section, Fig. 15, we find the most pronounced echoes in the Auditorium. If an observer generates a sound in the middle of the room directly under the center of the skylight, distinct echoes are set up. A bundle of sound passes to the concave surface which converges the sound to a focus, after which it spreads out again to the other concave surface and is again converged to a focus

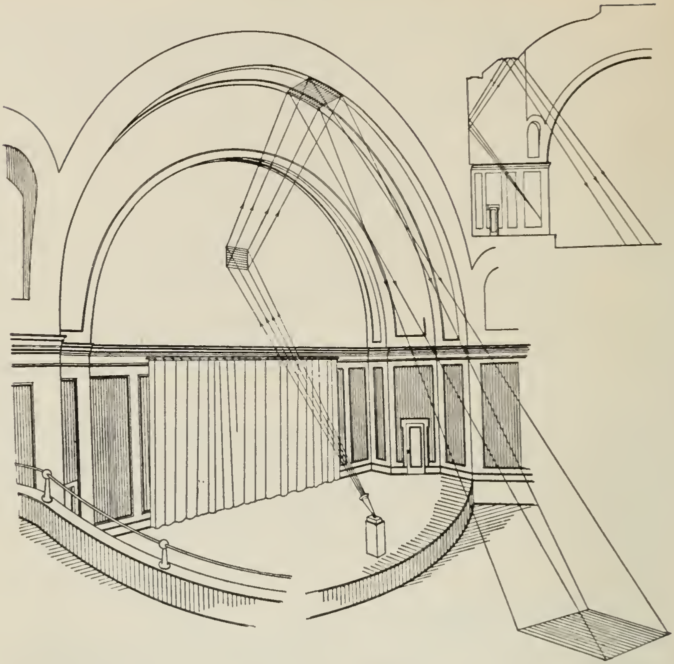

Fig. 14. Perspective of Stage Showing Focusing Action of Second Arch

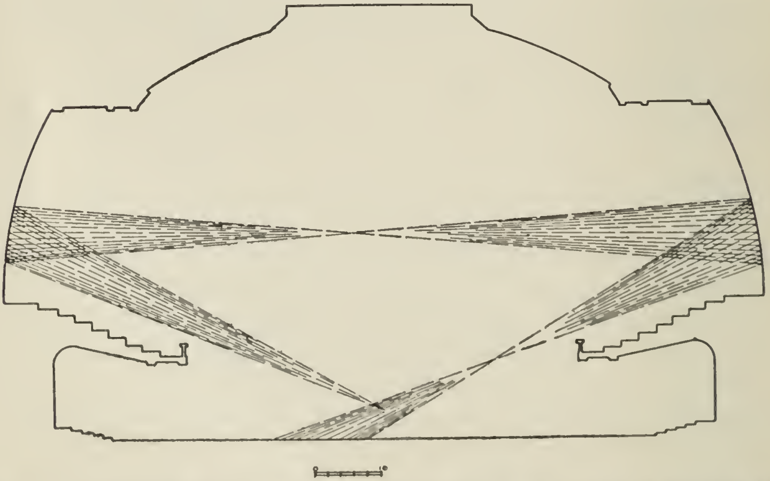

Fig. 15. Transverse Section Showing how Most Pronounced Echoes Are Set Up by the Two Concave Surfaces.

nearly at the starting point. The distance traveled is about 225 feet, taking about ¼ second, so that the conditions are right for setting up a strong echo. This echo is duplicated by the sound which goes in the reverse of the path just described. Another echo, somewhat less strong, is formed by the sound that goes to the dome overhead and which is reflected almost straight back, since the observer is nearly at the center of the sphere of which the dome is a part. These echoes repeat themselves, for the sound does not stop on reaching the starting point but is reflected from the floor and repeats the action just described. As many as ten distinct echoes have been generated by a single impulse of sound.

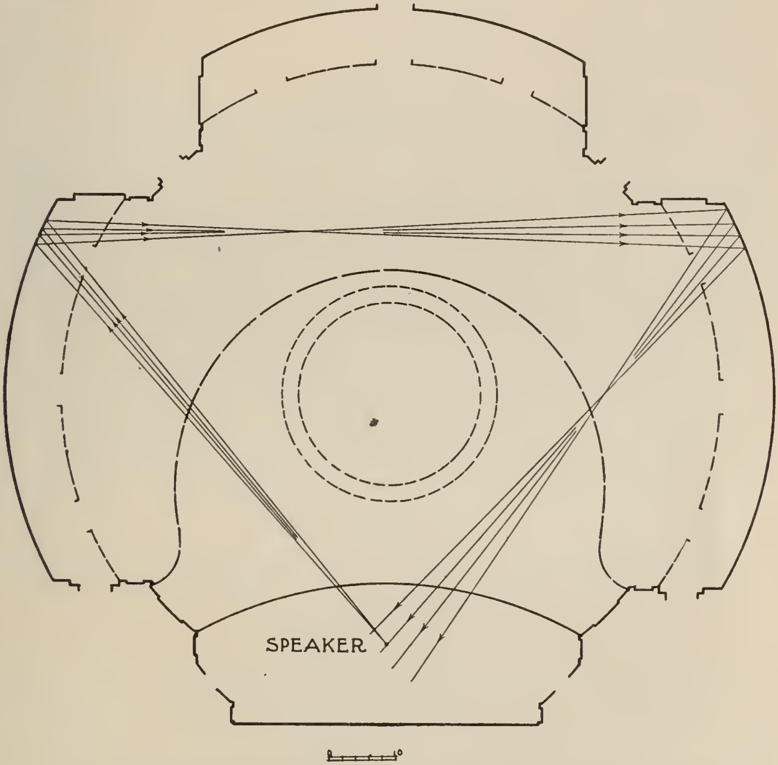

Fig. 16. Action of Sound in Causing Echo on the Stage.

The echo shown in Fig. 15 is repeated in a somewhat modified form for a sound generated on the stage by a speaker. Fig. 16 shows the path taken by the sound. This echo is duplicated by the sound that goes in the reverse direction of the arrows, so the speaker is greeted from both sides. Fig. 17 is a perspective showing the path. The sound does not confine itself closely to a geometrical pattern, as shown in the picture, but spreads out by diffraction. The main effect is shown by the figure.

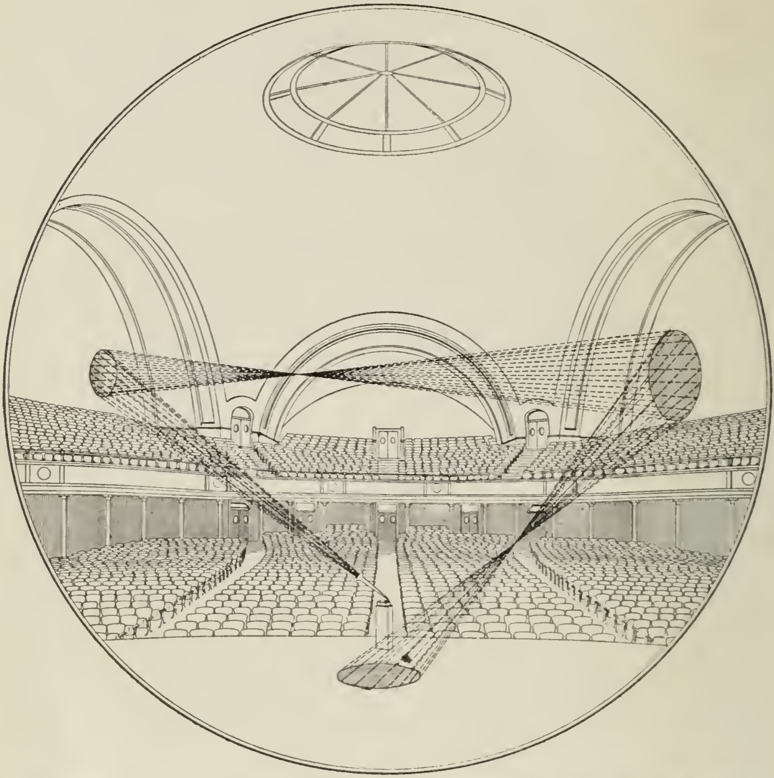

Fig. 17. Perspective Showing how an Echo Is Formed on the Stage by Two Reflections. Diffraction Effects Are Not Considered in this Drawing.

Thus far only the echoes that reached the stage have been described. Other echoes were found in other parts of the hall, and it seemed that few places were free from them. The side walls in the balcony, for instance, were instrumental in causing strong echoes in the rear of the balcony. Fig. 18 shows in perspective the action of one of these walls. These two surfaces were similar in shape and symmetrically placed. Each was the upper portion of a concave surface with its center of curvature in the center of the building under the dome. The general effect of the left hand wall was to concentrate the sound falling on it in the right hand seats in the balcony. Some of the sound struck the opposite wall and was reflected to the stage, as shown in Fig. 17. Auditors who sought the furthermost rear seats in the balcony to escape echoes were thus caught by this unexpected action of the sound. The right hand wall acted in a similar way to send the sound to the upper left balcony.

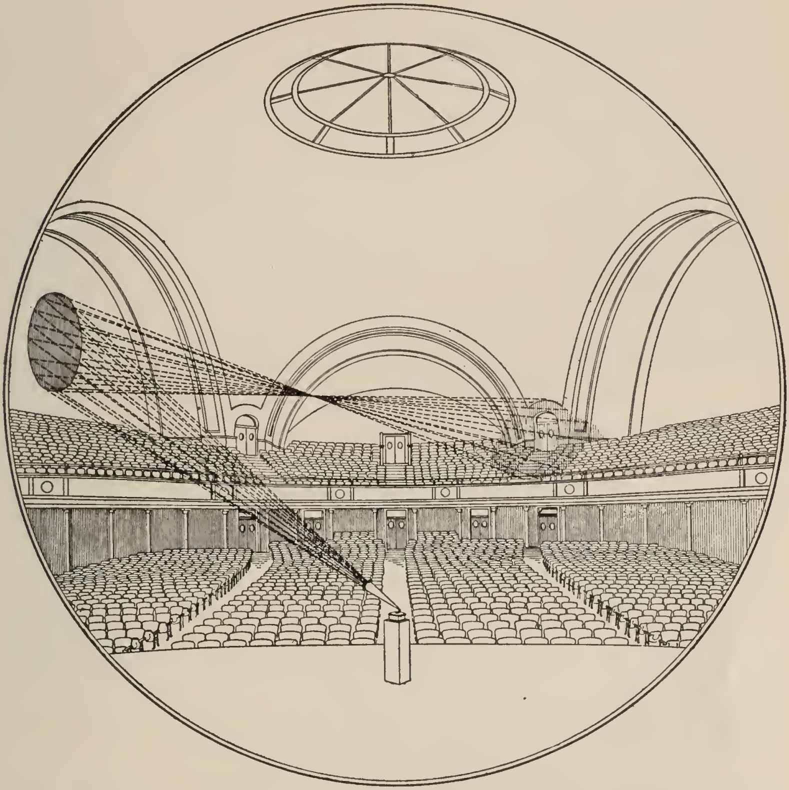

Fig. 18. Perspective Showing Sound Reflected from Concave Wall in Balcony. · Diffraction Not Considered.

The dome surface concentrates most of its sound near the front of the central portion of the balcony and the ground floor in front of the balcony in the form of a caustic cone. Figs. 7, 9 and 11 give some conception of how a concentration of sound is caused by this spherical surface. The echo in the front portion of the balcony was especially distinct. On one occasion, in this place, the author was able to hear the speaker more clearly from the echo than by listening to the direct sound.

Minor echoes were set up by the horizontal arch surfaces in the balcony. The sound from the stage was concentrated by reflection from these surfaces and then passed to a second reflection from the concave surfaces back of them. Auditors in the side balcony were thus disagreeably startled by having sound come from overhead from the rear.

### C. CONCLUSION DRAWN FROM THE ACOUSTICAL SURVEY.

The results of the survey show that curved walls are largely responsible for the formation of echoes because they concentrate the reflected sound. It seems desirable, therefore, to emphasize the danger of using such walls unless their action is annulled by absorbing materials or relief work. Large halls with curved walls are almost sure to have acoustical defects.

### D. METHODS EMPLOYED TO IMPROVE THE ACOUSTICS.

*Reflecting Boards.*—The provisional cure was brought about gradually by trying different devices suggested by the diagnosis. In one set of experiments sounding boards of various shapes and sizes were used. A flat board about five feet square placed at an incline over the position of the speaker produced little effect. A larger canvas surface, about 12 by 20 feet, was not much better. A parabolic reflector, however, gave a pronounced effect. This reflector was mounted over a pulpit at one end of the stage and served to intercept much of the sound that otherwise would have gone to the dome and produced echoes. The path of the reflected sound was parallel to the axis of the paraboloid of which the reflector was a quarter section. There was no difficulty in tracing out the reflected sound. Auditors in the path of the reflected rays reported an echo, but auditors in other parts of the Auditorium were remarkably free from the usual troubles. The device was not used permanently, since many speakers objected to the raised platform. Moreover, it was not a complete cure, since it was not suited for band concerts and other events, where the entire stage was used. Another reflector similar in shape to the one just described is shown in Figs. 21 and 22.

*Sabine's Method.*—The time of reverberation was determined by Sabine's method. An organ pipe making approximately 526 vibrations a second was blown for about three seconds and then stopped. An auditor listened to the decreasing sound, and when it died out made a record electrically on a chronograph drum. The time of reverberation was found to be 5.90 seconds, this being the mean of 19 sets of measurements, each of about 20 observations. The reverberation was found also by calculation from Sabine's equation (see Section III), taking the volume of the Auditorium as 11 800 cubic meters and calculating the

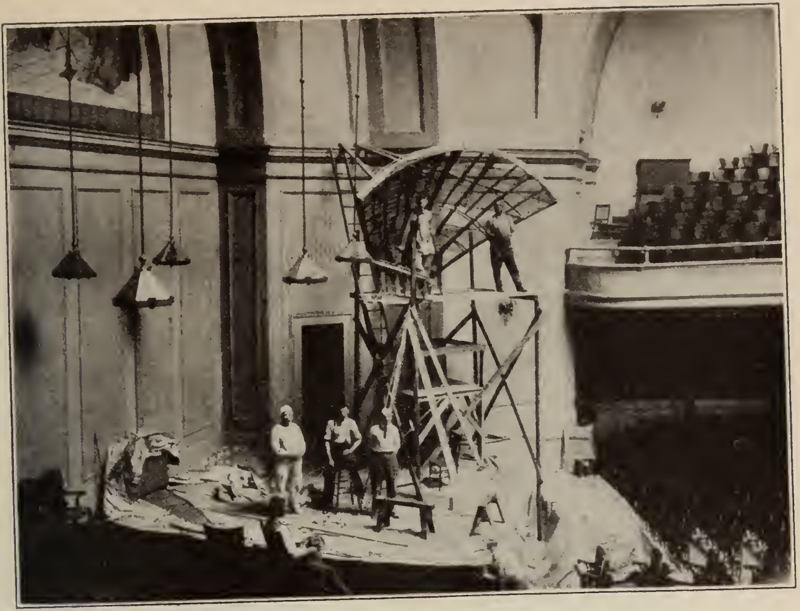

FIG. 19. REFLECTING BOARD IN PROCESS OF CONSTRUCTION.

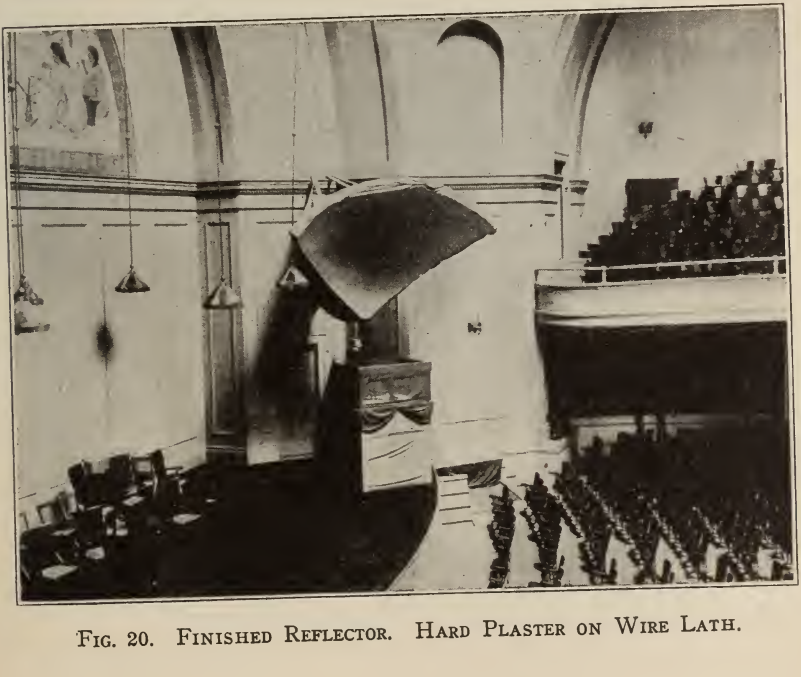

FIG. 20. FINISHED REFLECTOR. HARD PLASTER ON WIRE LATH.

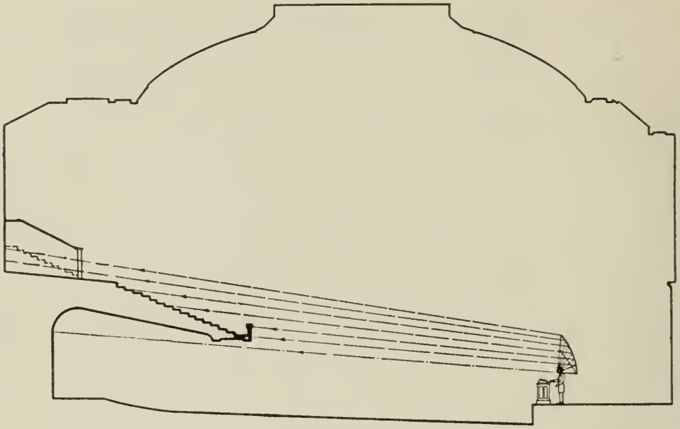

FIG. 21. PARABOLIC REFLECTOR SHOWING ITS ACTION ON SOUND.

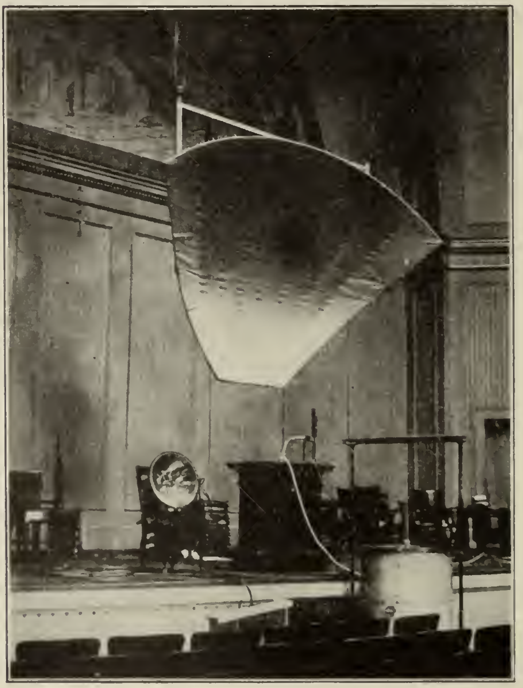

FIG. 22. PHOTOGRAPH OF PARABOLIC REFLECTOR.

absorbing power of all the surfaces in the room. This calculation gave 6.4 seconds. The agreement between the two results is as close as could be expected, since neither the intensity of the sound nor the pitch used by the author was the same as those used by Professor Sabine, and both of these factors affect the time of reverberation.

Several years later the time of reverberation was again determined after certain changes had been made. A thick carpet had been placed on the stage, heavy velour curtains 18 by 32 feet in area hung on the wall at the rear of the stage, a large canvas painting 400 square feet in area was installed, and the glass removed from the skylight in the ceiling. The time of reverberation was reduced to 4.8 seconds. With an audience present this value was reduced still more, and when the hall was crowded at commencement time the reverberation was not troublesome.

*Method of Eliminating Echoes.*—Although the time of reverberation was reduced to be fairly satisfactory, as just explained, the echoes still persisted, and were very annoying. Attempts were made to reduce individual echoes by hanging cotton flannel on the walls at critical points. Thus the shaded areas in Fig. 17 were covered and also the entire rear wall in the balcony. Pronounced echoes still remained, and it was evident that some drastic action was necessary to alleviate this condition. Four large canvases, shown in Figs. 23 and 24, were then hung in the dome in position suggested by the results of the diagnosis. A very decided improvement followed. For the first time the echoes were reduced to a marked degree and speakers on the stage could talk without the usual annoyance. This arrangement eliminated the echoes not only on the stage, but generally all over the house. A number of minor echoes were still left, but the conditions were much improved, especially when a large audience was present to reduce the reverberation.

*Proposed Final Cure.*—The state of affairs just described is the condition at the time of writing. Two propositions were considered in planning the final cure. One proposition involved a complete remodeling of the interior of the Auditorium. Plans of an interior were drawn in accordance with the results of the experimental work that would probably give satisfactory acoustics. This proposition was not carried out because of the expense and because it was thought desirable to attempt a cure without changing the shape of the room. The latter plan is the one now being followed. It is proposed to replace the present unsightly curtains with materials which will conform to the architectural features of the Auditorium and which will have a pleasing color scheme. At the same time, it will be necessary to hold to the features which have improved the acoustics.

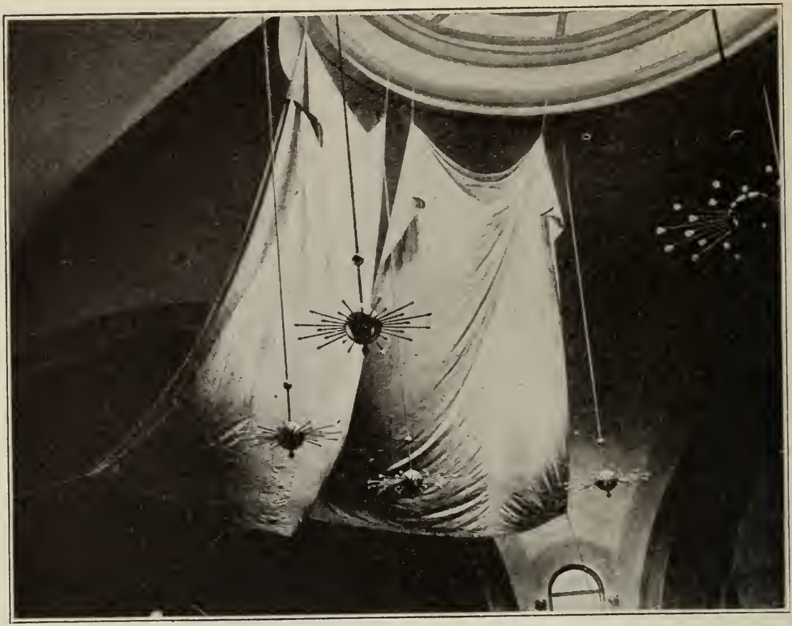

Fig. 23. Photograph of Two of the Canvas Curtains in the Dome of the Auditorium. Note also the Absorbing Materials under the Arches.

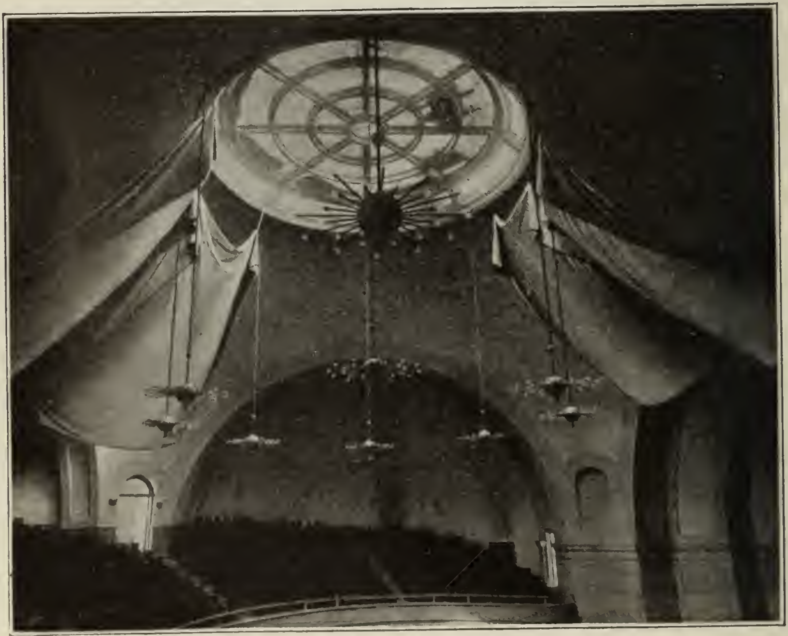

Fig. 24. Photograph of Dome of Auditorium Showing the Canvases Installed to Eliminate Echoes.

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*Eichhorn, A.* "Die Akustik Grosser Raüme nach Altgriechischer Theorie." Ernst and Korn, Publishers, Berlin, 1888. A discussion of Greek buildings and acoustics, with applications to modern conditions.

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*Franklin, W. S.* "Derivation of Equation of Decaying Sound in a Room and Definition of Open Window Equivalent of Absorbing Power." Physical Review, Vol. 16, pp. 372-374, 1903. A theoretical development of the formula found experimentally by Sabine.

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*Marage.* "Qualites acoustiques de certaines sailes pour la voix parlée." Comptes Rendus, Vol. 142, p. 878, 1906. An investigation of the acoustical properties of six halls in Paris.

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# THE TRACTIVE RESISTANCE OF A 28-TON ELECTRIC CAR

BY

HAROLD H. DUNN

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on plates

## BULLETIN No. 74

## ENGINEERING EXPERIMENT STATION

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 74 APRIL, 1914

# THE TRACTIVE RESISTANCE OF A 28-TON ELECTRIC CAR

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## CONTENTS

PAGE
- I. Introduction ... 3
  - 1. Purpose ... 3
  - 2. Definition of Terms ... 3
  - 3. Acknowledgments ... 3
- II. Summary ... 4
- III. Means Employed in Conducting the Tests ... 5
  - 4. The Electric Test Car ... 5
  - 5. The Recording Apparatus ... 5
  - 6. The Track ... 5
- IV. Test Conditions and Methods of Test ... 6
  - 7. Test Conditions ... 6
  - 8. Calibrations ... 7
  - 9. General Plan of the Tests ... 8
  - 10. Group A ... 11
  - 11. Group B ... 14
  - 12. Group C ... 15
- V. Results of the Tests ... 17
  - 13. Results of the Individual Tests ... 17
  - 14. Results of All the Tests ... 18
  - 15. Discussion of the Final Results ... 20
  - 16. Comparison with Other Experiments ... 22
- Appendix I. The Electric Test Car ... 23
- Appendix II. Methods Employed in Calculating the Results ... 30
- Appendix III. The Results of the Individual Tests ... 35

# THE TRACTIVE RESISTANCE OF A 28-TON ELECTRIC CAR.

## I. Introduction.

1. *Purpose.*—Three series of tests have been conducted by the Railway Engineering Department of the University of Illinois to determine the resistance offered to the motion of a 28-ton electric interurban car running on straight, level track, in still air at uniform speed; and to ascertain the relation existing between that resistance and the speed of the car. The first part of this bulletin describes the purpose, methods, and final results of the tests, but all details not essential to an understanding of the general methods and results are excluded. In the three appendices details are given concerning the apparatus, the methods of calculation, the test data, and the intermediate results.

2. *Definition of Terms.*—By the terms "resistance," "train resistance," and "car resistance" used in this bulletin is meant the number of pounds of tractive effort required at the rims of the driving wheels for each ton of train or car weight, to keep it moving at uniform speed on straight level track in still air. Such resistance data is employed in determining railway motor capacity, power consumption, and possible schedule speeds of a car or train of cars.

3. *Acknowledgments.*—The interest and co-operation of the officials of the Illinois Traction System have made it possible to carry out these tests over the lines of that road, and members of the Railway Engineering Department staff who have been concerned with this investigation thoroughly appreciate such interest and assistance.

The tests were planned by Professor Edward C. Schmidt, in charge of the Department of Railway Engineering, and they have been carried out under his general direction and supervision. He has also directed the work of the final computations, and reviewed the manuscript of the report. The tests of groups A and B were made under the direct supervision of Mr. Edgar I. Wenger, formerly Associate in the Department of Railway Engineering, and the preliminary calculations for these tests were made under his direction. The tests of group C and their preliminary calculations were made with the assistance of Mr. Daniel C. Faber, formerly a Research Fellow in the Engineering Experiment Station.

## II. Summary.

Eighteen tests made on selected sections of tangent track are described. The track upon which the tests were made is of good construction, such as one would expect to find on the better grade of interurban electric railways. The weather during the tests was generally fair. The average temperature was not below 25° F., and the wind velocity did not exceed 26 miles per hour. The test car used is described in detail in Appendix I.

The general plan was to run the test car backward and forward over the chosen track section at a variety of speeds, but maintaining approximately the same constant speed during each pair of opposing runs. The tests comprise in all 269 resistance determinations of which 131 have been calculated from runs during which the wind opposed the motion of the car, 130 from runs during which the wind helped the car, and 8 from runs made when no wind was blowing. During the two tests of group A, comprising 45 resistance determinations, opposing runs have been separated by a considerable time interval during which wind and weather conditions might change; but through changes made in the method of making the tests, opposing runs for the remaining tests have been separated by a small time interval.

Each test, consisting of a number of "runs," resulted in a number of values of average net car resistance which have been plotted with the corresponding average speed, giving diagrams such as Fig. 3. Through the two groups of points representing the results of runs in opposing directions, curves have been drawn between which a final mean curve has been decided upon. By this method wind resistance has been eliminated as nearly as may be from the results. The curves thus determined have been accepted as the results desired from the individual tests and have been grouped in Fig. 1. A final mean curve has been drawn there and repeated as Fig. 2, which represents the average relation between resistance and speed for all the tests.

The results of the individual tests are shown in Figs. 10 to 19 and Tables 3 to 12, inclusive, in Appendix III. The final results of the investigation are presented in Fig. 1, Table 2, and in formulae 1 and 2, page 20.

It is believed that Fig. 1, Table 2, and formulae 1 and 2 provide expressions for the mean relation between the speed and the resistance of the car to which these tests apply, when it runs on tangent level track of good construction, at uniform speed, and in still air. The greatest variation from these mean values under these conditions was about 9 per cent.

If application is made of these results to predict the resistance of any car of similar design and weight it should be borne in mind that the results apply to tests made with a car whose running-gear was in excellent condition, and that the tests were run in weather when the temperature was above 25° F.

## III. Means Employed in Conducting the Tests.

4. *The Electric Test Car.*—The data for this report have been obtained by means of the electric test car belonging to the Railway Engineering Department of the University of Illinois. It is a standard 45-foot interurban car of the double end type, weighing, complete with equipment, 55 150 pounds, subject to correction on account of changes in the equipment at different times. The cross sectional area of the car body and trucks is 90 square feet.

The motor equipment consists of four 50-horsepower, axle mounted, number 101-D Westinghouse direct current motors, geared to the axle in the ratio of 22 to 62. The Westinghouse unit switch system of multiple control is used.

The trucks, which are of the C-60 type of the Standard Motor Truck Company, are placed 23 ft. 3 in. between centers and have a wheel base of 6 ft. 4 in. The wheels are 33 in. in diameter and have the standard Master Car Builders' tread and flange, and the journals conform to the 4¼ in. × 8 in. standard of the same association. A more complete description of the car will be found in Appendix I.

5. *The Recording Apparatus.*—By means of the apparatus within the car a continuous graphical record of current, voltage, speed, time by 5 second intervals, distance traversed, brake cylinder pressure, and location on the road may be kept. These records are drawn upon a chart 40 inches wide which is made to pass under the recording pens at a rate proportional to time or distance. Other data taken during this investigation, but not recorded upon the chart, are described on page 11. Fig. 9 shows a copy of a portion of the chart from test 121. The transverse lines which mark one of the sections selected for calculation and some of the explanatory lettering do not appear on the original record, and the records as shown in Fig. 9 are grouped more closely together than on the original chart. A more complete description of the recording apparatus is to be found in Appendix I.

6. *The Track.*—This investigation was carried out on the tracks

of the Illinois Traction System between Danville, Urbana, Champaign, Decatur, and Springfield—a single track line of good construction over which an hourly service is maintained between the points named.

That part of the line between Danville and Urbana, 32 miles in length, was laid during 1903 and 1904 with 70-pound A. S. C. E. section rails except for 9 miles near Urbana, where 60-pound A. S. C. E. 1901 section rails were in use at the time these tests were made. All rails were laid on oak cross-ties spaced about 24 inches between centers and ballasted with gravel and cinders. This track had between 2 and 3 years in which to become settled before the tests were started. The track between Champaign and Decatur was laid in 1906 and 1907 with 70-pound A. S. C. E. section rails laid on oak, elm, and chestnut cross-ties spaced about 24 inches between centers and ballasted with gravel. When tests were made over this track it was well settled and in first-class condition. The line between Decatur and Springfield was laid in 1905 and is in all respects similar to that between Champaign and Decatur, except that part of the track is ballasted with cinders.

A survey of the Danville-Urbana line was made by the Railway Engineering Department of the University immediately preceding these tests and the results expressed in a profile drawn to a scale of ¼-inch to 100 feet. Elevations were taken on the north rail to 0.1 ft. at stations 300 feet apart, and turning points were taken at every fourth station where levels were read to 0.01 ft. The exact distances between all trolley line poles were recorded and incorporated in a table from which all distances, such as those between section limits, were determined.

For the purpose of this investigation certain long sections of comparatively level and well ballasted tangent track on the Champaign-Decatur-Springfield line were selected and surveyed by the Railway Engineering Department of the University. The results of these surveys were expressed in profiles, drawn to a scale of ¼-inch to 100 feet, which were used in making the calculations.

## IV. TEST CONDITIONS AND METHODS OF TEST.

7. *Test Conditions.*—The tests were all run during moderate weather. With four exceptions they were made on clear days only and on dry track. Data in regard to wind velocity and direction were obtained from United States Meteorological stations at Urbana, Springfield and Peoria, and during one group of tests, designated as group "C," these data were supplemented by wind determinations made by means of a portable wind vane and anemometer set up beside the test track. Tables 3 to 12 inclusive in Appendix III show that the approximate

average wind velocities prevailing during the tests varied from 0 to 26 miles per hour. The lowest average air temperature recorded during any test was 25° F., and the highest recorded temperature was 80° F.

TABLE 1

A SUMMARY OF TEST CONDITIONS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| | | | | Approximate Wind Data | | |
| Test No. | Weight of Car and Load | Approximate Average Air Temperature | Rail Condition | Average Direction | Average Velocity | Average Velocity Parallel to Test Track |
| | Pounds | Deg. F. | | | M. P. H. | M. P. H. |
| 15–16 | 56 600 | 40 | Dry | S. of W. | 7.2 | 7 |
| 23–24 | 56 250 | 70 | Dry | S. W. | 15.1 | 11 |
| 73–74 | 56 650 | 40 | Wet | N. of W. | 10.0 | 9 |
| 77–78 | 56 800 | 40 | Dry | N. W. | 12.0 | 12 |
| 91–92 | 56 350 | 45 | Dry | S. W. | 26.0 | 24 |
| 95–96 | 56 350 | 65 | Dry | N. W. | 16.0 | 15 |
| 109–110 | 56 950 | 55 | Wet | S. W. | 18.0 | 13 |
| 111–112 | 56 350 | 55 | Wet | S. W. | 18.0 | 13 |
| 113–114 | 56 200 | 55 | Dry | S. W. | 10.0 | 7 |
| 117–118 | 56 200 | 25 | Wet | S. W. | 12.0 | 9 |
| 119–120 | 56 750 | 45 | Dry | W. of S. | 3.5 | 2 |
| 121–122 | 56 750 | 30 | Dry | E. of N. | 3.0 | 2 |
| 123–124 | 56 750 | 30 | Dry | S. E. | 4.0 | 3 |
| 125–126 | 57 350 | 35 | Dry | S. W. | 15.0 | 10 |
| 127–128 | 57 500 | 25 | Wet | | 0 | 0 |
| 129–130 | 57 350 | 65 | Dry | S. E. | 15.0 | 10 |
| 141–142 | 57 800 | 40 | Dry | W. | 3.8 | 3 |
| 153–154 | 57 900 | 60 | Dry | | 0 | 0 |

The track sections selected for the tests are all straight and they vary in length from 250 to 10 000 feet. The majority of the tests were made on sections varying in length from 1000 to 1500 feet. Other requirements to be met by the sections were that the grade should be comparatively light and uniform, and that the track should be well ballasted and in good condition. While on the road the car was always in charge of a regular train crew provided by the Illinois Traction System.

8. *Calibrations.*—Preceding the commencement of the tests, all instruments were calibrated and at intervals during the investigation check readings were taken by means of indicating instruments in the same circuits as the recording instruments. Wherever possible, the results of these calibrations were expressed in the form of equations or tables as well as curves, and in making the calculations one of the two former was used in place of taking readings from curves. Owing to changes made in the apparatus during the tests, more than one calibration was necessary for several of the instruments.

9. *General Plan of the Tests.*—The test car records give quantities from which the gross resistance offered to motion of the car over a given section of track must first be calculated. The result desired from each run is that element of gross resistance which is always operating to retard a moving car or train; namely, the net resistance on straight and level track, at uniform speed, in still air. The other elements, one or more or none of which may be acting with the net resistance to form gross resistance, are: Resistance due to grade, resistance due to acceleration, curve resistance, and wind resistance (as distinguished from still air resistance). The tractive effort consumed by grade and acceleration may readily be determined by calculation, and the tests herein reported were so planned as to eliminate the two remaining elements. Curve resistance has been eliminated by selecting for calculation only those sections of chart made while the car was running on straight track. It is thought that by the procedure explained below the effect of wind resistance upon the final result has been reduced to a minimum.

The car was first run in one direction and then in the other over a given track section, at as nearly uniform speed as possible. A series of such runs, made at speeds varying through as wide a range as possible. constituted a test. Each run results in a value of average net car resistance which, when plotted with the corresponding average speed, becomes one point on a resistance-speed plot such as is shown in Fig. 3. By the to and fro motion of the car the wind alternately helps and opposes its motion and thereby decreases or increases its resistance. The resistance values from each test therefore fall into two groups, one of which represents values of resistance running with the wind, while the other represents values running against the wind.

In the figures, those points applying to runs with the wind are shown as solid dots, while the points applying to runs against the wind are shown as open circles. By methods which are described on pages 13 and 14, a mean curve has been drawn in each figure which represents the values of resistance with the influence of wind eliminated. These mean curves have been accepted as the desired resistance curves for the individual tests and have been grouped together as Fig. 1. An average curve obtained from this group, by the method described on page 20, has been repeated as Fig. 2 and accepted as the final curve of resistance for this 28-ton electric car under the conditions described.

As a result of slight variations in the methods of conducting the tests, they fall into three groups designated as A, B and C, which differ in the effect their conditions have on the assumptions regarding wind

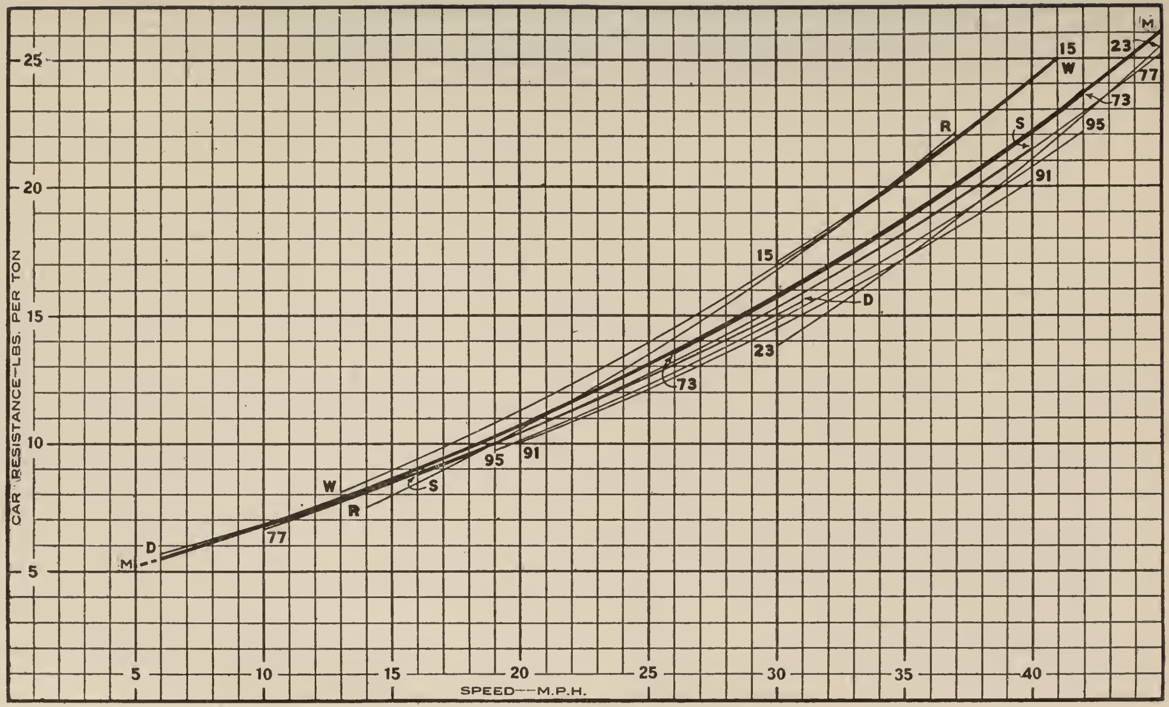

Fig. 1. The Relation of Resistance to Speed for All the Tests.

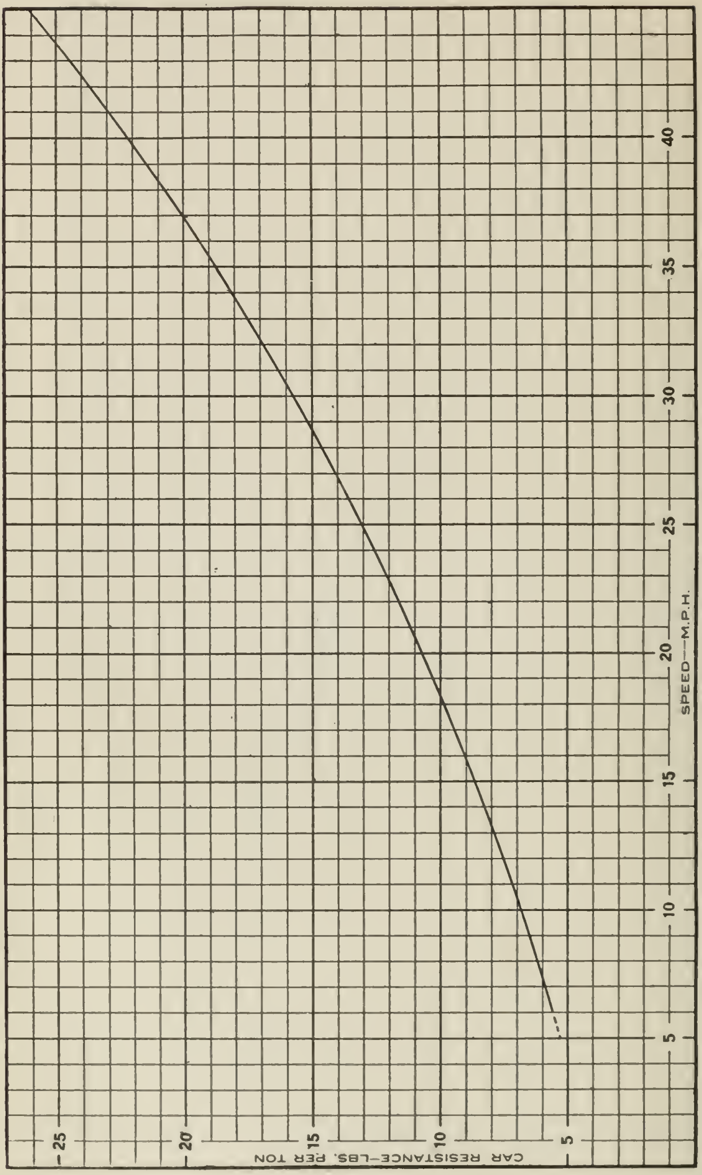

FIG. 2. THE MEAN RELATION BETWEEN RESISTANCE AND SPEED FOR ALL THE TESTS.

resistance, in the methods of obtaining certain data, and in the speed range.

10. *Group A.*—Those tests comprised in group A were through runs from Urbana to Danville, 32 miles, and return. During any test of this group only one set of opposing runs was made on any given section and such opposing runs were separated by a considerable time interval, since it took over an hour to run from Urbana to Danville or to return.

Test 23-24 is characteristic of this group. The results are shown in Table 4 and Fig. 11, which has here been repeated as Fig. 3. The recording apparatus was kept in operation throughout the trip and the following data recorded graphically upon the chart:

*a.* Current used by the motors.
*b.* Trolley line voltage.
*c.* Speed of the car by electrical recorder. (See App. I.)
*d.* Time by 5 second intervals.
*e.* Location upon the road.
*f.* Distance by 50 ft. intervals.
*g.* Brake cylinder pressure.

The following data were taken but not recorded graphically:

*h.* Average wind velocity and direction.
*k.* Air temperature.
*l.* Rail condition.
*m.* Weight of car and load.

Item *e,* the location of the car on the road, by means of which it is possible to correlate any position of the car with the profile of the road, was made by marking upon the chart the position of numbered trolley line poles, stations, or other markers as they were passed by a given point on the car. The brake cylinder pressure, item *g,* was recorded in order to make it possible to distinguish and to avoid those periods during the tests when the brakes were applied. Data regarding average wind velocity and direction, item *h,* were obtained from United States Weather Bureau Stations. Fig. 9, page 27; has been reproduced from a tracing of a portion of the chart for test No. 121.

Sections of chart were selected for calculation by comparison of the profile and chart as described in Appendix II. In general, the requirements to be met by the chosen chart sections are that the car shall be running on straight track, that no brake applications be made, that there be no heavy grades between the section limits, and that there be no considerable voltage, current or speed variation. They were chosen so that the speed range covered was as large as possible and so that the number of sections from an east bound trip (Urbana to Danville) would be balanced by the same or approximately the same number from the west bound trip. In this way the number of resistance determinations made from data taken during runs against the wind were balanced by an approximately equal number from runs with the wind. From test 23-24, for example, fourteen sections were chosen from the east bound and thirteen from the west bound trip.

In all the tests reported in this bulletin the variations in speed in passing the track sections have exceeded 2 miles per hour in only 28 per cent of the total number of resistance determinations, and in only 24 cases out of 269 has this speed variation exceeded 5 miles per hour. The maximum variation over any section was 9.95 miles per hour.

Each of the chosen chart sections has afforded, through calculations made with the data there recorded, a value of average net car resistance. The calculations are described in Appendix II. The steps in the process are here stated briefly. The average values of current and voltage were found for each section by determining the average ordinates of the curves. This data, with the time required for the car to pass the track section, and the efficiency of motors and gears at the above current and voltage, made it possible to calculate the energy in foot pounds delivered by the current to the driving wheels of the car. The energy thus determined was then corrected for grade and acceleration resistances, thereby giving the net energy absorbed by car resistance, from which the average net car resistance in pounds per ton was readily obtainable. In order to make the grade correction it was first necessary to determine the elevation of the center of gravity of the car as it entered and again as it left the track section. The correction for acceleration was calculated from the speeds determined by the heights of the speed curve at the points of entrance and exit. For this purpose determinations were made of the force required to produce accelerations in the rotation of the revolving parts as well as of the force required to produce the acceleration in the motion of translation of the car as a whole.

From each test a number of chart sections have been chosen and, as has been stated, each section has resulted in a value of average net car resistance which has been plotted with the corresponding average speed, thereby giving diagrams such as Fig. 3. Due to the method of selecting chart sections from opposing runs, the points on each such resistance-speed plot fall into two groups as previously explained. The numbers beside the points are the item numbers from the tables in Appendix III.

In drawing the curve to represent the results of the individual tests the two groups of points were first considered separately and a curve drawn for each. Thus, for example, curve *A*, Fig. 3, was drawn to represent the relation between resistance and speed for all the runs of test 23-24 during which the wind opposed the motion of the car, and curve *B* for the opposite condition. In order to draw these curves the plotted points were considered as being subdivided into a number of smaller groups, for each of which averages of speed and resistance were determined and plotted. Through the points representing these averages the curves were drawn. The groups of points were chosen so that the resulting average points would be distributed at about equal intervals throughout the speed range. The next step was to draw a mean curve such as *C*, Fig. 3, which would represent the relation existing between resistance and speed in still air. Since investigations indicate that the resistance offered by wind varies with the square of the speed, an arithmetical mean would not satisfy the conditions. Accordingly, an equation taking account of this fact has been derived (see Appendix II, page 34) and by means of it the ordinates of points on the mean curve have been calculated from the ordinates of the *A* and *B* curves at the same speed. The mean curves drawn by this method agree much more closely

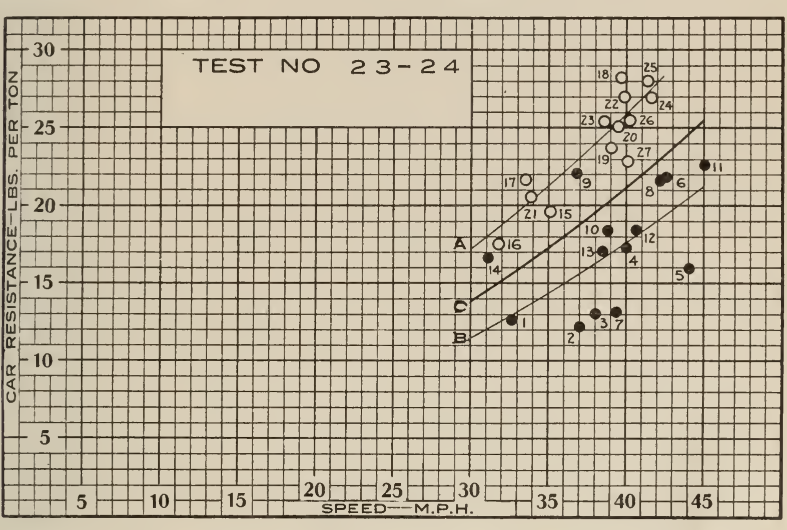

FIG. 3. THE RELATION OF RESISTANCE TO SPEED FOR TEST 23-24.

than do those curves drawn as arithmetical means between the *A* and *B* curves, and it has consequently been assumed that the method used has resulted in curves more accurately representing the resistance in still air than would have been obtained by using the arithmetical means.

The six curves thus obtained from tests of groups A and B, and the four from group C which were drawn by another method, were then superimposed upon each other in order to determine the characteristic form, and the individual curves were then modified to conform to that general shape. The curves drawn in this manner have been accepted as the results desired from the individual tests.

In group A there are two tests, namely 15-16 and 23-24, comprising together a total of 45 resistance determinations. The results are shown in Figs. 10 and 11 and Tables 3 and 4 in Appendix III.

11. *Group B.*—The tests belonging in group B differ from those in group A in the method of making the tests and in the effect this change in method has upon the assumptions regarding wind resistance. Whereas opposing runs from tests of group A were separated by a considerable time interval during which wind conditions might change, the tests of group B were planned so that the time interval between opposing runs would be as small as possible. A well kept section of track 1495 feet in length and located on the Urbana-Danville line was chosen for this purpose. The limits of this section, between which there was a difference in elevation of 0.56 feet, were defined by markers and offsets were made in the location record on the test chart as they were passed by the car. These points located the chart section to be calculated for each run. During any test the car was run back and forth over this section at a variety of speeds, each pair of opposing runs being made at approximately the same uniform speed. Data exactly similar to that obtained during the tests of group A were taken during each run. The reasons enumerated on pages 11 and 30 for the rejection of any chart section apply also to the runs of this group of tests. Through calculations made in the same way as has been described for the previous group, each run has resulted in an average value of net car resistance for the average speed at which the car passed the track section. These have been plotted and curves drawn by the method described for group A.

There are four tests in group B, namely, 73-74, 77-78, 91-92 and 95-96. The results are given in Figs. 12 to 15 and Tables 5 to 8, inclusive, in Appendix III.

12. *Group C.*—The principal points of difference between the tests of this and of preceding groups lie in the method of obtaining certain data, in the plan of grouping the results from two or more tests made on the same track section, and in the method of drawing the final curves.

Track sections, each comprising two or more tangents varying in length from 338 to 1320 feet, were chosen so as to meet the same requirements as those prescribed for the tests heretofore described. They have been designated by letters as follows:

*D:* Approximately 2680 feet long, located near Urbana on the Danville-Urbana line.

*W:* About 3700 feet in length, located near White Heath on the Champaign-Decatur line.

*S:* About 4100 feet long, located near White Heath on the Champaign-Decatur line.

*R* No. 1: Approximately 3000 feet long, located near Riverton on the Decatur-Springfield line.

*R* No. 2: About 4500 feet in length, located near Bement on the Champaign-Decatur line.

During each test the car was run back and forth over the chosen track section, as during the tests of group B, so that opposing runs were separated by a small time interval as in that group. Throughout each run the recording apparatus was kept in operation and data taken which were similar to those obtained during the tests that have been described, although the method of taking them was in some cases different. Changes made in the instrument equipment were as follows:

For the tests of groups A and B line voltage was recorded; but during the tests of group C connections were changed so that the record was that of voltage across the terminals of one motor. This change was made to facilitate calculations.

A Boyer mechanical speed recorder was installed in addition to the electric recorder described in Appendix I, and the record made by it has been used in the calculations. Both instruments were driven from the same shaft.

A portable windvane and anemometer were used during these tests to determine wind direction and velocity. These instruments were set up beside the test track and readings were made at intervals during the tests. By this means much more accurate determinations were made than could be obtained from the Weather Bureau reports.

It was found upon the completion of the tests of groups A and B that a low enough speed range had not been obtained and steps were therefore taken to supply this deficiency on subsequent tests. Two water rheostats were connected in the trolley circuit for the control of the motor voltage and the speed was held as nearly as possible at a constant predetermined value by the operator, who had for his guidance an ammeter in series with the speed-recording ammeter.

From the record of each run made over the test track, chart sections were chosen for calculation in accordance with the requirements stated on page 14 to conform to the characteristic shape. The resistance curves groups A and B. Each such chart section has resulted in one point on a resistance-speed plot such as Fig. 16, Appendix III. The results of all tests made on any given track section have been grouped together as, for example, track section D, on which tests 109-110, 111-112 and 113-114 were made. Inasmuch as the resistance-speed points in Fig. 16 have been derived from the results of tests made under somewhat different weather and wind conditions, it was found impracticable to draw the final curve in the same manner as for groups A and B. Consequently, in place of following the method described on page 13, the points on each resistance-speed plot, irrespective of the wind effect, were assumed to be divided into a number of groups, for each of which arithmetical averages of the values of speed and resistance were determined and plotted. These groups of points were chosen so that the resulting averages would be distributed at about equal intervals throughout the speed range. Through them the curve representing the relation between resistance and speed was drawn and later modified as described on page 14 to conform to the characteristic shape. The resistance curves thus derived, which are shown in Figs. 16 to 19 inclusive, have been accepted as the results desired from this group of tests.

It is recognized that this method of drawing the resistance curves is a deviation from the general plan followed for the preceding tests, but Fig. 1 shows that this deviation has brought about no greater variation of the individual curves from the mean curve *MM* than is found among those drawn as described on page 13. This may be accounted for by the fact that the wind velocities prevailing during the tests in question (see column 7, Table 1) were on the whole less than those prevailing during the preceding tests, and that consequently the resistance-speed points for runs with and against the wind were closely interwoven. Had the tests of group C lent themselves to the plan described on page 13, the curves with and against the wind would have been quite close together, and the mean curves determined by the two methods would not have differed greatly.

It will be noted that the numbers of the tests here included are not consecutive. Some of the tests bearing the intervening numbers were made for other purposes, but a considerable number were made for the purpose of this investigation and were later discarded for various reasons. Of the latter the greater number were omitted because an insufficient number of runs were made over the track sections to accurately determine a curve. This was occasioned by the necessity of running the car on sidings to allow regular cars to pass. Others were discarded because it was impossible to choose a sufficient number of chart sections that would meet the requirements, and still others were in the nature of preliminary runs.

## V. Results of the Tests.

13. *Results of the Individual Tests.*—The results of the individual tests are shown in the form of resistance-speed curves in Figs. 10 to 19 and Tables 3 to 12, inclusive, in Appendix III.

These tests, without exception, show a marked increase in resistance as the speed increases. The effect of wind resistance is clearly shown by a comparison of Figs. 10 to 15 inclusive, which have been derived from tests of groups A and B. For those tests during which a heavy wind alternately opposed and helped the motion of the car, the two curves, such as *A* and *B* (Fig. 3), drawn to represent the results of runs in opposite directions, are some distance apart, while for tests during which the wind was light these curves are quite close together. Figs. 15 and 10 illustrate this difference. Fig. 15 has been derived from test 95-96 during which the component of the average wind velocity parallel to the track was 15 miles per hour while Fig. 10 has been derived from test 15-16 during which that component was 7 miles per hour.

Had the wind velocity and direction remained precisely alike during all runs of a test, then the components of the wind velocity parallel to the track would have been of like value and the process of drawing the mean curve for the test would be valid in so far as the fundamental assumption underlying this process is in itself valid. The wind velocity and direction did not remain precisely constant during all runs of a test, however, and the resulting mean curves are consequently slightly in error. No method of eliminating this error or of evaluating it has presented itself.

The component of the wind velocity which is perpendicular to the test track increases gross resistance, irrespective of the direction in which the car is running, by pressing the flanges of the car wheels against the head of the rail, thereby increasing the frictional resistance. No method has been found for evaluating its effect upon the final resistance values and it is therefore included in the accepted results. Obviously it is one cause for the variations among the resistance curves as well as for the variations among the individual resistance-speed points.

Due in part to the reasons just mentioned, the points in any of the resistance-speed plots may vary considerably from the final mean curve for that test. The maximum variation of any point from the mean is approximately 90 per cent. Any comparison of variations from the mean made to determine the agreement of the results among themselves should, however, be confined to the results obtained from runs in one direction only. Such a comparison among the resistance determinations for tests of groups A and B shows variations which, though large in some cases, are not unusual for this class of experimental work. The maximum variation from the mean for the first two groups of tests occurred during test 23-24 (Fig. 3). Point 9 varies 42.5 per cent from mean curve *B*. The next largest variation is 24 per cent, while the average is much less than that. A very small percentage of this variation may be due to accumulated errors in instruments or in the calculations, although all reasonable precautions have been taken to avoid such errors. Each step in the process of making the calculations and producing the tables and curves has been duplicated at a different time and usually by a different person, and in all cases where the calculated value of resistance differed greatly from the mean all calculations leading thereto were repeated. The influence of such variable and uncontrollable elements of net resistance as flange friction, journal friction, instantaneous changes in velocity and direction of the wind, and changes in the lubrication of bearings and gears, is believed to be sufficient to account for the differences shown in the curves here presented. The variations of the points in Figs. 16 to 19, inclusive, are chargeable to the differences in the test conditions that are represented on those resistance-speed plots as well as to the above-mentioned causes.

14. *Results of All the Tests.*—The six resistance curves for the individual tests of groups A and B, and those representing the results obtained on the four track sections of group C, have all been brought together in Fig. 1.* In view of the number and character of the elements

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*Each curve has been marked with the test number or letter corresponding thereto.

of net resistance that can not be controlled, the differences among the curves there shown are not greater than might be expected. For speeds up to 20 miles per hour the maximum variation among the resistance values from the different curves is about 13 per cent, between 20 and 30 miles per hour this variation is approximately 17 per cent, while above 30 miles per hour it approximates only 20 per cent.

TABLE 2

VALUES OF RESISTANCE AT VARIOUS SPEEDS, DERIVED FROM THE FINAL CURVE AND THE CURVES FOR THE INDIVIDUAL TESTS. THIS TABLE PROVIDES THE CO-ORDINATES OF THE CURVES IN FIGURE 2.

| Speed — Miles Per Hour | Car Resistance—Pounds Per Ton | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Final Mean Curve | Test 15–16 | Test 23–24 | Test 73–74 | Test 77–78 | Test 91–92 | Test 95–96 | Tests on Section D | Tests on Section W | Tests on Section S | Tests on Section R |
| 5 | 5.25 | | | | | | | | | | |
| 6 | 5.52 | | | | | | | 5.65 | | | |
| 7 | 5.81 | | | | | | | | | | |
| 8 | 6.12 | | | | | | | | | | |
| 9 | 6.48 | | | | | | | | | | |
| 10 | 6.80 | | | | 6.62 | | | 6.87 | | | |
| 11 | 7.12 | | | | | | | | | | |
| 12 | 7.50 | | | | | | | | | | |
| 13 | 7.87 | | | | | | | | 8.12 | | |
| 14 | 8.25 | | | | | | | | | | 7.55 |
| 15 | 8.62 | | | | 8.48 | | | 8.62 | 9.00 | | 8.02 |
| 16 | 9.05 | | | | | | | | | 8.75 | |
| 17 | 9.48 | | | | | | | | | | |
| 18 | 9.87 | | | | | | | | | | |
| 19 | 10.32 | | | | | | 9.75 | | | | |
| 20 | 10 75 | | | | 10.50 | 10.02 | 10.18 | 10.55 | 11.30 | 10.44 | 10.60 |
| 21 | 11.22 | | | | | | | | | | |
| 22 | 11.66 | | | | | | | | | | |
| 23 | 12.13 | | | | | | | | | | |
| 24 | 12.62 | | | | | | | | | | |
| 25 | 13.03 | | | | 12.75 | 12.13 | 12.37 | 12.65 | 13.93 | 12.75 | 13.45 |
| 26 | 13.62 | | | 13.55 | | | | | | | |
| 27 | 14.12 | | | | | | | | | | |
| 28 | 14.65 | | | | | | | | | | |
| 29 | 15.22 | | | | | | | | | | |
| 30 | 15.75 | 17.10 | 13.84 | 15.73 | 15.34 | 14.54 | 14.85 | 15.00 | 16.88 | 15.30 | 16.70 |
| 31 | 16.36 | | | | | | | 15.50 | | | |
| 32 | 16.90 | | | | | | | | | | |
| 33 | 17.50 | | | | | | | | | | |
| 34 | 18.12 | | | | | | | | | | |
| 35 | 18.75 | 20.44 | 17.15 | 18.75 | 18.22 | 17.22 | 17.62 | | 20.25 | 18.21 | 20.47 |
| 36 | 19.37 | | | | | | | | | | |
| 37 | 20.04 | | | | | | | | | | 22.13 |
| 38 | 20.75 | | | | | | | | | | |
| 39 | 21.44 | | | | | | | | | | |
| 40 | 22.13 | 24.20 | 21.00 | 22.16 | 21.45 | 20.25 | 20.75 | | 24.15 | 21.50 | |
| 41 | 22.87 | 25.02 | | | | | | | 25.00 | | |
| 42 | 23.66 | | | 23.68 | | | 22.12 | | | | |
| 43 | 24.45 | | | | | | | | | | |
| 44 | 25.26 | | | | | | | | | | |
| 45 | 26.12 | | 25.56 | | 25.25 | | | | | | |

The mean curve *MM* shown in Fig. 1 and reproduced as Fig. 2 has been drawn to express the average relation between resistance and speed for all the tests. To determine this curve, the resistances corresponding to a given speed, say 10 m.p.h., were taken from the individual curves of Fig. 2 and averaged. In this process each resistance value was given a weight corresponding to the total number of resistance determinations represented by the curve from which it was taken. These averages were calculated at speeds varying by steps of 5 miles per hour and plotted in Figs. 1 and 2. Through them the mean curve *MM* has been drawn. The curve thus determined represents the final results obtained from this investigation. The values of resistance at various speeds have been determined from all the curves shown in Fig. 1 and they are presented in Table 2. The data there given are sufficient to accurately reproduce all the final resistance-speed curves.

The final results have also been expressed in the form of an equation which is given as formula 1. This equation makes it possible to calculate resistance values that do not vary more than one-half of one per cent from those obtained from the curve *MM*. In this formula $R$ is the resistance expressed in pounds per ton and $S$ is the speed in miles per hour.

$$R = 4 + 0.222\,S + 0.00582\,S^2 \ldots\ldots\ldots (1)$$

This equation has been modified as shown in formula 2 to take more definite account of air resistance. In formula 2, $A$ is the cross-sectional area of the car expressed in square feet and $W$ is the car weight in tons.

$$R = 4 + 0.222\,S + 0.00181\,\frac{A}{W}\,S^2 \ldots\ldots (2)$$

These formulae are the equations of a parabola which corresponds closely to the curve *MM*. In using these formulae care should be taken not to extend their use to speeds much beyond the limits of the curve *MM*.

15. *Discussion of the Final Results.*—The final results of this investigation are presented in Fig. 2, Table 2, and formulae 1 and 2, all of which express the mean relation between speed and resistance for the 28-ton car when running at uniform speed on tangent and level track of good construction, during weather when the temperature is not lower than 25° F., and when the wind velocity does not exceed about 26 miles per hour. The curve *MM* (Figs. 1 and 2) is an average of curves derived from ten individual tests that have been run under conditions which, though they differ somewhat, are not at all unusual. Notwithstanding this difference in test conditions, the maximum variation of the individual curves from the mean curve *MM* is only 12 per cent (see Fig. 1). With the exception of that portion of the curve for

test 23-24, which lies between 30 and 34 miles per hour, the maximum variation of the individual curves from the mean curve is only 9 per cent for all speeds above 30 miles per hour. At speeds below 30 miles per hour this variation is considerably less than 9 per cent. In other words, all curves from the individual tests lie within a narrow belt. All points within this belt may very properly be considered to represent a relation between car resistance and speed which is true under some usual conditions.

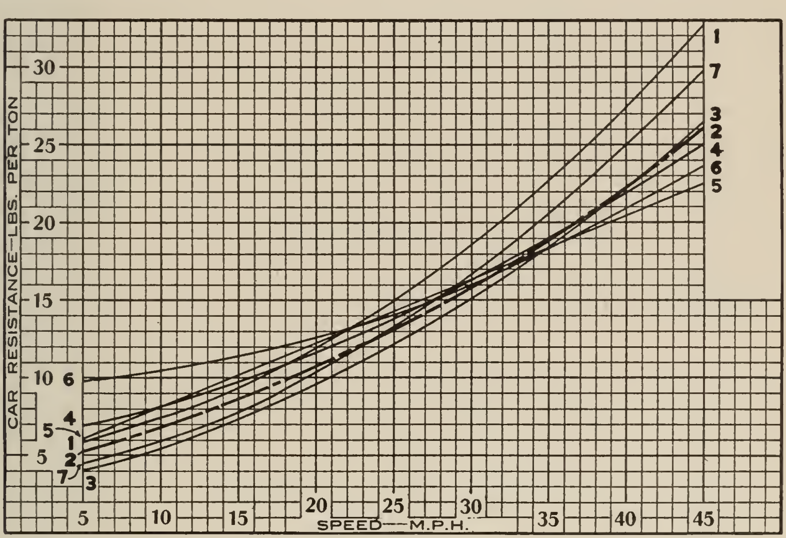

FIG. 4. COMPARATIVE DIAGRAM OF RESISTANCE-SPEED CURVES CALCULATED BY MEANS OF SEVEN FORMULAE APPLICABLE TO ELECTRIC RAILWAY CONDITIONS.

W = 28 TONS. A = 90 SQ. FT. N = 1.

1. DAVIS.
2. UNIVERSITY OF ILLINOIS.
3. SMITH.
4. BLOOD.
5. LUNDIE.
6. ARMSTRONG.
7. MAILLOUX.

If it is desired to estimate the resistance of any car which in weight and general design is similar to the car used during these tests, it is believed that the curve *MM* offers a basis for the prediction of such resistance, the accuracy of which will lie within the limits just stated; that is, it is believed that by the use of the curve *MM* net resistance may be predicted within about 9 per cent. Obviously, in using the curve for such a purpose the conditions surrounding these tests must be fully appreciated.

In addition to the expression given for the resistance in still air, the tables and curves in Appendix III provide information as to the excess resistance due to wind. They show that during moderate weather and for wind velocities up to 24 miles per hour which directly oppose the motion of the car, the resistance may exceed that in still air by as much as 70 per cent at speeds below 30 miles per hour, and by 50 per cent at speeds above 30 miles per hour.

16. *Comparison with Other Experiments.*—For purposes of comparison with the results of other investigations the following formulae are given, together with Fig. 4:

Davis[1] $R = 5 + 0.13\,S + \frac{0.3\,S^2}{W}[1 + 0.1\,(N - 1)]$

University of Illinois $R = 4 + 0.222\,S + 0.00582\,S^2.$

$R = 4 + 0.222\,S + 0.00181\frac{A}{W}S^2.$

Smith[2] $R = 3 + 0.167\,S + 0.0025\,\frac{A}{W}\,S^2.$

Blood[3] $R = 6 + 0.13\,S + (0.0014 + \frac{0.35}{W}\cdot)\,S^{1.8}.$

Lundie[4] $R = 4 + 0.24\,S + \frac{4.8\,S}{W}$

Armstrong[5] $R = \frac{50}{\sqrt{W}} + 0.03\,S + \frac{0.002AS^2}{W}[1 + 0.1\,(N - 1)]$

Mailloux[6] $R = 3.5 + 0.15\,S + \frac{0.02\,N + 0.25}{N\,W}\,S^2$

Notation:—$R$ = Car resistance in pounds per ton.
$S$ = Speed of the car in miles per hour.
$W$ = Weight of the car in tons.
$N$ = Number of cars in the train.
$A$ = Effective cross-sectional area of the car body and trucks in sq. ft.

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1. Street Railway Journal, 1904, v. 24, p. 1003.
2. Proc. Amer. Inst. Elec. Eng., 1904, v. 23, p. 696.
3. Proc. Amer. Soc. Mech. Eng., v. 24, p. 945.
4. Street Railway Journal, 1902, v. 19, p. 557.
5. Standard Handbook for Electrical Engineers, Section on Electric Traction, by A. H. Armstrong.
6. Proc. Amer. Inst. Elec. Eng., 1904, v. 23, p. 731.

## APPENDIX I.

### THE ELECTRIC TEST CAR.

The electric test car, by means of which the tests reported in this bulletin were made, was built under the direction of the University of Illinois and placed in operation in 1906. The equipment, which is for 500-volt direct current operation, was placed so as to provide facilities for the instruction of students as well as for conducting investigations concerning various features of electric car operation. For this purpose the switch group, circuit breaker, limit switch and certain other parts of the equipment were placed within the car where their action under all conditions might readily be seen. The weight of the car with its equipment, subject to certain corrections, is 55 150 pounds. The corrections referred to are necessitated by additions to the equipment and changes made therein at different times. The general design of the car is shown in Figs. 5, 6 and 7.

*Car Body.*—The car body, which is of the double-end type commonly used in moderately heavy interurban service, was built by the Jewett Car Co., of Newark, Ohio. Its principal dimensions are given in Fig. 7. The cross-sectional area of the body and trucks is 90 square feet. The vestibules are of the round end type. Ball bearing center plates were installed between tests 29 and 30.

*Trucks.*—The car body is mounted upon Standard Motor Truck Company C-60 type trucks whose main dimensions are:—

Distance from center to center of trucks 23 ft. 3 in.
Wheel base, 6 ft. 4 in.
Lateral play of axle journals, 3/16 in.
Axle journals, 4¼ in. × 8 in.
Diameter of wheels, 33 in.
Weight of truck exclusive of electrical equipment, 7824 pounds.

On one truck there are four rolled steel wheels, while those on the other truck are chilled cast iron. Both sets have the standard M. C. B. tread and flange.

*Motors.*—On each truck are two number 101-D Westinghouse 500-volt, direct current motors, each one having a commercial rating of 50 H. P. They are mounted on the axles and geared thereto in the ratio of 22:62. The characteristic curves are shown in Fig. 8.

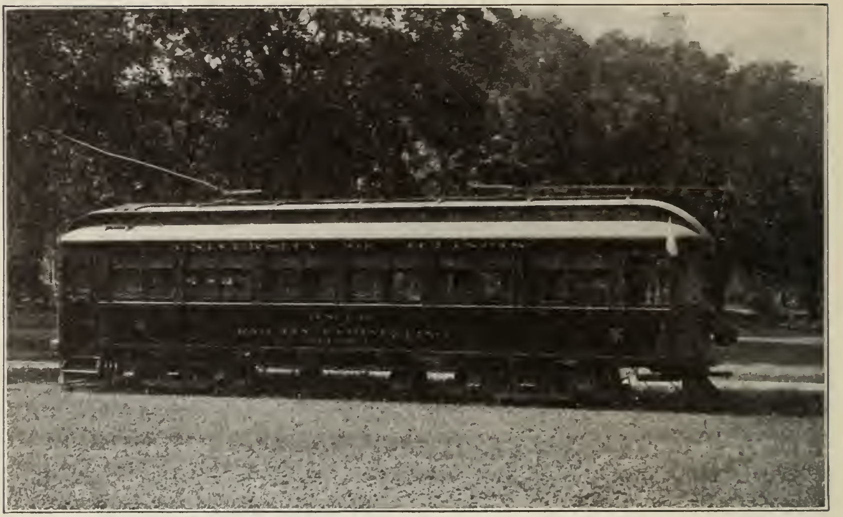


FIG. 5. THE ELECTRIC RAILWAY TEST CAR.

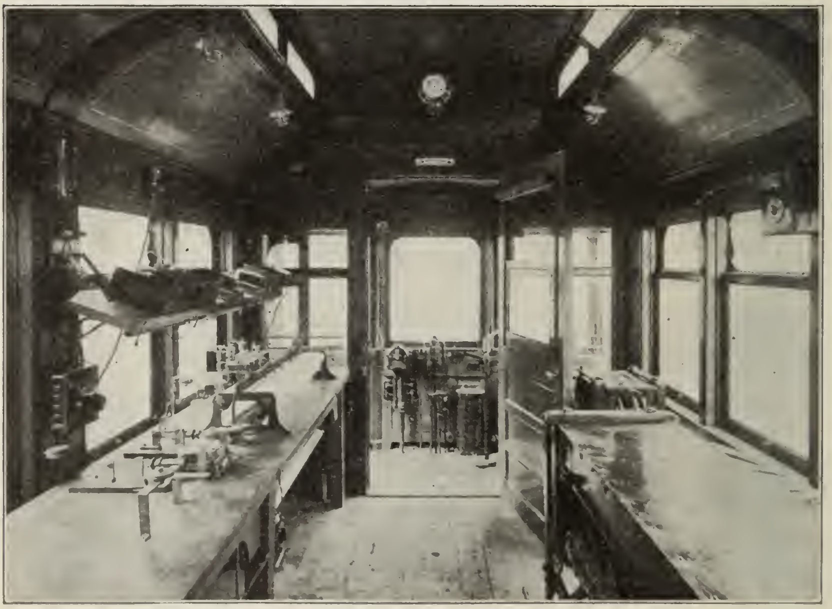


FIG. 6. INTERIOR OF ELECTRIC RAILWAY TEST CAR.

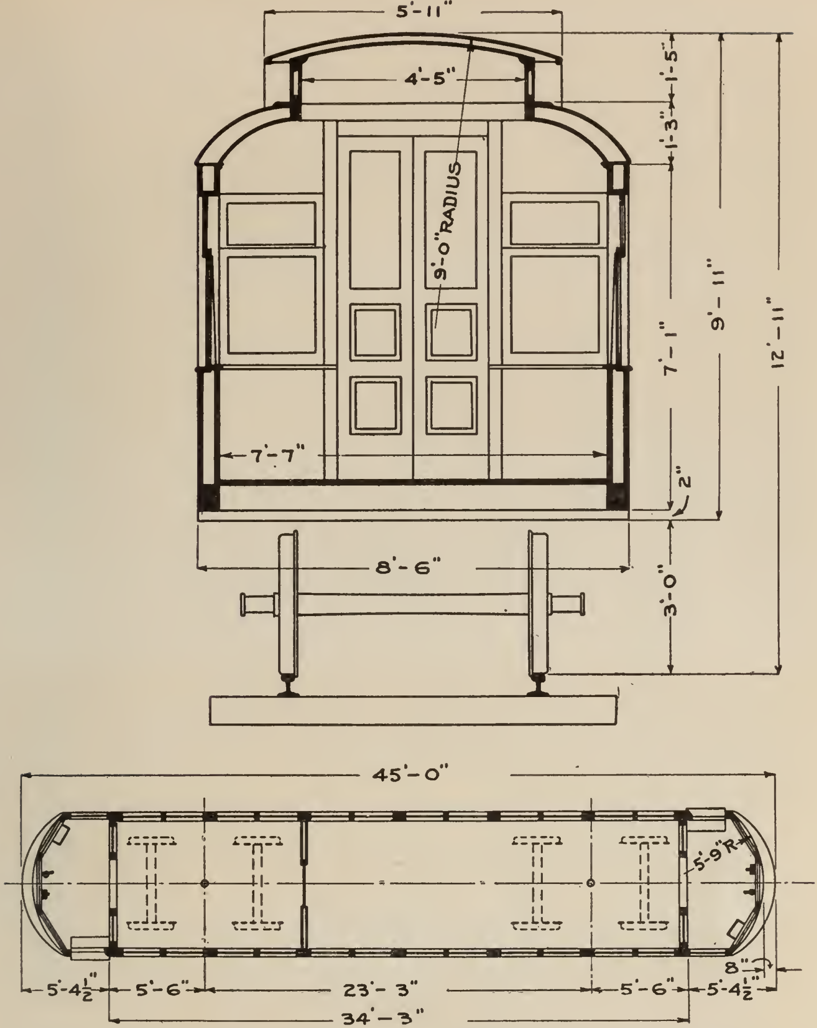


FIG. 7. PLAN AND CROSS SECTION OF THE ELECTRIC RAILWAY TEST CAR.

*Control Equipment.*—The motor control equipment is that known as the Westinghouse unit switch system of multiple control. In this type of control the switches which make the necessary connections and short circuit the starting resistances are operated by compressed air from the brake reservoir. The valves which control the air are operated from the master controller through the medium of magnets and a low voltage storage battery circuit. Acceleration is governed by a limit switch the armature of which is weighted to give the desired rate.

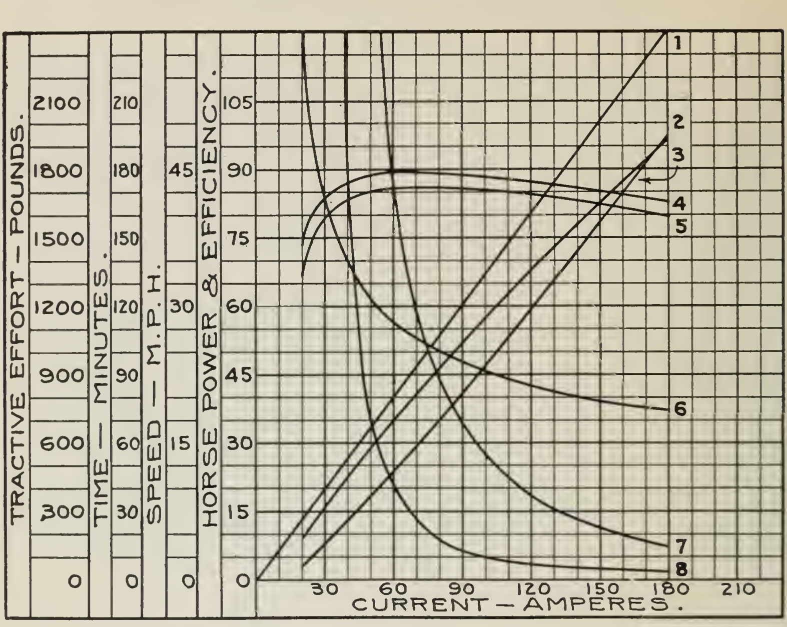


FIG. 8. THE CHARACTERISTIC CURVES OF THE TEST CAR MOTORS.

WESTINGHOUSE NO. 101-D, 500 VOLT D. C. RAILWAY MOTOR.
GEAR RATIO 22:62. 33 INCH WHEELS.
CONTINUOUS CAPACITY 46 AMPERES AT 300 VOLTS.
CONTINUOUS CAPACITY 42 AMPERES AT 400 VOLTS.

1. ELECTRICAL HORSE POWER.
2. BRAKE HORSE POWER WITH GEARS.
3. TRACTIVE EFFORT.
4. EFFICIENCY WITHOUT GEARS.
5. APPROXIMATE EFFICIENCY WITH GEARS.
6. SPEED.
7. TIME TO RISE 75° C. FROM 25° C.
8. SAFE TIME FOR LOAD IN SERVICE FOR 20° C. RISE FROM 75° C.

## THE RECORDING APPARATUS.

Within the car there is apparatus by means of which a continuous graphical record may be kept of motor current, voltage, speed, time, distance, location, and brake cylinder pressure upon a chart 40 inches in width.

*The Chart.*—Fig. 9 has been reproduced from a tracing of a portion of the chart for test No. 121. The records there shown are grouped more closely together than in the original record, and the transverse lines which mark one of the sections selected for calculation and some of the explanatory lettering do not appear thereon. This chart is

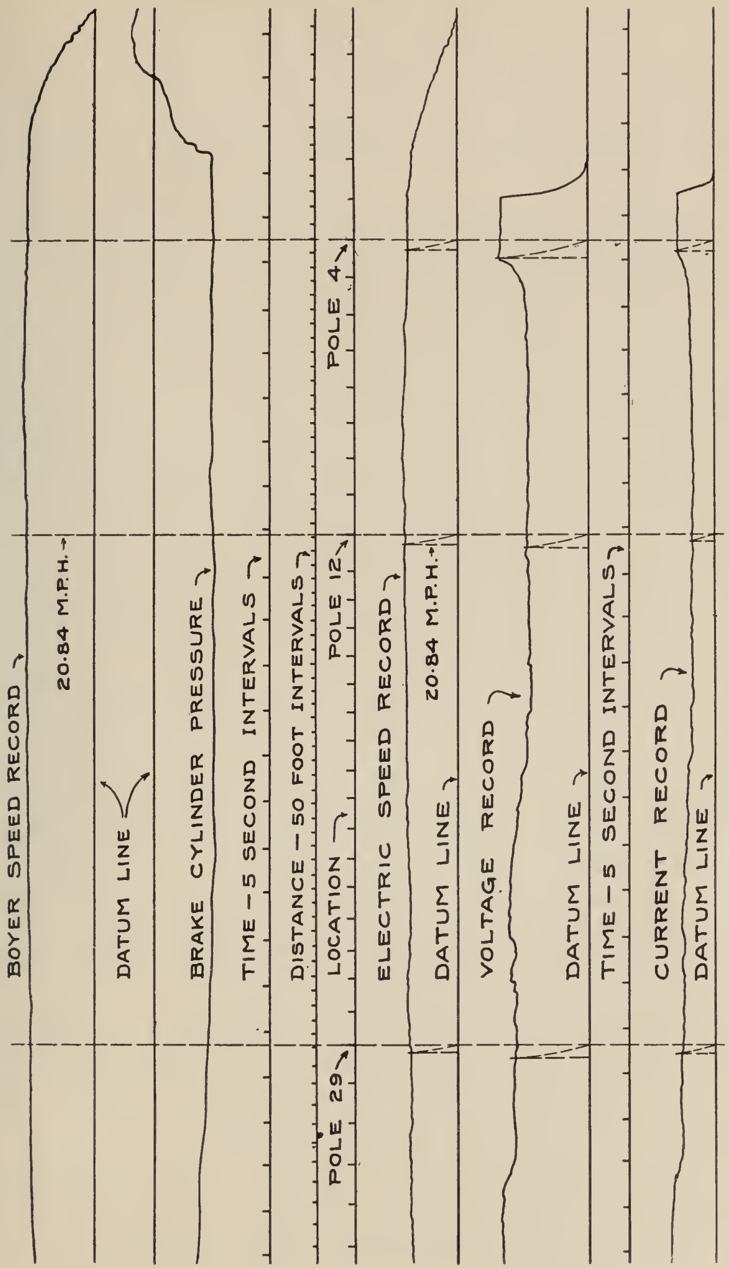


Fig. 9. A Portion of the Chart from Test 121.

caused to pass from a supply roll, over a series of tension and guide rolls and the recording table, and is then rewound upon a detachable record roll. As the paper travels over the table the instruments draw on it continuous records of the above mentioned data. The paper may be driven by a motor, thus making the records on a time base, or by a flexible shaft geared to the axle, thereby making them on a distance base.

*Current Record.*—The current taken by the motors is recorded by a General Electric graphic recording ammeter which is excited from a storage battery. A rheostat is provided for the adjustment of the exciting current which should be kept at a constant predetermined value. Check readings may be made by means of an indicating ammeter in the same circuit as the recording meter.

*Voltage Record.*—The voltage record is made by a General Electric graphic recording voltmeter which is likewise excited by a constant current from the storage battery. The same provision is made for making check readings as described for the current record.

Prior to test 121-122 these instruments were connected so as to give a record of line voltage, but for that and all subsequent tests the connections were changed so that the record is that of voltage across the terminals of one motor.

*Speed Record.*—One speed record is made by a General Electric graphic recording ammeter connected in the armature circuit of a ½ kilowatt low voltage generator which is excited by a constant storage battery current and driven from the axle by a flexible shaft. The current generated is proportional to speed and therefore the recording ammeter when properly calibrated gives a record of speed.

A Boyer mechanical speed recorder was installed just prior to test 103-104 and its record has been used on all subsequent tests.

Instantaneous values of speed are obtained from an "Autometer," which also is driven from the flexible shaft.

*Time Record.*—By means of a contacting device on a clock, electrical connections are made every 5 seconds so as to operate two magnets to the armatures of which the time recording pens are attached. Offsets are thus made in the two time lines every 5 seconds.

*Location Record.*—An observer, by means of a telegrapher's key, battery circuit, and magnet to the armature of which a recording pen is attached, causes offsets to be made in the line giving a record of location as a given point on the test car passes numbered trolley line poles, stations, or other markers. This record enables one to correlate any position of the car with the profile.

*Distance Record.*—A pen operated electrically from a contacting device on one of the car wheels makes an offset in a line for every 50 feet of track passed over by the car. This record is used merely as a check on the distances obtained from a table giving the exact distance between any two trolley line poles.

*Brake Cylinder Pressure.*—A pressure gage connected to the brake cylinder has been modified so as to make a record upon the chart. This record is used merely to determine and to avoid those portions of tests during which the brakes were applied.

*Auxiliary Apparatus.*—The power consumed by the motors may be obtained from an integrating wattmeter. A separate wattmeter indicates the power consumed by the pump motors. For a part of the tests herein reported two water rheostats were placed in the trolley circuit for the purpose of cutting down the voltage and thereby obtaining lower speeds than it was otherwise possible to maintain.

# APPENDIX II.

## METHODS EMPLOYED IN CALCULATING THE RESULTS.

The steps in the process by which the final results have been reached are here explained in some detail. The description of the general process which has been given in Section IV of the body of the report is to be considered as supplementary to this Appendix. The data taken and the methods by which they were obtained have been described there and in Appendix I. Fig. 9 has been reproduced from a portion of the chart made by the recording apparatus during test number 121.

*Selection of the Sections.*—From the chart produced during each test, sections have been chosen for calculation by comparison of the profile and chart. As has been stated, only those sections were considered where the car was running on straight track, and over which no brake applications were made. They were selected so that the average speeds over the sections chosen from any particular test should cover as wide a range as possible, and so that the number of sections from east bound runs would be balanced by the same or approximately the same number from west bound runs. In this way the number of resistance determinations made from data obtained during runs against the wind were balanced by an approximately equal number for the opposite condition. In addition, the following were considered sufficient reasons for the rejection of any section of chart:

a. Any considerable variation of the current or voltage from the average value.

b. Heavy grades between the section limits. Level track or light uniform grades are of course the preferable conditions.

c. A large difference in the speed of the car when entering and when leaving the track section or at any points between the section limits.

Aside from other considerations it is highly desirable to choose both track and chart sections so that the energy consumed by acceleration and grade shall be as small as possible, in order that errors in their calculation shall have slight effect on the accuracy of the values of average net car resistance. When the variation in speed is large the energy required to produce acceleration is alone frequently greater than that required to overcome all other resistances combined.

Fig. 9 shows one of the chart sections selected from a run made over a track section lying between trolley line poles numbered 4 and 12. These limiting points are designated on Fig. 9 by numbered offsets in the location record. Parallel lines drawn through them and perpendicular to the location record line intercept all but four of the records at the points occupied by the pens at the moment when the car entered and left the track section. The four records in question, namely, current, voltage, brake cylinder pressure, and speed by the electrical recorder, are made by pens which move along arcs of circles and which are adjusted so that in their zero position their recording points lie in the same straight line as the other pens. For any but the zero reading, however, these pens take positions ahead of the others and consequently allowance must be made for this fact in determining their positions at the moments when the car passed the points of entrance and exit. This matter has been cared for by means of templates, as shown in Fig. 9.

*The Calculations.*—The result desired from each such section of chart is a value of average net car resistance. The following explanation of the derivation of each column of data in the tables should make clear all the processes involved in the production of the final results.

Table 1 gives a summary of test conditions. Tables 3 to 12 inclusive present original data, and intermediate and final results as follows:

*Column 1.* (Item No.) is self explanatory.

*Column 2.* (Test No.) is self explanatory.

*Column 3.* (Wind opposing or helping) is from original data.

*Column 4.* (Section limits) defines the track section as well as the chart section by stating the limiting points which have been designated by trolley line pole numbers. The location of each line pole or other marker is shown accurately upon the profiles.

*Column 5.* (Grade) is derived from column 4 and the profiles, and expresses the grade as the difference in elevation in feet between the entrance and exit points. A positive sign indicates that the car is going up grade.

*Column 6.* (Length of section) is derived from column 4 and a table giving the exact distance between any two trolley line poles.

*Column 7.* (Time) is derived from the time record on the chart by determining the number of whole and decimal parts of 5-second intervals included between the parallel lines drawn through the limiting points of the section.

*Column 8.* (Motor data) is from the original data.

*Column 9.* (Efficiency) is derived from columns 8, 13 and 14, and a table of efficiencies.

Tables 3 to 12 inclusive, in Appendix III, show that the voltage (column 13) varied from about 100 to 600 volts. The low voltages were due largely to the method of using rheostats on the car for the purpose of obtaining low speeds. It was thought that the efficiency curve for the motors at 500 volts was not accurate enough under such conditions and therefore one of the motors was removed from the truck and a series of efficiency tests made upon it at five different voltages, namely, 120, 200, 300, 400, and 500. From the results of these tests a set of efficiency curves for the test car motors with gears was constructed, the Westinghouse Electric & Machine Company's standard gear losses being used. The 500-volt curve, thus determined, corresponds with the curve provided by the manufacturer. From these curves the above mentioned table was compiled.

*Column 10.* (Item No.) is self explanatory.

*Column 11.* (Speed at entrance to the section) The height of the speed curve above its base line at the point occupied by the recording pen at the moment the car entered the track section represents the speed of the car at that moment, to some scale determined by a calibration curve. Such a curve was made for each test by plotting average speeds calculated from columns 6 and 7 against the corresponding average heights of the speed curve in sufficient number to determine a curve. Straight lines were found to satisfy the conditions and for each one the equation was found and used in the calculations.

*Column 12.* (Speed at exit from the section) is determined from the height of the speed curve at the point of exit, and the equation of the calibration curve as explained above.

*Column 13.* (Average voltage) is determined by first obtaining the average height of the voltage curve between the section limits. From a calibration table the voltage corresponding to the average height may be found and it is recorded in column 13.

*Column 14.* (Average current) is determined from the current curve in the same manner as column 13 from the voltage curve.

*Column 15.* (Energy imparted to the car by the current) is calculated by means of the equation—

$$E_c = W \times e \times T \times 2655.$$

in which $E_c$ = the energy in foot pounds imparted to the car by the current.

$W$ = Average watts, calculated from columns 13 and 14.

$e$ = The efficiency of the motors and gears for the given current and voltage (Column 9).

$T$ = The time in hours for the car to traverse the section = $\frac{\text{column 7}}{3600}$

2655 = foot pounds in one watt hour.

*Column 16.* (Energy imparted to the car by the change in kinetic energy between the entrance and exit points) is determined from the entrance and exit speeds (columns 11 and 12), and an equation expressing the relation between kinetic energy and the speed of the car. This equation was derived from a series of determinations of the force required to produce acceleration in the rotation of the revolving parts as well as of the force required to produce the acceleration in the motion of translation of the car as a whole. The equation follows:

$$E_k = (1995 + 5P)\ (S_1^2 - S_2^2)$$

in which $E_k$ = The energy in foot pounds imparted to the car by the change in kinetic energy between the entrance and exit points.

$S_1$ = The speed of the car in miles per hour at the moment of entering the track section (column 11).

$S_2$ = The speed of the car in miles per hour at the moment of leaving the track section (column 12).

$P$ = The number of additional 150 pounds above the weight of the car and its regular equipment, that is above 55 150 pounds. This item includes live load and special equipment.

1995 and 5 are constants derived from the inertia calculations referred to above.

A negative sign before the figures in column 16 indicates that the speed of the car had increased in passing over the track section and consequently that the kinetic energy possessed by the car at the moment it left the section was greater than when it entered it by the amount given. This energy had to be supplied by the current, by the grade, or by both current and grade.

*Column 17.* (Energy imparted to the car by the grade) is the product of the weight of the car (column 2, Table 1) and the difference in elevation in feet between the points of entrance and exit. A negative sign indicates that the car is going up grade and is consequently requiring additional energy for that purpose.

*Column 18.* (Net car resistance in pounds per ton)—The net energy used in overcoming car resistance alone while traversing the track section is first found from columns 15, 16 and 17 by correcting the energy delivered by the current, for changes in speed and grade. From the net energy the average value for the section is found by dividing by the length of that section in feet (column 6), and this in turn is reduced to net car resistance in pounds per ton of car weight by dividing by the weight of the car in tons. This is the result desired from each chart section.

*Column 19.* (Average speed) is calculated from the length of the track section (column 6) and the time required to traverse that distance (column 7). Columns 18 and 19 provide the co-ordinates of the points plotted in Figs. 10 to 19.

*Method of Obtaining the Mean Curves of Car Resistance for Tests of Groups A and B.*—As described on page 13, the ordinates of points on the mean curves for the individual tests of groups A and B have been calculated by means of an equation which is based on the assumption that wind resistance varies as the square of the speed. The derivation of the equation follows:

Let $R_s$ = the resistance at speed $s$ excluding air resistance

$$R = \text{the total resistance at speed } s = R_s + cs^2 \tag{1}$$

in which $c$ = some constant quantity

$R_a$ = the resistance for a run against the wind at speed $s$

$$= R_s + c\,(s + v)^2 \tag{2}$$

in which $v$ = the component of the wind velocity parallel to the track.

$R_w$ = the resistance for a run at speed $s$ with the wind

$$= R_s + c\,(s - v)^2 \tag{3}$$

Subtracting equation (3) from (2) and solving for $c$

$$c = \frac{R_a - R_w}{4\,sv} \tag{4}$$

Substituting (4) in equation (2) and solving for $R_s$

$$R_s = R_a - \frac{R_a - R_w}{4\,sv}\,(s + v)^2 \tag{5}$$

Substituting equation (5) in equation (1)

$$R = R_a - \frac{R_a - R_w}{4\,s}\,(2s + v) \tag{6}$$

Equation (6) is the one sought. The mean curves of groups A and B, calculated by means of this equation, and those of group C, determined as described on page 16, after being modified as described on page 14 to conform to the general shape, were accepted as the results desired from the individual tests.

# APPENDIX III.

## The Results of the Individual Tests.

Note.—In the figures solid dots are for runs with the wind, and open circles are for runs against the wind.

## TABLE 3

### TEST NO. 15-16.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Motor Data | |
| Item No. | Test No. | Wind Opposing or Helping | Section Limits. Trolley Line Pole Numbers. | Grade. Rise or Fall Over Section. + Up — Down | Length of Track Section | Time to Run Over Section | Number in Use and Connection * | Efficiency of Motors and Gears |
| | | O or H | | Feet | Feet | Sec's. | | % |
| 1 | 15 | H | 1 620–1 575 | +13.20 | 4 009 | 77.21 | 4 M | 85.1 |
| 2 | " | " | 1 470–1 440 | + 4.17 | 2 991 | 72.14 | " | 78.4 |
| 3 | " | " | 1 440–1 380 | — 5.34 | 5 994 | 127.94 | " | 79.6 |
| 4 | " | " | 1 380–1 345 | —10.15 | 3 503 | 63.14 | " | 77.5 |
| 5 | " | " | 1 335–1 305 | — 3.45 | 2 986 | 53.30 | " | 81.0 |
| 6 | " | " | 1 295–1 270 | + 4.05 | 2 495 | 42.50 | " | 81.4 |
| 7 | " | " | 1 250–1 230 | — 3.74 | 1 987 | 32.20 | " | 82.4 |
| 8 | " | " | 1 070– 970 | — 9.92 | 9 987 | 173.99 | " | 81.2 |
| 9 | 16 | O | 970–1 000 | + 4.23 | 2 958 | 61.88 | " | 82.0 |
| 10 | " | " | 1 000–1 035 | + 3.53 | 3 480 | 69.21 | " | 82.2 |
| 11 | " | " | 1 035–1 070 | + 2.15 | 3 549 | 66.02 | " | 82.9 |
| 12 | " | " | 1 115–1 135 | — 1.19 | 1 988 | 38.47 | " | 85.4 |
| 13 | " | " | 1 270–1 295 | — 4.05 | 2 495 | 42.82 | " | 82.4 |
| 14 | " | " | 1 305–1 330 | + 3.41 | 2 488 | 43.66 | " | 81.8 |
| 15 | " | " | 1 400–1 430 | + 1.57 | 2 998 | 67.76 | " | 80.7 |
| 16 | " | " | 1 455–1 480 | + 0.42 | 2 492 | 52.40 | " | 81.0 |
| 17 | " | " | 1 480–1 535 | +17.12 | 5 473 | 110.77 | " | 81.4 |
| 18 | " | " | 1 580–1 620 | —14.74 | 3 576 | 67.54 | " | 81.4 |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

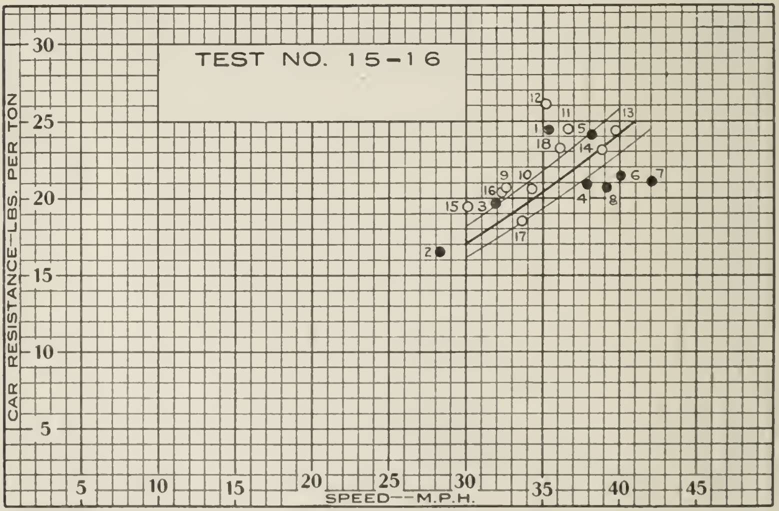


FIG. 10.

## TABLE 3 (Continued)

### TEST NO. 15-16.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed | | Average Voltage | Average Current | Energy Imparted to Car | | | Net Car Resistance | Average Speed Over the Section |
| | At Entrance to Section | At Exit From Section | | | By the Current | By the Change in Kinetic Energy | By the Grade | | |
| | M. P. H. | M. P. H. | Volts | Amp's. | Ft. Lbs. | Ft. Lbs. | Ft. Lbs. | Lbs. Per Ton | M. P. H. |
| 1 | 33.46 | 38.32 | 510 | 171.7 | 4 243 500 | — 720 400 | —747 100 | 24.46 | 35.42 |
| 2 | 26.65 | 29.24 | 351 | 132.1 | 1 934 200 | — 298 900 | —236 000 | 16.51 | 28.27 |
| 3 | 29.24 | 35.08 | 394 | 128.4 | 3 800 000 | — 775 700 | +302 200 | 19.60 | 31.95 |
| 4 | 35.08 | 36.37 | 411 | 113.4 | 1 682 000 | — 190 300 | +574 500 | 20.84 | 37.83 |
| 5 | 37.99 | 38.64 | 469 | 129.9 | 1 940 000 | — 102 860 | +195 300 | 24.05 | 38.20 |
| 6 | 39.94 | 39.29 | 494 | 129.9 | 1 637 000 | + 106 350 | —229 200 | 21.44 | 40.04 |
| 7 | 37.99 | 40.58 | 534 | 133.3 | 1 392 800 | — 420 200 | +211 700 | 21.06 | 42.07 |
| 8 | 34.10 | 41.23 | 467 | 131.5 | 6 398 500 | —1 109 100 | +561 500 | 20.69 | 39.15 |
| 9 | 29.57 | 32.16 | 436 | 141.1 | 2 302 000 | — 330 150 | —239 400 | 20.69 | 32.59 |
| 10 | 32.16 | 35.40 | 452 | 141.1 | 2 676 000 | — 452 000 | —199 800 | 20.55 | 34.28 |
| 11 | 35.40 | 37.02 | 492 | 142.0 | 2 820 000 | — 242 300 | —121 700 | 24.45 | 36.64 |
| 12 | 29.89 | 36.37 | 554 | 170.1 | 2 283 000 | — 886 600 | + 67 350 | 26.01 | 35.24 |
| 13 | 36.70 | 38.96 | 523 | 135.1 | 1 839 000 | — 353 100 | +229 200 | 24.28 | 39.73 |
| 14 | 37.99 | 37.34 | 492 | 132.6 | 1 718 000 | + 101 100 | —193 000 | 23.09 | 38.85 |
| 15 | 27.62 | 31.19 | 388 | 138.8 | 2 172 000 | — 433 550 | — 88 860 | 19.44 | 30.18 |
| 16 | 28.60 | 31.19 | 422 | 134.6 | 1 778 000 | — 319 800 | — 23 770 | 20.34 | 32.43 |
| 17 | 31.19 | 32.16 | 436 | 136.6 | 3 961 000 | — 126 900 | —969 000 | 18.49 | 33.68 |
| 18 | 29.24 | 36.37 | 456 | 134.4 | 2 485 100 | — 966 000 | +834 300 | 23.25 | 36.10 |

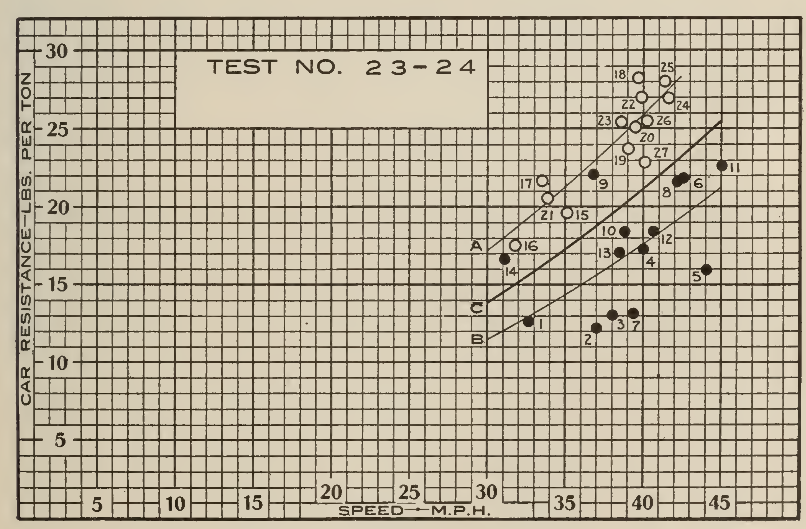


FIG. 11.

## TABLE 4

## TEST NO. 23-24.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
|  |  |  |  | Grade |  |  | Motor Data |  |
| Item No. | Test No. | Wind Opposing or Helping | Section Limits. — Trolley Line Pole Numbers. | Rise or Fall Over Section — + Up — Down | Length of Track Section | Time to Run Over Section | Number in Use and Connection * | Efficiency of Motors and Gears |
|  |  | O or H |  | Feet | Feet | Sec's |  | % |
| 1 | 23 | H | 1 471 -1 462 | +0.2 | 897 | 18.71 | 4 M | 83.5 |
| 2 | “ | “ | 1 462 -1 453 | +0.6 | 896 | 16.50 | “ “ | 82.6 |
| 3 | “ | “ | 1 402.7-1 396.7 | —1.94 | 600 | 10.75 | “ “ | 76.6 |
| 4 | “ | “ | 1 394.2-1 388.2 | —0.16 | 594 | 10.12 | “ “ | 80.8 |
| 5 | “ | “ | 1 361 -1 343 | —1.49 | 1 804 | 27.91 | “ “ | 79.3 |
| 6 | “ | “ | 1 291 -1 282 | —0.06 | 901 | 14.42 | “ “ | 80.0 |
| 7 | “ | “ | 1 081 -1 075 | +0.78 | 582 | 10.07 | “ “ | 84.5 |
| 8 | “ | “ | 1 012 - 970 | —4.76 | 4 144 | 66.90 | “ “ | 81.8 |
| 9 | “ | “ | 917.2- 901.75 | +1.02 | 1 529 | 28.33 | “ “ | 83.3 |
| 10 | “ | “ | 871.5- 853.4 | +2.15 | 1 803 | 31.65 | “ “ | 79.9 |
| 11 | “ | “ | 780.4- 720.5 | —6.74 | 5 970 | 90.30 | “ “ | 83.4 |
| 12 | “ | “ | 567.4- 540.5 | +0.43 | 2 689 | 45.07 | “ “ | 82.9 |
| 13 | “ | “ | 489.3- 468.5 | +0.50 | 2 083 | 36.89 | “ “ | 81.2 |
| 14 | “ | “ | 190.5- 150.5 | —3.96 | 3 948 | 86.45 | “ “ | 80.0 |
| 15 | 24 | O | 168.8- 190.5 | +3.19 | 2 090 | 40.56 | “ “ | 83.2 |
| 16 | “ | “ | 403.1- 420.2 | +1.33 | 1 664 | 35.60 | “ “ | 83.1 |
| 17 | “ | “ | 459.5- 492.3 | +0.53 | 3 276 | 66.55 | “ “ | 80.5 |
| 18 | “ | “ | 550.5- 570.3 | —0.62 | 1 977 | 33.94 | “ “ | 82.5 |
| 19 | “ | “ | 625.3- 687.5 | +2.29 | 6 270 | 109.49 | “ “ | 84.3 |
| 20 | “ | “ | 726.3- 780.4 | +5.57 | 5 392 | 92.98 | “ “ | 84.0 |
| 21 | “ | “ | 1 039 -1 075 | +2.53 | 3 678 | 74.00 | “ “ | 83.3 |
| 22 | “ | “ | 1 111 -1 141 | —0.98 | 2 982 | 50.94 | “ “ | 84.5 |
| 23 | “ | “ | 1 226.5-1 251.5 | +5.48 | 2 483 | 43.86 | “ “ | 83.9 |
| 24 | “ | “ | 1 273 -1 297.5 | —2.46 | 2 443 | 40.00 | “ “ | 83.8 |
| 25 | “ | “ | 1 306.8-1 334 | +2.99 | 2 707 | 44.56 | “ “ | 83.7 |
| 26 | “ | “ | 1 343 -1 373 | +4.97 | 3 004 | 50.88 | “ “ | 82.2 |
| 27 | “ | “ | 1 406 -1 465 | —5.11 | 5 889 | 100.11 | “ “ | 82.0 |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

## TABLE 5

## TEST NO. 73-74.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  | Motor Data |  |
| Item No. | Test No. | Wind O or H | Section Limits | Grade | Length of Section | Time | No. and Connection * | Efficiency |
| 1 | 73 | H | 1 470-1 455 | +0.56 | 1 495 | 23.78 | 4 M | 81.1 |
| 2 | “ | “ | “ “ | “ | “ | 24.60 | 4 M | 81.9 |
| 3 | “ | “ | “ “ | “ | “ | 29.00 | 4 M | 80.4 |
| 4 | “ | “ | “ “ | “ | “ | 28.58 | 4 S | 84.7 |
| 5 | “ | “ | “ “ | “ | “ | 28.31 | 4 S | 84.7 |
| 6 | “ | “ | “ “ | “ | “ | 37.80 | C | —— |
| 7 | 74 | O | 1 455-1 470 | —0.56 | “ | 26.91 | 4 M | 82.7 |
| 8 | “ | “ | “ “ | “ | “ | 28.40 | 4 M | 83.3 |
| 9 | “ | “ | “ “ | “ | “ | 30.35 | 4 S | 85.5 |
| 10 | “ | “ | “ “ | “ | “ | 31.94 | 4 S | 85.8 |
| 11 | “ | “ | “ “ | “ | “ | 38.58 | C | —— |

*S=Series-multiple. M=Multiple. Sw=Switchicg. C=Coasting.

## TABLE 4 (Continued)

### TEST NO. 23-24.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| | Speed | | | | Energy Imparted to Car | | | | |
| Item No. | At Entrance to Section | At Exit from Section | Average Voltage | Average Current | By the Current | By the Change in Kinetic Energy | By the Grade | Net Car Resistance | Average Speed Over Section |
| | M. P. H. | M. P. H. | Volts | Amp's. | Ft. Lbs. | Ft. Lbs. | Ft. Lbs. | Lbs. Per Ton | M. P. H |
| 1 | 31.33 | 34.97 | 464 | 153.8 | 822 230 | — 493 520 | — 11 250 | 12.58 | 32.69 |
| 2 | 34.97 | 37.16 | 455 | 145.3 | 664 520 | — 323 040 | — 33 750 | 12.21 | 37.02 |
| 3 | 38.61 | 39.71 | 435 | 108.5 | 286 630 | — 176 180 | +109 120 | 13.01 | 38.05 |
| 4 | 39.52 | 40.07 | 482 | 127.0 | 369 150 | — 89 520 | + 9 000 | 17.28 | 40.02 |
| 5 | 43.53 | 44.62 | 477 | 118.5 | 922 650 | — 196 490 | + 83 810 | 15.96 | 44.05 |
| 6 | 46.99 | 46.81 | 503 | 120.3 | 514 820 | + 34 530 | + 3 380 | 21.81 | 42.60 |
| 7 | 39.34 | 40.98 | 528 | 159.4 | 528 160 | — 269 380 | — 43 870 | 13.13 | 39.40 |
| 8 | 41.71 | 44.62 | 529 | 129.5 | 2 764 800 | — 513 750 | +267 750 | 21.61 | 42.23 |
| 9 | 37.34 | 39.16 | 515 | 144.0 | 1 290 700 | — 284 730 | — 57 380 | 22.06 | 36.80 |
| 10 | 39.16 | 39.16 | 459 | 123.0 | 1 052 900 | 0 | —120 900 | 18.38 | 38.84 |
| 11 | 42.26 | 47.90 | 605 | 132.6 | 4 455 700 | —1 039 900 | +379 120 | 22.60 | 45.08 |
| 12 | 37.16 | 40.98 | 536 | 137.5 | 2 030 800 | — 610 420 | — 24 190 | 18.46 | 40.68 |
| 13 | 35.15 | 37.34 | 465 | 131.5 | 1 350 900 | — 324 650 | — 28 130 | 17.07 | 38.50 |
| 14 | 29.14 | 37.34 | 422 | 127.2 | 2 737 900 | —1 114 800 | +222 700 | 16.62 | 31.14 |
| 15 | 34.24 | 36.97 | 464 | 149.6 | 1 727 500 | — 397 550 | —179 400 | 19.57 | 35.13 |
| 16 | 29.14 | 33.70 | 448 | 151.4 | 1 479 800 | — 586 000 | — 74 810 | 17.50 | 31.85 |
| 17 | 32.42 | 33.70 | 426 | 130.8 | 2 201 500 | — 173 080 | — 29 810 | 21.69 | 33.57 |
| 18 | 38.61 | 38.25 | 526 | 136.0 | 1 477 200 | + 56 580 | + 34 880 | 28.21 | 39.71 |
| 19 | 33.15 | 41.89 | 548 | 151.6 | 5 655 200 | —1 341 200 | —128 800 | 23.73 | 39.04 |
| 20 | 37.34 | 40.62 | 548 | 146.9 | 4 636 900 | — 522 920 | —313 310 | 25.06 | 39.50 |
| 21 | 30.05 | 37.16 | 476 | 150.0 | 3 245 900 | — 977 230 | —142 300 | 20.55 | 33.89 |
| 22 | 37.89 | 40.98 | 571 | 149.4 | 2 708 100 | — 498 380 | + 55 130 | 27.00 | 39.89 |
| 23 | 39.16 | 39.52 | 536 | 147.1 | 2 139 800 | — 57 920 | —308 300 | 25.40 | 38.61 |
| 24 | 41.53 | 42.99 | 560 | 142.0 | 1 965 800 | — 252 350 | +138 400 | 26.95 | 41.64 |
| 25 | 42.80 | 41.89 | 548 | 142.4 | 2 146 500 | + 157 600 | —168 200 | 28.05 | 41.41 |
| 26 | 42.26 | 40.44 | 517 | 133.3 | 2 125 700 | + 307 800 | —279 560 | 25.49 | 40.26 |
| 27 | 38.25 | 42.07 | 515 | 132.2 | 4 121 800 | — 627 450 | +287 440 | 22.83 | 40.11 |

## TABLE 5 (Continued)

### TEST NO. 73-74.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| | Speed | | | | Energy Imparted to Car | | | | |
| Item No. | Entrance | Exit | Average Voltage | Average Current | By Current | By Kinetic Energy | By Grade | Net Car Resistance | Average Speed |
| 1 | 41.35 | 42.21 | 516 | 125.8 | 923 200 | —146 950 | —31 720 | 17.58 | 42.87 |
| 2 | 40.83 | 41.52 | 520 | 130.5 | 1 008 000 | —116 200 | " | 20.31 | 41.45 |
| 3 | 33.56 | 35.29 | 443 | 127.7 | 972 800 | —243 600 | " | 16.47 | 35.13 |
| 4 | 36.16 | 36.85 | 528 | 80.9 | 762 700 | —103 000 | " | 14.83 | 35.72 |
| 5 | 35.12 | 35.99 | 530 | 81.2 | 761 100 | —126 500 | " | 14.23 | 36.00 |
| 6 | 30.10 | 24.09 | — | — | — | +666 000 | " | 14.98 | 26.96 |
| 7 | 36.68 | 37.20 | 507 | 138.6 | 1 153 000 | — 78 560 | +31 720 | 26.12 | 37.87 |
| 8 | 34.60 | 36.33 | 506 | 144.9 | 1 279 000 | —250 950 | " | 25.02 | 35.89 |
| 9 | 32.52 | 33.39 | 530 | 92.1 | 934 100 | —117 250 | " | 20.02 | 33.58 |
| 10 | 30.97 | 32.87 | 528 | 98.0 | 1 046 000 | —248 050 | " | 19.59 | 31.91 |
| 11 | 30.10 | 24.05 | — | — | — | +669 950 | " | 16.56 | 26.42 |

## TABLE 6

## TEST NO. 77-78.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | Grade | | | Motor Data | |
| Item No. | Test No. | Wind Opposing or Helping | Section Limits. — Trolley Line Pole Numbers | Rise or Fall Over Section — + Up — Down | Length of Track Section | Time to Run Over Section | Number in Use and Connection * | Efficiency of Motors and Gears |
| | | O or H | | Feet | Feet | Sec's. | | % |
| 1 | 77 | H | 1 470–1 455 | +0.56 | 1 495 | 23.03 | 4 M | 78.5 |
| 2 | 78 | O | 1 455–1 470 | —0.56 | “ | 27.68 | 4 M | 82.3 |
| 3 | 77 | H | 1 470–1 455 | +0.56 | “ | 32.87 | 2 M | 85.0 |
| 4 | 78 | O | 1 455–1 470 | —0.56 | “ | 32.57 | 2 M | 85.7 |
| 5 | 77 | H | 1 470–1 455 | +0.56 | “ | 35.48 | 2 M | 81.6 |
| 6 | 78 | O | 1 455–1 470 | —0.56 | “ | 36.26 | 2 M | 78.9 |
| 7 | 77 | H | 1 470–1 455 | +0.56 | “ | 38.01 | 2 M | 79.3 |
| 8 | 77 | H | 1 470–1 455 | +0.56 | “ | 57.22 | C | |
| 9 | 78 | O | 1 455–1 470 | —0.56 | “ | 95.00 | C | |
| 10 | 77 | H | 1 470–1 455 | +0.56 | “ | 85.00 | C | |
| 11 | 78 | O | 1 455–1 470 | —0.56 | “ | 79.45 | C | |
| 12 | 77 | H | 1 470–1 455 | +0.56 | “ | 77.58 | C | |
| 13 | 78 | O | 1 455–1 470 | —0.56 | “ | 80.06 | C | |
| 14 | 78 | O | 1 455–1 470 | —0.56 | “ | 29.82 | 4 M | 82.2 |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

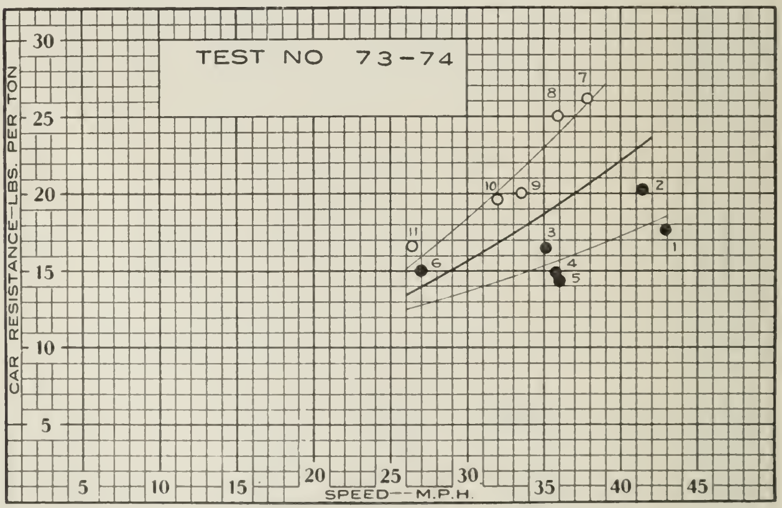


FIG. 12.

## TABLE 6 (Continued)

### TEST NO. 77-78.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed | | Average Voltage | Average Current | Energy Imparted to the Car | | | Net Car Resistance | Average Speed Over the Section |
| | At Entrance to Section | At Exit from Section | | | By the Current | By the Change in Kinetic Energy | By the Grade | | |
| | M. P. H. | M. P. H. | Volts | Amp's. | Ft. Lbs. | Ft. Lbs. | Ft. Lbs. | Lbs. Per Ton | M. P. H. |
| 1 | 45.22 | 44.46 | 507 | 112.4 | 759 750 | +139 720 | —31 810 | 20.44 | 44.25 |
| 2 | 35.48 | 37.01 | 499 | 136.8 | 1 146 960 | —227 360 | +31 810 | 22.41 | 36.82 |
| 3 | 29.94 | 31.85 | 480 | 89.5 | 885 210 | —241 940 | —31 810 | 14.40 | 31.01 |
| 4 | 34.34 | 36.05 | 507 | 98.7 | 1 029 860 | —246 750 | +31 810 | 19.19 | 31.29 |
| 5 | 28.61 | 29.94 | 510 | 64.9 | 706 830 | —159 640 | —31 810 | 12.14 | 28.72 |
| 6 | 27.84 | 26.31 | 499 | 57.6 | 606 320 | +169 840 | +31 810 | 19.03 | 28.11 |
| 7 | 26.12 | 25.17 | 420 | 61.1 | 570 050 | + 99 890 | —31 810 | 15.03 | 26.82 |
| 8 | 19.06 | 12.37 | | | | +431 050 | —31 810 | 9.40 | 17.81 |
| 9 | 14.85 | 6.64 | | | | +361 690 | +31 810 | 9.27 | 10.73 |
| 10 | 15.43 | 10.46 | | | | +263 780 | —31 810 | 5.46 | 11.99 |
| 11 | 17.15 | 8.55 | | | | +453 090 | +31 810 | 11.42 | 12.83 |
| 12 | 15.04 | 9.89 | | | | +263 200 | —31 810 | 5.45 | 13.14 |
| 13 | 17.72 | 8.93 | | | | +480 220 | +31 810 | 12.06 | 12.73 |
| 14 | 31.66 | 34.72 | 412 | 146.0 | 1 086 840 | —416 400 | +31 810 | 16.54 | 34.18 |

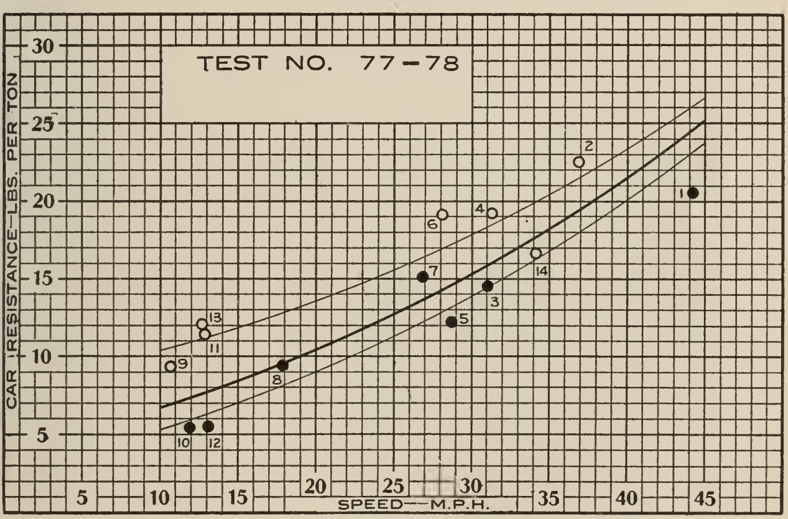


FIG. 13.

## TABLE 7

## TEST NO. 91-92.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Motor Data | |
| Item No. | Test No. | Wind Opposing or Helping | Section Limits. — Trolley Line Pole Numbers | Grade. — Rise or Fall Over Section. — + Up — Down | Length of Track Section | Time to Run Over Section | Number in Use and Connection * | Efficiency of Motors and Gears |
| | | O or H | | Feet | Feet | Sec's. | | % |
| 1 | 92 | O | 1 455–1 470 | —0.56 | 1 495 | 31.50 | 4 M | 81.5 |
| 2 | 91 | H | 1 470–1 455 | +0.56 | " | 25.61 | 4 M | 78.0 |
| 3 | 92 | O | 1 455–1 470 | —0.56 | " | 29.80 | 4 M | 82.0 |
| 4 | 91 | H | 1 470–1 455 | +0.56 | " | 30.35 | C | |
| 5 | 92 | O | 1 455–1 470 | —0.56 | " | 43.89 | C | |
| 6 | 91 | H | 1 470–1 455 | +0.56 | " | 36.46 | C | |
| 7 | 92 | O | 1 455–1 470 | —0.56 | " | 37.86 | C | |
| 8 | 91 | H | 1 470–1 455 | +0.56 | " | 44.96 | C | |
| 9 | 92 | O | 1 455–1 470 | —0.56 | " | 45.94 | C | |
| 10 | 91 | H | 1 470–1 455 | +0.56 | " | 41.47 | C | |
| 11 | 92 | O | 1 455–1 470 | —0.56 | " | 63.71 | C | |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

## TABLE 8

## TEST NO. 95-96.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Motor Data | |
| Item No. | Test No. | Wind O or H | Section Limits | Grade | Length of Section | Time | No. and Connection * | Efficiency |
| 1 | 95 | H | 1 470–1 455 | +0.56 | 1 495 | 21.97 | 4 M | 79.6 |
| 2 | 96 | O | 1 455–1 470 | —0.56 | " | 27.55 | " | 82.6 |
| 3 | 95 | H | 1 470–1 455 | +0.56 | " | 23.90 | " | 81.2 |
| 4 | 96 | O | 1 455–1 470 | —0.56 | " | 27.42 | " | 82.0 |
| 5 | 95 | H | 1 470–1 455 | +0.56 | " | 24.70 | " | 81.2 |
| 6 | 96 | O | 1 455–1 470 | —0.56 | " | 28.40 | " | 82.6 |
| 7 | 95 | H | 1 470–1 455 | +0.56 | " | 43.37 | 4 S | 72.9 |
| 8 | 96 | O | 1 455–1 470 | —0.56 | " | 58.10 | 4 S | 73.6 |
| 9 | 95 | H | 1 470–1 455 | +0.56 | " | 32.00 | 4 M | 83.0 |
| 10 | 96 | O | 1 455–1 470 | —0.56 | " | 33.10 | " | 83.1 |
| 11 | 95 | H | 1 470–1 455 | +0.56 | " | 27.50 | " | 80.5 |
| 12 | 96 | O | 1 455–1 470 | —0.56 | " | 30.20 | " | 82.7 |
| 13 | 95 | H | 1 470–1 455 | +0.56 | " | 38.05 | C | |
| 14 | 96 | O | 1 455–1 470 | —0.56 | " | 33.30 | C | |
| 15 | 95 | H | 1 470–1 455 | +0.56 | " | 37.25 | C | |
| 16 | 96 | O | 1 455–1 470 | —0.56 | " | 35.20 | C | |
| 17 | 95 | H | 1 470–1 455 | +0.56 | " | 48.05 | C | |
| 18 | 96 | O | 1 455–1 470 | —0.56 | " | 39.75 | C | |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

## TABLE 7 (Continued)

### TEST NO. 91-92.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed |  | Average Voltage | Average Current | Energy Imparted to the Car |  |  | Net Car Resistance | Average Speed Over the Section |
|  | At Entrance to Section | At Exit from Section |  |  | By the Current | By the Change in Kinetic Energy | By the Grade |  |  |
|  | M. P. H. | M. P. H. | Volts | Amp's. | Ft. Lbs. | Ft. Lbs. | Ft. Lbs. | Lbs. Per Ton | M. P. H. |
| 1 | 32.71 | 33.24 | 454 | 134.7 | 1 157 860 | — 71 130 | —31 560 | 25.05 | 32.36 |
| 2 | 40.52 | 41.41 | 472 | 112.5 | 782 280 | —148 390 | +31 560 | 15.80 | 39.82 |
| 3 | 34.13 | 34.48 | 472 | 137.1 | 1 166 190 | — 48 870 | —31 560 | 25.78 | 34.21 |
| 4 | 35.01 | 32.00 |  |  |  | +410 460 | +31 560 | 10.50 | 33.59 |
| 5 | 27.56 | 20.10 |  |  |  | +723 530 | —31 560 | 16.43 | 23.22 |
| 6 | 28.80 | 25.78 |  |  |  | +335 430 | +31 560 | 8.71 | 27.96 |
| 7 | 30.93 | 22.76 |  |  |  | +892 650 | —31 560 | 20.44 | 26.92 |
| 8 | 22.76 | 18.85 |  |  |  | +331 080 | +31 560 | 8.61 | 22.67 |
| 9 | 27.38 | 17.43 |  |  |  | +907 320 | —31 560 | 20.79 | 22.19 |
| 10 | 26.67 | 22.23 |  |  |  | +441 830 | +31 560 | 11.24 | 24.58 |
| 11 | 20.63 | 12.10 |  |  |  | +568 150 | —31 560 | 12.74 | 16.00 |

## TABLE 8 (Continued)

### TEST NO. 95-96.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed |  | Average Voltage | Average Current | Energy Imparted to Car |  |  | Net Car Resistance | Average Speed |
|  | Entrance | Exit |  |  | By Current | By Kinetic Energy | By Grade |  |  |
| 1 | 46.44 | 46.13 | 518 | 117.5 | 785 010 | + 58 398 | —31 556 | 19.27 | 46.40 |
| 2 | 37.87 | 37.24 | 499 | 138.7 | 1 161 600 | + 96 295 | +31 556 | 30.61 | 37.00 |
| 3 | 41.21 | 43.11 | 516 | 126.6 | 934 980 | —326 020 | —31 556 | 13.71 | 42.65 |
| 4 | 37.24 | 37.24 | 482 | 135.4 | 1 082 200 | 0 | +31 556 | 26.44 | 37.17 |
| 5 | 39.94 | 42.00 | 510 | 127.0 | 958 050 | —343 500 | —31 556 | 13.84 | 41.27 |
| 6 | 35.34 | 36.76 | 488 | 140.0 | 1 182 000 | —208 350 | +31 556 | 23.87 | 35.89 |
| 7 | 22.96 | 25.18 | 539 | 45.6 | 573 090 | —217 480 | —31 556 | 7.69 | 23.50 |
| 8 | 17.40 | 18.20 | 522 | 46.8 | 770 440 | — 57 957 | +31 556 | 17.66 | 17.54 |
| 9 | 27.72 | 33.75 | 450 | 150.4 | 1 325 700 | —754 300 | —31 556 | 12.82 | 31.85 |
| 10 | 29.62 | 33.27 | 452 | 151.2 | 1 386 400 | —467 130 | +31 556 | 22.57 | 30.80 |
| 11 | 35.97 | 37.56 | 477 | 125.4 | 976 580 | —237 920 | —31 556 | 16.79 | 37.07 |
| 12 | 32.16 | 33.75 | 477 | 142.7 | 1 253 800 | —213 260 | +31 556 | 25.45 | 33.75 |
| 13 | 30.26 | 26.45 |  |  |  | +439 690 | —31 556 | 9.69 | 26.79 |
| 14 | 34.54 | 26.61 |  |  |  | +986 810 | +31 556 | 24.18 | 30.61 |
| 15 | 29.31 | 25.02 |  |  |  | +474 310 | —31 556 | 10.51 | 27.36 |
| 16 | 32.32 | 25.34 |  |  |  | +819 020 | +31 556 | 20.19 | 28.96 |
| 17 | 22.96 | 18.83 |  |  |  | +351 230 | —31 556 | 7.59 | 21.21 |
| 18 | 28.35 | 20.89 |  |  |  | +747 520 | +31 556 | 18.50 | 25.64 |

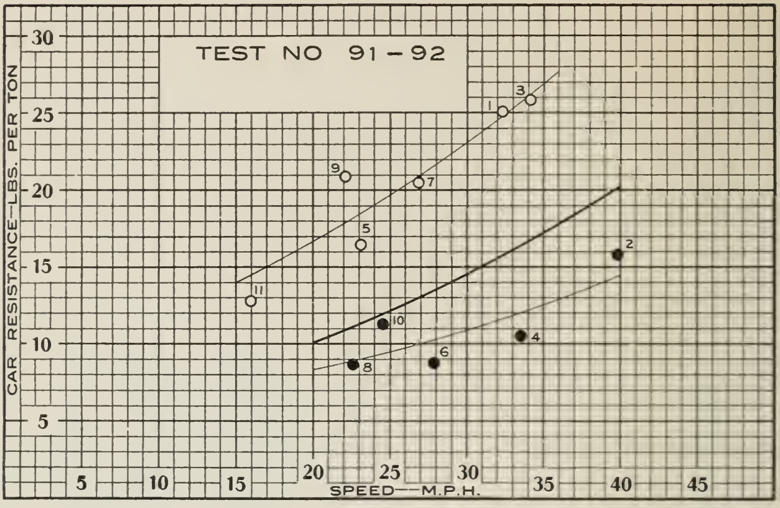

TEST NO 91 – 92
CAR RESISTANCE–LBS. PER TON
30
25
20
15
10
5
SPEED—M.P.H.
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11

FIG. 14.

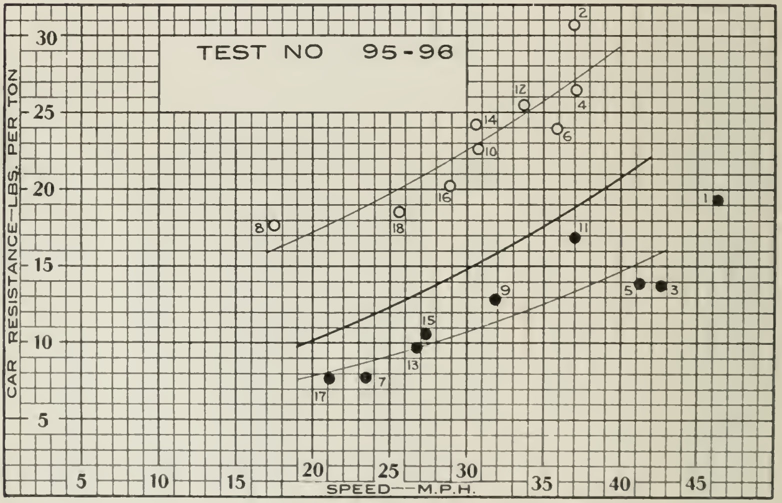
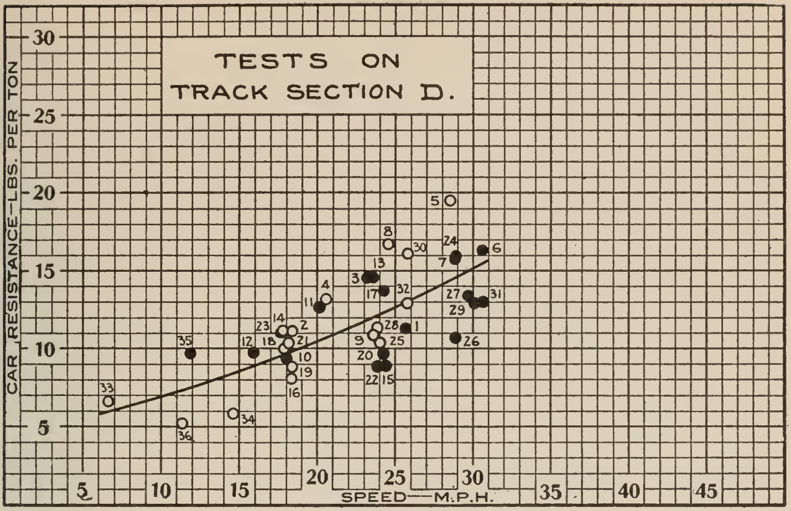

TEST NO 95-96
CAR RESISTANCE–LBS. PER TON
30
25
20
15
10
5
SPEED—M.P.H.
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18

FIG. 15.


TESTS ON
TRACK SECTION D.
CAR RESISTANCE—LBS. PER TON
30
25
20
15
10
5
SPEED—M.P.H.
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
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29
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32
33
34
35
36

FIG. 16.

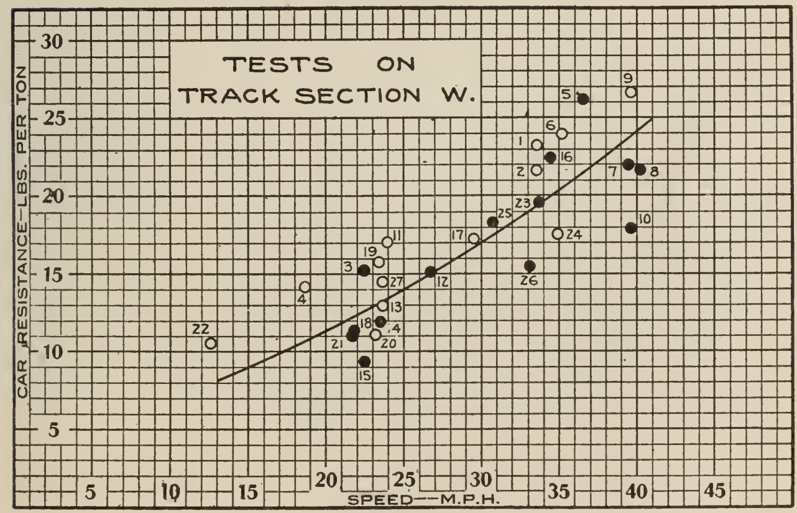
TESTS ON
TRACK SECTION W.
CAR RESISTANCE—LBS. PER TON
30
25
20
15
10
5
SPEED—M.P.H.
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27

FIG. 17.

## TABLE 9

## TESTS ON TRACK SECTION D.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | Grade | | | Motor Data | |
| Item No. | Test No. | Wind Opposing or Helping | Section Limits. — Trolley Line Pole Numbers | Rise or Fall Over Section — + Up — Down | Length of Track Section | Time to Run Over Section | Number in Use and Connection * | Efficiency of Motors and Gears |
| | | O or H | | Feet | Feet | Sec's. | | % |
| 1 | 109 | H | 1 571–1 566 | +0.37 | 445 | 11.8 | 4 S | 70.1 |
| 2 | 110 | O | 1 575–1 581 | +2.54 | 522 | 19.3 | 4 S | 81.4 |
| 3 | 109 | H | 1 571–1 563 | +0.42 | 714 | 20.9 | 4 S | 71.6 |
| 4 | 110 | O | 1 575–1 581 | +2.54 | 522 | 17.3 | 4 S | 82.5 |
| 5 | 110 | O | 1 575–1 581 | +2.54 | 522 | 12.5 | 4 M | 84.5 |
| 6 | 109 | H | 1 571–1 563 | +0.42 | 714 | 15.9 | 4 M | 79.9 |
| 7 | 109 | H | 1 571–1 563 | +0.42 | 714 | 16.9 | 4 M | 74.5 |
| 8 | 110 | O | 1 575–1 583 | +4.60 | 700 | 19.4 | 4 M | 80.9 |
| 9 | 110 | O | 1 575–1 583 | +4.60 | 700 | 20.2 | 1 | 86.0 |
| 10 | 109 | H | 1 571–1 566 | +0.37 | 445 | 16.8 | C | —— |
| 11 | 111 | H | 1 581–1 575 | —2.54 | 522 | 17.6 | C | —— |
| 12 | 111 | H | 1 581–1 575 | —2.54 | 522 | 22.3 | C | —— |
| 13 | 111 | H | 1 571–1 563 | +0.42 | 714 | 20.6 | 4 S | 73.9 |
| 14 | 112 | O | 1 575–1 581 | +2.54 | 522 | 20.0 | 4 S | 79.4 |
| 15 | 111 | H | 1 571–1 566 | +0.37 | 445 | 12.4 | 4 S | 70.5 |
| 16 | 112 | O | 1 575–1 581 | +2.54 | 522 | 19.4 | 4 S | 76.6 |
| 17 | 111 | H | 1 571–1 566 | +0.37 | 445 | 12.5 | 4 S | 68.7 |
| 18 | 112 | O | 1 575–1 583 | +4.60 | 700 | 26.7 | 4 S | 80.8 |
| 19 | 112 | O | 1 575–1 583 | +4.60 | 700 | 25.9 | 4 S | 79.5 |
| 20 | 111 | H | 1 571–1 566 | +0.37 | 445 | 12.5 | 4 S | 61.5 |
| 21 | 112 | O | 1 575–1 583 | +4.60 | 700 | 26.3 | 4 S | 79.6 |
| 22 | 111 | H | 1 571–1 566 | +0.37 | 445 | 12.7 | 4 S | 66.0 |
| 23 | 112 | O | 1 575–1 583 | +4.60 | 700 | 26.9 | 4 S | 78.5 |
| 24 | 111 | H | 1 571–1 566 | +0.37 | 445 | 10.5 | 4 M | 72.6 |
| 25 | 112 | O | 1 575–1 581 | +2.54 | 522 | 14.8 | 4 M | 82.1 |
| 26 | 111 | H | 1 571–1 566 | +0.37 | 445 | 10.5 | 4 M | 67.8 |
| 27 | 111 | H | 1 571–1 566 | +0.37 | 445 | 10.2 | 4 M | 73.8 |
| 28 | 112 | O | 1 575–1 583 | +4.60 | 700 | 20.0 | 4 M | 74 5 |
| 29 | 111 | H | 1 571–1 566 | +0.37 | 445 | 10.1 | 4 M | 73.6 |
| 30 | 112 | O | 1 525–1 583 | +4.60 | 700 | 18.5 | 4 M | 80.8 |
| 31 | 111 | H | 1 571–1 566 | +0.37 | 445 | 9.9 | 4 M | 73.8 |
| 32 | 112 | O | 1 575–1 583 | +4.60 | 700 | 18.5 | 4 M | 81.5 |
| 33 | 114 | O | 1 575–1 578 | +0.48 | 256 | 26.3 | 2 M | 71.0 |
| 34 | 114 | O | 1 566–1 571 | —0.37 | 445 | 20.7 | 2 M | 65.0 |
| 35 | 113 | H | 1 571–1 566 | +0.37 | 445 | 25.5 | 2 M | 66.7 |
| 36 | 114 | O | 1 566–1 571 | —0.37 | 445 | 26 7 | 2 M | 70.5 |

*S = Series-multiple. M = Multiple. Sw = Switching. C = Coasting.

## TABLE 9 (Continued)

## TESTS ON TRACK SECTION D.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed | | Average Voltage | Average Current | Energy Imparted to Car | | | Net Car Resistance | Average Speed Over the Section |
| | At Entrance to the Section | At Exit from the Section | | | By the Current | By the Change in Kinetic Energy | By the Grade | | |
| | M. P. H. | M. P. H. | Volts | Amp's. | Ft. Lbs. | Ft. Lbs. | Ft. Lbs. | Lbs. Per Ton | M. P. H. |
| 1 | 24.30 | 23.94 | 492 | 42.9 | 128 600 | + 35 700 | — 21 070 | 11.30 | 25.72 |
| 2 | 16.74 | 17.64 | 496 | 65.0 | 373 400 | — 63 580 | —144 700 | 11.11 | 18.44 |
| 3 | 22.50 | 21.60 | 472 | 45.5 | 237 200 | + 81 560 | — 23 920 | 14.50 | 23.23 |
| 4 | 19.80 | 20.34 | 565 | 64.8 | 385 300 | — 44 530 | —144 700 | 13.19 | 20.57 |
| 5 | 27.36 | 28.98 | 460 | 174.0 | 622 000 | —187 600 | —144 700 | 19.49 | 28.56 |
| 6 | 31.50 | 32.76 | 449 | 124.0 | 521 600 | —166 400 | — 23 920 | 16.29 | 30.60 |
| 7 | 27.90 | 27.90 | 344 | 107.5 | 343 200 | 0 | — 23 920 | 15.71 | 28.80 |
| 8 | 23.94 | 24.30 | 368 | 148.0 | 630 300 | — 35 700 | —262 000 | 16.68 | 24.59 |
| 9 | 22.86 | 22.86 | 484 | 77.2 | 478 600 | 0 | —262 000 | 10.86 | 23.61 |
| 10 | 19.80 | 18.00 | | | | +139 800 | — 21 070 | 9.36 | 18.03 |
| 11 | 19.62 | 19.08 | | | | + 42 510 | +143 100 | 12.62 | 20.22 |
| 12 | 15.30 | 15.30 | | | | 0 | +143 100 | 9.72 | 15.95 |
| 13 | 23.04 | 22.50 | 496 | 47.8 | 266 100 | + 50 040 | — 23 670 | 14.54 | 23.62 |
| 14 | 17.64 | 18.00 | 475 | 59.9 | 333 000 | — 26 110 | —143 100 | 11.13 | 17.78 |
| 15 | 24.12 | 24.12 | 467 | 43.9 | 132 100 | 0 | — 20 850 | 8.88 | 24.46 |
| 16 | 18.00 | 18.00 | 443 | 54.0 | 262 000 | 0 | —143 100 | 8.09 | 18.33 |
| 17 | 24.12 | 23.40 | 460 | 42.2 | 122 800 | + 69 620 | — 20 850 | 13.68 | 24.26 |
| 18 | 17.64 | 18.00 | 477 | 63.6 | 482 400 | — 26 110 | —259 200 | 10.00 | 17.87 |
| 19 | 18.36 | 18.36 | 475 | 59.9 | 431 900 | 0 | —259 200 | 8.76 | 18.42 |
| 20 | 23.22 | 22.68 | 417 | 38.8 | 91 680 | + 50 450 | — 20 850 | 9.67 | 24.27 |
| 21 | 18.00 | 17.64 | 467 | 60.6 | 436 800 | + 26 110 | —259 200 | 10.32 | 18.13 |
| 22 | 23.94 | 23.76 | 463 | 39.8 | 113 800 | + 17 460 | — 20 850 | 8.81 | 23.88 |
| 23 | 18.36 | 17.46 | 454 | 58.1 | 410 500 | + 65 600 | —259 200 | 11.00 | 17.74 |
| 24 | 28.44 | 28.26 | 337 | 105.5 | 200 000 | + 20 800 | — 20 850 | 15.94 | 28.88 |
| 25 | 22.14 | 24.66 | 372 | 160.4 | 535 000 | —240 000 | —143 100 | 10.32 | 24.03 |
| 26 | 27.72 | 27.72 | 306 | 96.2 | 154 400 | 0 | — 20 850 | 10.65 | 28.88 |
| 27 | 30.06 | 30.24 | 365 | 104.0 | 210 800 | — 22 100 | — 20 850 | 13.38 | 29.72 |
| 28 | 23.22 | 24.30 | 358 | 149.0 | 586 400 | —104 400 | —259 200 | 11.30 | 23.85 |
| 29 | 29.34 | 29.52 | 352 | 106.0 | 204 600 | — 21 550 | — 20 850 | 12.93 | 30.02 |
| 30 | 25.20 | 25.38 | 381 | 141.5 | 594 500 | — 18 520 | —259 200 | 16.06 | 25.79 |
| 31 | 30.78 | 30.96 | 365 | 105.0 | 206 500 | — 22 600 | — 20 850 | 13.00 | 30.63 |
| 32 | 24.66 | 25.74 | 387 | 145.0 | 624 000 | —110 800 | —259 200 | 12.88 | 25.80 |
| 33 | 5.76 | 6.66 | 113 | 62.3 | 97 020 | — 22 700 | — 26 980 | 6.58 | 6.64 |
| 34 | 13.32 | 13.68 | 160 | 45.1 | 71 640 | — 19 730 | + 20 790 | 5.81 | 14.64 |
| 35 | 11.34 | 10.08 | 142 | 48.9 | 87 180 | + 54 810 | — 20 790 | 9.69 | 11.89 |
| 36 | 8.64 | 10.44 | 149 | 55.4 | 114 300 | — 69 710 | + 20 790 | 5.23 | 11.35 |

## TABLE 10

## TESTS ON TRACK SECTION W.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Motor Data | |
| Item No. | Test No. | Wind O or H | Section Limits | Grade | Length of Section | Time | Number and Connection * | Efficiency |
| 1 | 118 | O | 7-16 | +1.37 | 1 189 | 24.1 | 4 M | 78.4 |
| 2 | 118 | O | 21-30 | —0.40 | 1 154 | 23.4 | 4 M | 81.0 |
| 3 | 117 | H | 30-21 | +0.40 | 1 154 | 35.1 | 4 S | 63.1 |
| 4 | 118 | O | 7-16 | +1.37 | 1 189 | 43.4 | 4 S | 70.0 |
| 5 | 117 | H | 16- 7 | —1.37 | 1 189 | 22.1 | 4 M | 83.5 |
| 6 | 118 | O | 21-30 | —0.40 | 1 154 | 22.3 | 4 M | 80.1 |
| 7 | 117 | H | 16- 7 | —1.37 | 1 189 | 20.5 | 4 M | 78.2 |
| 8 | 117 | H | 16- 7 | —1.37 | 1 189 | 20.1 | 4 M | 79.2 |
| 9 | 118 | O | 21-30 | —0.40 | 1 154 | 19.8 | 4 M | 83.0 |
| 10 | 117 | H | 16- 7 | —1.37 | 1 189 | 20.4 | 4 M | 78.3 |
| 11 | 118 | O | 21-30 | —0.40 | 1 154 | 32.7 | 4 S | 73.7 |
| 12 | 117 | H | 30-21 | +0.40 | 1 154 | 29.4 | 4 S | 70.0 |
| 13 | 123 | O | 30-21 | +0.40 | 1 154 | 33.2 | 4 S | 70.6 |
| 14 | 124 | H | 21-30 | —0.40 | 1 154 | 33.4 | 4 S | 65.4 |
| 15 | 124 | H | 7-16 | +1.37 | 1 189 | 35.9 | 4 S | 66.8 |
| 16 | 124 | H | 21-30 | —0.40 | 1 154 | 22.8 | 4 M | 81.9 |
| 17 | 123 | O | 30-21 | +0.40 | 1 154 | 36.6 | 4 M | 82.9 |
| 18 | 124 | H | 21-30 | —0.40 | 1 154 | 36.0 | 4 S | 67.4 |
| 19 | 123 | O | 16- 7 | —1.37 | 1 189 | 34.5 | 4 S | 72.0 |
| 20 | 123 | O | 30-21 | +0.40 | 1 154 | 33.8 | 4 S | 74.7 |
| 21 | 124 | H | 21-30 | —0.40 | 1 154 | 36.2 | 4 S | 66.9 |
| 22 | 123 | O | 16- 7 | —1.37 | 1 189 | 64.1 | 4 Sw | 61.3 |
| 23 | 124 | H | 21-30 | —0.40 | 1 154 | 32.3 | 4 M | 77.9 |
| 24 | 123 | O | 30-21 | +0.40 | 1 154 | 22.5 | 4 M | 78.5 |
| 25 | 124 | H | 21-30 | —0.40 | 1 154 | 25.5 | 4 M | 74.3 |
| 26 | 124 | H | 21-30 | —0.40 | 1 154 | 23.7 | 4 M | 74.9 |
| 27 | 123 | O | 16- 7 | —1.37 | 1 189 | 34.2 | 4 S | 70.7 |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

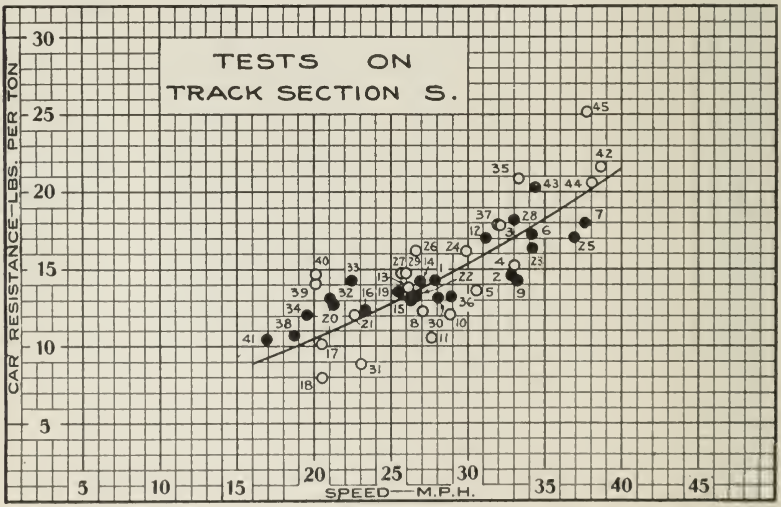

FIG. 18.

## TABLE 10 (Continued)

## TESTS ON TRACK SECTION W.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed | | Average Voltage | Average Current | Energy Imparted to the Car | | | Net Car Resistance | Average Speed |
| | Entrance | Exit | | | By Current | By Kinetic Energy | By Grade | | |
| 1 | 35.28 | 34.02 | 407 | 119.0 | 674 850 | +177 260 | —76 990 | 23.20 | 33.61 |
| 2 | 33.48 | 34.56 | 449 | 131.7 | 826 550 | —149 170 | +22 480 | 21.58 | 33.60 |
| 3 | 23.76 | 21.24 | 415 | 42.5 | 288 140 | +230 200 | —22 480 | 15.22 | 22.41 |
| 4 | 19.44 | 18.54 | 399 | 52.0 | 464 910 | + 69 390 | —76 990 | 14.10 | 18.66 |
| 5 | 36.18 | 38.34 | 537 | 153.5 | 1 121 800 | —326 750 | +76 990 | 26.10 | 36.65 |
| 6 | 36.00 | 36.00 | 454 | 125.8 | 752 340 | 0 | +22 480 | 23.89 | 35.27 |
| 7 | 40.68 | 40.50 | 467 | 113.5 | 626 610 | + 29 660 | +76 990 | 21.95 | 39.51 |
| 8 | 42.12 | 42.30 | 492 | 116.8 | 674 720 | — 30 850 | +76 990 | 21.58 | 40.31 |
| 9 | 40.32 | 41.04 | 555 | 142.5 | 958 580 | —118 920 | +22 480 | 26.59 | 39.70 |
| 10 | 40.32 | 41.04 | 473 | 114.5 | 638 020 | —118 920 | +76 990 | 17.84 | 39.72 |
| 11 | 23.76 | 24.30 | 555 | 59.0 | 582 090 | — 52 680 | +22 480 | 17.01 | 24.05 |
| 12 | 27.18 | 26.64 | 592 | 50.3 | 451 990 | + 59 000 | —22 480 | 15.06 | 26.75 |
| 13 | 23.40 | 23.58 | 256 | 52.3 | 462 930 | — 17 340 | —22 700 | 12.92 | 23.69 |
| 14 | 23.04 | 22.50 | 225 | 44.7 | 324 130 | + 42 510 | +22 700 | 11.89 | 23.54 |
| 15 | 20.70 | 20.34 | 220 | 46.6 | 362 570 | + 30 290 | —77 750 | 9.34 | 22.56 |
| 16 | 34.20 | 35.28 | 460 | 136.7 | 865 940 | —153 830 | +22 700 | 22.44 | 34.47 |
| 17 | 25.74 | 29.88 | 427 | 152.5 | 1 059 000 | —472 050 | —22 700 | 17.23 | 29.55 |
| 18 | 20.70 | 21.06 | 222 | 47.5 | 377 400 | — 30 820 | +22 700 | 11.28 | 21.83 |
| 19 | 22.50 | 23.40 | 268 | 54.8 | 538 040 | — 84 690 | +77 750 | 15.74 | 23.47 |
| 20 | 20.34 | 23.40 | 290 | 60.9 | 657 690 | —274 380 | —22 700 | 11.01 | 23.25 |
| 21 | 20.52 | 20.88 | 220 | 46.8 | 367 750 | — 30 550 | +22 700 | 10.99 | 21.72 |
| 22 | 11.34 | 11.70 | 119 | 42.5 | 293 270 | — 17 000 | +77 750 | 10.49 | 12.63 |
| 23 | 32.76 | 32.76 | 397 | 116.0 | 616 430 | 0 | +22 700 | 19.52 | 33.74 |
| 24 | 33.30 | 33.66 | 417 | 118.8 | 645 310 | — 49 420 | —22 700 | 17.50 | 34.95 |
| 25 | 30.96 | 30.60 | 346 | 109.6 | 529 860 | + 45 430 | +22 700 | 18.26 | 30.83 |
| 26 | 32.04 | 32.40 | 372 | 108.6 | 528 900 | — 47 560 | +22 700 | 15.39 | 33.17 |
| 27 | 22.68 | 23.40 | 254 | 52.7 | 477 370 | — 68 010 | +77 750 | 14.43 | 23.69 |

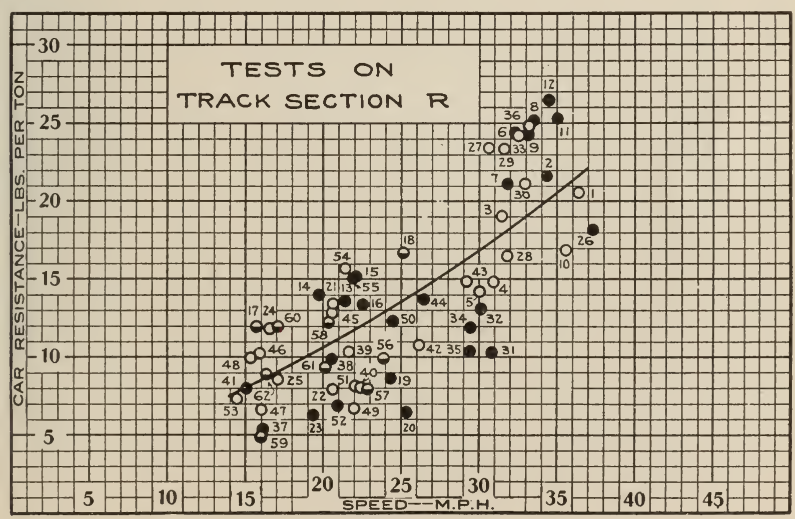

FIG. 19.

Note.—Solid dots are for runs with the wind, open circles are for runs against the wind, and circles with the lower half black are for runs in no wind.

## TABLE 11

## TESTS ON TRACK SECTION S.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| Item No. | Test No. | Wind Opposing or Helping | Section Limits. — Trolley Line Pole Numbers | Grade — Rise or Fall Over Section — + Up — Down | Length of Section | Time to Run Over Section | Motor Data | |
| | | | | | | | Number in Use and Connection * | Efficiency of Motors and Gears |
| | | O or H | | Feet | Feet | Sec's. | | % |
| 1 | 119 | H | 37–29 | +0.76 | 1 057 | 25.9 | 4 M | 68.3 |
| 2 | 119 | H | 12– 4 | —3.40 | 1 054 | 21.9 | “ | 60.4 |
| 3 | 120 | O | 29–37 | —0.76 | 1 057 | 22.4 | “ | 80.2 |
| 4 | 120 | O | 4–12 | +3.40 | 1 054 | 21.8 | “ | 60.1 |
| 5 | 120 | O | 29–36 | —0.54 | 928 | 20.7 | “ | 76.0 |
| 6 | 119 | H | 37–29 | +0.76 | 1 057 | 21.1 | “ | 85.1 |
| 7 | 119 | H | 12– 4 | —3.40 | 1 054 | 19.1 | “ | 67.2 |
| 8 | 120 | O | 29–37 | —0.76 | 1 057 | 26.6 | “ | 70.4 |
| 9 | 119 | H | 12– 4 | —3.40 | 1 054 | 21.6 | “ | 63.4 |
| 10 | 120 | O | 29–37 | —0.76 | 1 057 | 25.0 | “ | 71.0 |
| 11 | 120 | O | 4–12 | +3.40 | 1 054 | 26.0 | “ | 67.8 |
| 12 | 119 | H | 12– 4 | —3.40 | 1 054 | 23.0 | 2 M | 73.1 |
| 13 | 120 | O | 29–37 | —0.76 | 1 057 | 27.5 | “ | 84.7 |
| 14 | 119 | H | 12– 4 | —3.40 | 1 054 | 26.7 | “ | 74.3 |
| 15 | 119 | H | 12– 4 | —3.40 | 1 054 | 27.3 | 4 S | 70.5 |
| 16 | 119 | H | 37–29 | +0.76 | 1 057 | 30.9 | “ | 77.2 |
| 17 | 120 | O | 29–37 | —0.76 | 1 057 | 35.1 | “ | 74.9 |
| 18 | 120 | O | 4–12 | +3.40 | 1 054 | 35.0 | “ | 72.0 |
| 19 | 119 | H | 12– 4 | —3.40 | 1 054 | 28.2 | “ | 67.8 |
| 20 | 119 | H | 37–29 | +0.76 | 1 057 | 33.9 | “ | 79.3 |
| 21 | 120 | O | 4–12 | +3.40 | 1 054 | 31.7 | “ | 76.8 |
| 22 | 119 | H | 12– 4 | —3.40 | 1 054 | 26.9 | “ | 69.9 |
| 23 | 119 | H | 12– 4 | —3.40 | 1 054 | 21.0 | 4 M | 63.3 |
| 24 | 120 | O | 29–37 | —0.76 | 1 057 | 24.1 | “ | 78.1 |
| 25 | 119 | H | 12– 4 | —3.40 | 1 054 | 19.5 | “ | 68.7 |
| 26 | 120 | O | 29–37 | —0.76 | 1 057 | 26.7 | 2 M | 82.8 |
| 27 | 120 | O | 4–12 | +3.40 | 1 054 | 28.0 | “ | 84.8 |
| 28 | 119 | H | 12– 4 | —3.40 | 1 054 | 21.8 | “ | 80.8 |
| 29 | 120 | O | 4–12 | +3.40 | 1 054 | 27.7 | “ | 84.9 |
| 30 | 119 | H | 12– 4 | —3.40 | 1 054 | 25.6 | 4 S | 64.6 |
| 31 | 120 | O | 4–12 | +3.40 | 1 054 | 31.3 | “ | 71.9 |
| 32 | 122 | H | 29–37 | —0.76 | 1 057 | 34.3 | “ | 68.7 |
| 33 | 122 | H | 4–12 | +3.40 | 1 054 | 32.0 | “ | 70.7 |
| 34 | 122 | H | 29–37 | —0.76 | 1 057 | 36.9 | “ | 70.2 |
| 35 | 121 | O | 12– 4 | —3.40 | 1 054 | 21.6 | 4 M | 72.5 |
| 36 | 122 | H | 29–37 | —0.76 | 1 057 | 25.0 | “ | 73.6 |
| 37 | 121 | O | 12– 4 | —3.40 | 1 054 | 22.5 | “ | 62.2 |
| 38 | 122 | H | 29–37 | —0.76 | 1 057 | 38.5 | 4 Sw | 67.0 |
| 39 | 121 | O | 37–29 | +0.76 | 1 057 | 35.8 | “ | 72.6 |
| 40 | 121 | O | 12– 4 | —3.40 | 1 054 | 35.7 | “ | 56.0 |
| 41 | 122 | H | 29–37 | —0.76 | 1 057 | 42.5 | 4 S | 68.8 |
| 42 | 121 | O | 12– 4 | —3.40 | 1 054 | 18.6 | 4 M | 80.9 |
| 43 | 122 | H | 4–12 | +3.40 | 1 054 | 20.9 | “ | 82.7 |
| 44 | 121 | O | 12– 4 | —3.40 | 1 054 | 18.9 | “ | 66.6 |
| 45 | 121 | O | 37–29 | +0.76 | 1 057 | 19.1 | “ | 82.3 |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

## TABLE 11 (Continued)

### TESTS ON TRACK SECTION S.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| | Speed | | | | Energy Imparted to the Car | | | | |
| Item No. | At Entrance to the Section | At Exit from the Section | Average Voltage | Average Current | By the Current | By the Change in Kinetic Energy | By the Grade | Net Car Resistance | Average Speed Over the Section |
| | M. P. H. | M. P. H. | Volts | Amps. | Ft. Lbs. | Ft. Lbs. | Ft. Lbs. | Lbs. Per Ton | M. P. H. |
| 1 | 28.05 | 26.98 | 280 | 95.7 | 349 380 | +120 710 | — 43 130 | 14.23 | 27.84 |
| 2 | 32.20 | 32.01 | 282 | 79.2 | 217 740 | + 25 000 | +192 900 | 14.57 | 32.85 |
| 3 | 31.47 | 33.10 | 405 | 131.5 | 705 910 | —215 760 | + 43 130 | 17.78 | 32.20 |
| 4 | 34.36 | 31.11 | 284 | 77.0 | 211 130 | +436 200 | —192 900 | 15.19 | 32.99 |
| 5 | 29.12 | 30.40 | 361 | 115.0 | 482 080 | —156 180 | + 30 650 | 13.55 | 30.56 |
| 6 | 31.48 | 35.97 | 524 | 170.0 | 1 180 600 | —620 840 | — 43 130 | 17.23 | 34.15 |
| 7 | 37.40 | 37.04 | 354 | 86.6 | 289 940 | + 54 940 | +192 900 | 17.98 | 37.62 |
| 8 | 26.08 | 26.98 | 300 | 102.0 | 422 600 | — 97 900 | + 43 130 | 12.26 | 27.10 |
| 9 | 33.10 | 33.28 | 310 | 82.0 | 256 530 | — 24 490 | +192 900 | 14.21 | 33.27 |
| 10 | 27.88 | 28.77 | 316 | 102.0 | 422 170 | —103 360 | + 43 130 | 12.07 | 28.81 |
| 11 | 28.22 | 26.80 | 284 | 94.0 | 346 860 | +160 160 | —192 900 | 10.50 | 27.67 |
| 12 | 31.11 | 30.40 | 350 | 52.2 | 226 290 | + 89 530 | +192 900 | 17.01 | 31.22 |
| 13 | 24.62 | 27.32 | 407 | 94.1 | 657 240 | —287 490 | + 43 130 | 13.77 | 26.20 |
| 14 | 25.89 | 26.25 | 319 | 57.9 | 270 080 | — 38 480 | +192 900 | 14.20 | 26.91 |
| 15 | 25.52 | 26.61 | 488 | 45.1 | 312 270 | —116 480 | +192 900 | 13.00 | 26.31 |
| 16 | 22.82 | 23.55 | 512 | 53.6 | 482 400 | — 69 390 | — 43 130 | 12.33 | 23.32 |
| 17 | 19.76 | 21.73 | 434 | 50.7 | 426 170 | —167 560 | + 43 130 | 10.06 | 20.52 |
| 18 | 20.68 | 19.76 | 405 | 46.9 | 352 740 | + 76 270 | —192 900 | 7.89 | 20.52 |
| 19 | 24.99 | 25.52 | 452 | 41.7 | 265 660 | — 54 880 | +192 900 | 13.50 | 25.50 |
| 20 | 20.30 | 22.11 | 501 | 58.5 | 580 900 | —157 360 | — 43 130 | 12.68 | 21.26 |
| 21 | 22.82 | 21.91 | 490 | 53.3 | 468 620 | + 83 440 | —192 900 | 12.01 | 22.68 |
| 22 | 26.08 | 26.80 | 468 | 43.3 | 280 810 | — 78 050 | +192 900 | 13.23 | 26.72 |
| 23 | 33.81 | 33.45 | 308 | 81.6 | 246 270 | + 49 640 | +192 900 | 16.35 | 34.24 |
| 24 | 29.11 | 30.75 | 370 | 125.0 | 642 430 | —201 250 | + 43 130 | 16.15 | 29.92 |
| 25 | 36.33 | 36.33 | 356 | 90.0 | 316 350 | 0 | +192 900 | 17.03 | 36.89 |
| 26 | 26.25 | 26.80 | 393 | 78.6 | 503 150 | — 59 810 | + 43 130 | 16.22 | 26.72 |
| 27 | 24.44 | 24.99 | 417 | 94.4 | 689 070 | — 55 730 | —192 900 | 14.72 | 25.69 |
| 28 | 32.38 | 32.55 | 432 | 66.3 | 371 790 | — 22 630 | +192 900 | 18.13 | 33.00 |
| 29 | 24.62 | 25.18 | 419 | 95.0 | 689 940 | — 57 170 | —192 900 | 14.71 | 25.98 |
| 30 | 26.61 | 26.80 | 463 | 39.0 | 220 150 | — 20 800 | +192 900 | 13.12 | 28.10 |
| 31 | 22.63 | 21.39 | 454 | 45.8 | 344 890 | +111 900 | —192 900 | 8.82 | 23.08 |
| 32 | 20.48 | 20.48 | 202 | 49.7 | 348 960 | 0 | + 43 130 | 13.07 | 21.02 |
| 33 | 23.19 | 20.83 | 229 | 53.1 | 405 450 | +212 970 | —192 900 | 14.23 | 22.48 |
| 34 | 18.50 | 19.40 | 193 | 52.5 | 387 050 | — 69 930 | + 43 130 | 12.01 | 19.54 |
| 35 | 33.09 | 32.91 | 338 | 104.0 | 406 300 | + 24 350 | +192 900 | 20.85 | 33.30 |
| 36 | 26.98 | 28.05 | 301 | 116.0 | 473 860 | —120 710 | + 43 130 | 13.21 | 28.86 |
| 37 | 32.55 | 31.66 | 266 | 81.2 | 222 730 | +117 150 | +192 900 | 17.81 | 31.98 |
| 38 | 17.78 | 18.50 | 174 | 50.0 | 331 000 | — 53 550 | + 43 130 | 10.69 | 18.74 |
| 39 | 19.22 | 19.76 | 229 | 57.8 | 507 570 | — 43 150 | — 43 130 | 14.05 | 20.14 |
| 40 | 19.94 | 19.04 | 159 | 37.0 | 173 390 | + 71 920 | +192 900 | 14.65 | 20.13 |
| 41 | 15.62 | 17.24 | 174 | 50.8 | 381 270 | —109 130 | + 43 130 | 10.51 | 16.96 |
| 42 | 36.50 | 37.95 | 475 | 128.0 | 675 280 | —221 300 | +192 900 | 21.63 | 38.69 |
| 43 | 34.00 | 34.35 | 465 | 143.0 | 848 320 | — 49 040 | —192 900 | 20.27 | 34.42 |
| 44 | 38.68 | 37.78 | 355 | 85.4 | 281 280 | +141 070 | +192 900 | 20.57 | 38.08 |
| 45 | 38.31 | 38.31 | 505 | 136.0 | 796 440 | 0 | — 43 130 | 25.12 | 37.75 |

## TABLE 12

## TESTS ON TRACK SECTION R.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | Motor Data | |
| Item No. | Test No. | Wind O or H | Section Limits | Grade | Length of Section | Time | No. and Connection * | Efficiency |
| 1 | 125 | O | 14 - 4 | —2.62 | 1 320 | 24.7 | 4 M | 80.3 |
| 2 | 126 | H | 4 - 14 | +2.62 | 1 320 | 26.2 | 4 M | 78.9 |
| 3 | 125 | O | 45 - 38 | —2.93 | 920 | 19.9 | 4 M | 84.3 |
| 4 | 125 | O | 14 - 4 | —2.62 | 1 320 | 29.1 | 4 M | 73.7 |
| 5 | 125 | O | 45 - 38 | —2.93 | 920 | 20.9 | 4 M | 80.5 |
| 6 | 126 | H | 38 - 45 | +2.93 | 920 | 19.4 | 4 M | 83.8 |
| 7 | 126 | H | 4 - 14 | +2.62 | 1 320 | 28.3 | 4 M | 75.9 |
| 8 | 126 | H | 38 - 45 | +2.93 | 920 | 18.7 | 4 M | 85.1 |
| 9 | 126 | H | 4 - 14 | +2.62 | 1 320 | 27.1 | 4 M | 77.4 |
| 10 | 125 | O | 45 - 38 | —2.93 | 920 | 17.6 | 4 M | 83.8 |
| 11 | 126 | H | 38 - 45 | +2.93 | 920 | 17.9 | 4 M | 85.0 |
| 12 | 126 | H | 4 - 14 | +2.62 | 1 320 | 26.1 | 4 M | 80.2 |
| 13 | 126 | H | 38 - 45 | +2.93 | 920 | 29.3 | 4 S | 77.3 |
| 14 | 126 | H | 4 - 14 | +2.62 | 1 320 | 45.6 | 4 S | 71.4 |
| 15 | 126 | H | 4 - 14 | +2.62 | 1 320 | 40.9 | 4 S | 75.3 |
| 16 | 126 | H | 38 - 45 | +2.93 | 920 | 27.8 | 4 S | 76.3 |
| 17 | 128 | — | 4 - 14 | +2.62 | 1 320 | 57.1 | 4 S | 72.4 |
| 18 | 127 | — | 14 - 4 | —2.62 | 1 320 | 35.8 | 4 M | 62.4 |
| 19 | 129 | H | 14 - 4 | —2.62 | 1 320 | 36.9 | 4 S | 66.6 |
| 20 | 129 | H | 45 - 38 | —2.93 | 920 | 24.7 | 4 S | 73.5 |
| 21 | 130 | O | 4 - 14 | +2.62 | 1 320 | 43.6 | 4 S | 72.0 |
| 22 | 130 | O | 38 - 45 | +2.93 | 920 | 30.4 | 4 S | 63.3 |
| 23 | 129 | H | 14 - 4 | —2.62 | 1 320 | 46.5 | 4 S | 38.0 |
| 24 | 130 | O | 4 - 14 | +2.62 | 1 320 | 54.4 | 4 S | 72.5 |
| 25 | 130 | O | 38 - 45 | +2.93 | 920 | 36.8 | 4 S | 70.7 |
| 26 | 129 | H | 14 - 4 | —2.62 | 1 320 | 24.1 | 4 M | 76.0 |
| 27 | 130 | O | 4 - 14 | +2.62 | 1 320 | 29.4 | 4 M | 82.5 |
| 28 | 130 | O | 38 - 45 | +2.93 | 920 | 19.7 | 4 M | 82.9 |
| 29 | 130 | O | 38 - 45 | +2.93 | 920 | 19.8 | 4 M | 83.2 |
| 30 | 130 | O | 4 - 14 | +2.62 | 1 320 | 27.3 | 4 M | 84.0 |
| 31 | 129 | H | 45 - 38 | —2.93 | 920 | 20.3 | 4 M | 72.7 |
| 32 | 129 | H | 14 - 4 | —2.62 | 1 320 | 29.9 | 4 M | 46.7 |
| 33 | 130 | O | 4 - 14 | +2.62 | 1 320 | 27.7 | 4 M | 82.7 |
| 34 | 129 | H | 45 - 38 | —2.93 | 920 | 21.3 | 4 M | 45.8 |
| 35 | 129 | H | 14 - 4 | —2.62 | 1 320 | 30.6 | 4 M | 65.4 |
| 36 | 130 | O | 4 - 14 | +2.62 | 1 320 | 27.1 | 4 M | 83.4 |
| 37 | 129 | H | 27 - 24 | —0.26 | 372 | 16.7 | 4 S | 49.5 |
| 38 | 142 | H | 1 725 -1 730 | +1.63 | 519 | 17.2 | 4 S | 74.0 |
| 39 | 141 | O | 1 741 -1 737.5 | —1.90 | 338 | 10.6 | 4 S | 55.0 |
| 40 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 15.8 | 4 S | 65.4 |
| 41 | 142 | H | 1 737.5-1 741 | +1.90 | 338 | 15.3 | 4 S | 71.4 |
| 42 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 13.5 | 4 M | 44.0 |
| 43 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 12.1 | 4 M | 43.0 |
| 44 | 142 | H | 1 737.5-1 741 | +1.90 | 338 | 8.7 | 4 M | 69.4 |
| 45 | 141 | O | 1 741 -1 737.5 | —1.90 | 338 | 11.2 | 4 S | 69.3 |
| 46 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 22.2 | 4 S | 53.6 |
| 47 | 141 | O | 1 741 -1 737.5 | —1.90 | 333 | 14.4 | 4 S | 66.5 |
| 48 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 23.0 | 4 S | 43.0 |
| 49 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 16.1 | 4 S | 60.3 |
| 50 | 142 | H | 1 737.5-1 741 | +1.90 | 338 | 9.4 | 4 M | 77.7 |
| 51 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 16.0 | 4 S | 49.6 |
| 52 | 142 | H | 1 737.5-1 741 | +1.90 | 338 | 11.0 | 4 M | 63.1 |
| 53 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 24.4 | 4 S | 55.1 |
| 54 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 16.5 | 4 S | 58.4 |
| 55 | 142 | H | 1 737.5-1 741 | +1.90 | 338 | 10.5 | 4 M | 73.2 |
| 56 | 141 | O | 1 730 -1 725 | —1.63 | 519 | 14.8 | 4 M | 47.8 |
| 57 | 153 | — | 1 730 -1 725 | —1.63 | 519 | 15.5 | 4 S | 41.0 |
| 58 | 153 | — | 1 730 -1 725 | —1.63 | 519 | 17.4 | 4 S | 31.0 |
| 59 | 153 | — | 1 741 -1 737.5 | —1.90 | 338 | 14.4 | 4 S | 42.1 |
| 60 | 153 | — | 1 730 -1 725 | —1.63 | 519 | 15.7 | 4 S | 33.5 |
| 61 | 153 | — | 1 730 -1 725 | —1.63 | 519 | 17.5 | 4 S | 43.4 |
| 62 | 153 | — | 1 741 -1 737.5 | —1.90 | 338 | 14.1 | 4 S | 37.5 |

*S=Series-multiple. M=Multiple. Sw=Switching. C=Coasting.

## Tablf 12 (Continued)

## Tests on Track Section R.

| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|
| Item No. | Speed | | Average Voltage | Average Current | Energy Imparted to the Car | | | Net Car Resistance | Average Speed |
| | Entrance | Exit | | | By the Current | By the Kinetic Energy | By the Grade | | |
| 1 | 35.29 | 36.52 | 431 | 128.4 | 809 490 | —182 840 | +150 260 | 20.53 | 36.43 |
| 2 | 34.73 | 33.11 | 380 | 127.7 | 739 860 | +227 500 | —150 260 | 21.59 | 34.35 |
| 3 | 29.16 | 33.67 | 441 | 168.7 | 920 480 | —586 560 | +168 040 | 19.03 | 31.50 |
| 4 | 29.68 | 30.96 | 321 | 112.2 | 569 730 | —160 670 | +150 260 | 14.78 | 30.92 |
| 5 | 26.80 | 30.60 | 365 | 145.2 | 657 630 | —451 510 | +168 040 | 14.18 | 30.00 |
| 6 | 32.02 | 30.78 | 387 | 139.7 | 648 150 | +161 200 | —168 040 | 24.31 | 32.31 |
| 7 | 32.58 | 30.22 | 340 | 119.2 | 642 040 | +306 800 | —150 260 | 21.10 | 31.81 |
| 8 | 32.58 | 34.01 | 490 | 178.8 | 1 028 200 | —197 110 | —168 040 | 25.13 | 33.51 |
| 9 | 35.47 | 32.75 | 361 | 122.6 | 684 670 | +384 110 | —150 260 | 24.27 | 33.21 |
| 10 | 32.76 | 36.52 | 480 | 156.2 | 815 570 | —539 220 | +168 040 | 16.84 | 35.61 |
| 11 | 34.38 | 35.28 | 496 | 173.2 | 964 010 | —129 780 | —168 040 | 25.25 | 35.02 |
| 12 | 36.19 | 33.82 | 397 | 132.1 | 809 540 | +343 460 | —150 260 | 26.49 | 34.50 |
| 13 | 20.32 | 21.41 | 264 | 70.3 | 620 020 | — 94 160 | —168 040 | 13.56 | 21.39 |
| 14 | 21.22 | 19.61 | 205 | 55.1 | 542 410 | +136 070 | —150 260 | 13.96 | 19.74 |
| 15 | 21.78 | 22.31 | 262 | 64.8 | 771 350 | — 48 370 | —150 260 | 15.13 | 22.00 |
| 16 | 20.88 | 21.41 | 266 | 68.1 | 566 770 | — 46 400 | —168 040 | 13.36 | 22.55 |
| 17 | 15.42 | 16.31 | 182 | 59.8 | 663 740 | — 58 600 | —150 650 | 11.98 | 15.77 |
| 18 | 27.48 | 25.51 | 193 | 84.0 | 267 060 | +216 610 | +150 650 | 16.71 | 25.12 |
| 19 | 23.78 | 25.58 | 214 | 46.4 | 359 960 | —183 920 | +150 260 | 8.62 | 24.39 |
| 20 | 22.13 | 26.29 | 265 | 59.0 | 418 670 | —417 000 | +168 040 | 6.43 | 25.38 |
| 21 | 21.22 | 19.97 | 214 | 55.6 | 551 010 | +106 580 | —150 260 | 13.40 | 20.63 |
| 22 | 20.52 | 18.54 | 175 | 43.4 | 216 130 | +160 100 | —168 040 | 7.89 | 20.62 |
| 23 | 18.54 | 18.54 | 112 | 29.4 | 85 830 | 0 | +150 260 | 6.24 | 19.35 |
| 24 | 15.29 | 16.02 | 185 | 59.8 | 643 660 | — 47 310 | —150 260 | 11.78 | 16.54 |
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| 43 | 29.05 | 28.32 | 176 | 59.1 | 39 910 | + 87 320 | + 94 210 | 14.77 | 29.22 |
| 44 | 26.52 | 25.28 | 246 | 100.0 | 109 540 | +133 920 | —109 820 | 13.68 | 26.49 |
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| 46 | 15.18 | 15.36 | 109 | 36.5 | 69 830 | — 11 460 | + 94 210 | 10.17 | 15.94 |
| 47 | 13.92 | 16.08 | 129 | 49.2 | 89 620 | —135 110 | +109 820 | 6.58 | 16.00 |
| 48 | 15.90 | 15.54 | 84 | 24.8 | 30 390 | + 23 890 | + 94 210 | 9.90 | 15.37 |
| 49 | 20.78 | 21.83 | 170 | 40.7 | 99 090 | — 93 280 | + 94 210 | 6.66 | 21.98 |
| 50 | 26.35 | 26.35 | 298 | 143.0 | 229 520 | 0 | —109 820 | 12.25 | 24.51 |
| 51 | 22.02 | 22.38 | 155 | 33.6 | 60 990 | — 33 330 | + 94 210 | 8.12 | 22.10 |
| 52 | 21.12 | 20.04 | 191 | 85.6 | 83 700 | + 92 680 | —109 820 | 6.81 | 20.96 |
| 53 | 13.02 | 14.28 | 117 | 37.5 | 87 010 | — 71 720 | + 94 210 | 7.30 | 14.50 |
| 54 | 21.12 | 20.58 | 169 | 39.1 | 93 950 | + 46 950 | + 94 210 | 15.67 | 21.42 |
| 55 | 22.57 | 21.49 | 236 | 118.0 | 157 860 | + 99 210 | —109 820 | 15.08 | 21.92 |
| 56 | 23.47 | 23.47 | 159 | 65.1 | 54 160 | 0 | + 94 210 | 9.89 | 23.90 |
| 57 | 21.57 | 21.92 | 223 | 27.4 | 57 270 | — 31 740 | + 94 380 | 7.98 | 22.82 |
| 58 | 19.94 | 19.03 | 94 | 20.5 | 15 330 | + 73 940 | + 94 380 | 12.22 | 20.34 |
| 59 | 14.71 | 16.16 | 113 | 30.8 | 31 120 | — 93 330 | +110 010 | 4.88 | 16.00 |
| 60 | 22.11 | 21.39 | 115 | 22.5 | 20 070 | + 65 300 | + 94 380 | 11.96 | 17.12 |
| 61 | 19.94 | 19.94 | 131 | 31.0 | 45 490 | 0 | + 94 380 | 9.31 | 20.22 |
| 62 | 16.16 | 16.70 | 74 | 24.1 | 13 850 | — 37 000 | +110 010 | 8.88 | 16.33 |

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# THERMAL PROPERTIES OF STEAM

BY

G. A. GOODENOUGH

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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# THERMAL PROPERTIES OF STEAM

BY G. A. GOODENOUGH, PROFESSOR OF THERMODYNAMICS.

## CONTENTS

PAGE

I. Introduction ... 3
  1. Experimental Data on the Properties of Steam ... 3
  2. Purpose and Scope of the Investigation ... 4
  3. Acknowledgment ... 5
II. Thermodynamic Relations ... 5
  4. Notation ... 5
  5. Relations between Certain Thermal Properties ... 6
  6. General Equations ... 6
  7. Development of a General Theory ... 8
III. Review of Earlier Investigations ... 9
  8. Zeuner's Theory ... 9
  9. Callendar's Theory ... 11
  10. Davis's Investigations ... 13
  11. Jakob's Investigation ... 14
  12. Heck's Theory ... 15
  13. Other Investigations ... 16
IV. Development of a General Theory of the Properties of Steam ... 17
  14. Units and Constants Employed ... 17
 A. Relation between Temperature and Pressure of Saturated Steam ... 17
  15. Experimental Data ... 17
  16. Pressure-Temperature Formulas ... 19
    a. Dupré-Hertz Formula ... 20
    b. Callendar's Formula ... 20
    c. Bertrand's Formulas ... 20
    d. Marks' Formula ... 21
  17. Comparison of Formulas ... 23

Page
B. Volume of Superheated and Saturated Steam. Characteristic Equations ... 24
  18. Experimental Data ... 24
  19. Characteristic Equations ... 25
  20. Tests of the Characteristic Equation ... 29
C. Specific Heat of Superheated Steam ... 29
  21. Experimental Data ... 33
  22. Systems of Specific Heat Curves ... 36
  23. Equation for Specific Heat ... 37
D. Total Heat, Latent Heat, Heat of Liquid ... 42
  24. Equation for Heat Content ... 42
  25. Tests of the *i*-equation ... 43
  26. Specific Heat of Water. Heat of Liquid ... 48
  27. Latent Heat of Saturated Steam ... 52
  28. Experiments on Latent Heat ... 54
  29. Entropy ... 56
  30. Intrinsic Energy ... 57
E. Joule-Thomson Effect ... 58
  31. The Joule-Thomson Coefficient ... 58
  32. Comparison with the Throttling Experiments ... 58
F. Thermal Properties Near the Critical Temperature ... 61
  33. Previous Investigations ... 61
  34. Thiesen's Exponential Formula ... 61
  35. Heat Content of Liquid and Vapor ... 62
  36. Specific Volume of Steam ... 62
  37. Table of Properties at High Temperatures ... 64
V. Resumé of Formulas ... 64
VI. Conclusion ... 66
  38. Comparison of Tables and Formulations ... 66
  39. Effect of Future Experiments ... 68
  40. Summary ... 69

# THERMAL PROPERTIES OF STEAM.

## I. INTRODUCTION.

1. *Experimental Data on the Properties of Steam.*—The classic researches of Regnault gave the first reliable knowledge of the properties of steam. Although these experiments were made more than sixty-five years ago (most of Regnault's reports on the properties of steam bear the date 1847) with apparatus and methods incapable of the highest degree of accuracy, they are still deservedly held in high esteem, and until recently, Regnault's data formed the basis of all tabulations of the thermal properties of steam. Within the last ten years there has been a revival of interest in the subject and a number of important investigations, experimental and analytical, have been prosecuted. As a result, we have at present data of a high degree of accuracy covering nearly every phase of the work. The following is a summary of the more important of the experimental investigations:

a. The relation between the pressure and temperature of saturated steam has been established definitely by three series of experiments made respectively by Holborn and Henning, Holborn and Baumann, and Scheel and Heuse. The three series taken together cover the range 32° F. to the critical temperature. These experiments were conducted at the Reichsanstalt with all the resources afforded by modern apparatus and methods of precise measurement.

b. The relation between volume, pressure, and temperature of superheated steam has been determined by the experiments of Knoblauch, Linde, and Klebe at the Munich laboratory. These experiments afford satisfactory data for the range of pressure and superheat covered by them, but an extension over a much wider range should be made.

c. A number of experiments have been made to determine the specific heat of superheated steam. Of these, the experiments conducted in the Munich laboratory, first by Knoblauch and Jakob and afterward by Knoblauch and Mollier, are justly accepted as the most reliable. At the present time similar experiments covering a wider range of pressure are being made by Lanz and Schmidt.

d. The direct experiments of Griffiths, Joly, Smith, Henning, and Dieterici furnish data on the latent heat of saturated steam.

e. The variation of the specific heat of water has been the subject of several investigations. For the range 32°-212° F. the experiments of Barnes have been verified by those of Callendar, and they are generally accepted. Above 212° F. precise measurements of this important property are lacking. The only available experiments are those of Regnault and Dieterici, and neither of these can be accepted as thoroughly reliable.

f. Four sets of experiments on the throttling of steam by Grindley, Griessmann, Peake, and Dodge, respectively, furnish valuable data that may be used for various purposes. Thus Davis has made effective use of them in establishing the curve of total heat of saturated steam; and in the present investigation they are used as a check on the heat content of superheated steam.

2. *Purpose and Scope of the Investigation.*—The various thermal properties of a vapor are not independent. On the contrary, the equations that express the variations of these properties are related through certain well-known thermodynamic laws. The development of a general theory applicable to a vapor, as water vapor, involves the following steps: 1. The establishment of a system of equations to represent the various thermal magnitudes. The forms of the equations chosen must be such that it is possible by proper selection of arbitrary functions to make the equations satisfy the thermodynamic relations. 2. The comparison of the equations with experimental data and the adjustment of constants to give the best possible agreement. 3. The satisfaction of the various thermodynamic relations. With sufficiently accurate and complete experimental data it should be possible to work out such a theory and thus deduce a set of consistent equations giving all the thermal properties as functions of the independent variables chosen. At the present time the character of the experimental evidence is such as to justify a fresh attack on the problem. It is the purpose of this paper to exhibit the development of a general theory which apparently gives with extreme accuracy the properties of saturated and superheated steam over a range of pressure and temperature far wider than the range covered in technical applications.

In order to make the presentation more useful, and perhaps more intelligible, it has been considered advisable to include (1) a section on thermodynamic relations, and (2) a section devoted to a brief historical review of some of the more important analytical investigations.

3. *Acknowledgment.*—The very considerable labor of computation has been lightened by the assistance of a number of the writer's associates in the Department of Mechanical Engineering. In the earlier phases of the work Mr. A. Schaller and Mr. John A. Dent offered valuable suggestions and cheerfully assumed a considerable part of the work of computation. In the final revision, Mr. S. L. Simmering, fellow in the Engineering College, contributed largely to the progress of the investigation. Mr. G. Meyer, Jr., and Mr. T. E. Maury, senior students, assisted in some of the final computations, and Mr. Meyer made the drawings for the illustrations. The valuable assistance of these men is hereby gratefully acknowledged.

## II. Thermodynamic Relations.

4. *Notation.*—Throughout the discussion the following notation is used:

$J$ = mechanical equivalent of heat
$A = 1/J$, reciprocal of mechanical equivalent
$t$ = temperature on F. or C. scale
$T$ = absolute temperature
$p$ = pressure
$v$ = volume of unit weight (1 lb.) of fluid
$\gamma$ = weight of unit volume
$c_v$ = specific heat at constant volume
$c_p$ = specific heat at constant pressure
$u$ = intrinsic energy per unit weight
$q$ = heat absorbed by fluid per unit weight
$q'$ = heat of liquid
$q''$ = total heat of saturated steam
$i$ = heat content $= u + Apv$
$r$ = latent heat of vaporization
$\rho$ = internal latent heat
$\psi = Ap\,(v'' - v')$ = external latent heat
$s$ = entropy
$\mu$ = Joule-Thomson coefficient

The liquid state is characterized by a symbol with a prime, the state of saturated vapor by the same symbol with a double prime. Thus $s'$, $u'$, and $i'$ denote respectively the entropy, energy, and heat content of the liquid, $s''$, $u''$, $i''$, the same properties of the saturated vapor.

5. *Relations Between Certain Thermal Properties.*—The thermal properties of the liquid and saturated vapor are connected by relations that follow directly from the definition of the properties. Thus

$$r = \rho + \psi = \rho + Ap\,(v'' - v')$$

$$u'' = q' + \rho \qquad q'' = q' + r$$

$$s'' = s' + \frac{r}{T} \qquad i'' = i' + r$$

The heat content $i$ is defined by the relation

$$i = u + Apv$$

$u$ being expressed in thermal units; hence

$$i' = u' + Apv' \qquad i'' = u'' + Apv''.$$

The heat of the liquid is given by

$$q' = \int_{32}^{t} c'dt,$$

in which $c'$ denotes the specific heat of the liquid. If the heat is absorbed by the water at constant atmospheric pressure from 32° to 212°, then for this range

$$q' = i' = \int_{32}^{t} c'dt.$$

The entropy of the liquid is given by

$$s' = \int \frac{dq'}{T} = \int \frac{c'dt}{T},$$

the integral being taken between proper limits.

Above 212° F. a distinction between $q''$ and $i''$ must be observed. By definition

$$i'' = u'' + Apv''$$

and $$q'' = u'' + \psi = u'' + Apv'' - Apv'.$$

Hence $$i'' - q'' = Apv',$$

similarly $$i' - q' = Apv'.$$

At low temperatures the difference $Apv'$ is small and negligible. The term *total heat* is often used indiscriminately for $q''$ and $i''$; in this discussion, however, total heat refers to $q''$ and the name *heat content* is reserved for the property denoted by $i$.

6. *General Equations.*[1]—From the two laws of thermodynamics the following principal equations are deduced.

---

[1] Goodenough's Principles of Thermodynamics, Chap. IV.

$$dq = c_v dT + AT\left(\frac{\partial p}{\partial T}\right)_v dv, \tag{1}$$

$$dq = c_p dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp. \tag{2}$$

The energy equation

$$du = dq - Apdv$$

combined with (1) leads to the fundamental equation

$$du = c_v dT + A\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv; \tag{3}$$

and the equation of definition

$$di = dq + Avdp$$

combined with (2) gives an analogous equation

$$di = c_p dT - A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]dp. \tag{4}$$

The application of the criterion of an exact differential to (4) leads to the important Clausius relation

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p. \tag{5}$$

In the case of an isothermal process $dT = 0$ and eq. (4) becomes

$$\left(\frac{di}{dp}\right)_T = -A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]; \tag{6}$$

while for a throttling process, in which $i$ remains constant, (4) becomes

$$\left(\frac{dT}{dp}\right)_i = \frac{A}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]. \tag{7}$$

The derivative in the first member of (7) is the Joule-Thomson coefficient $\mu$.

At the saturation limit the thermal properties are connected by the Clapeyron-Clausius relation, namely

$$r = AT\,(v'' - v')\left(\frac{dp}{dt}\right)_{sat}. \tag{8}$$

The derivative $\frac{dp}{dt}$ is obtained from the relation $p = f(t)$ connecting the pressure and temperature of saturated steam.

The relations (5) and (8) are specially important in the development of any general theory of vapors. The equations that express the various thermal properties cannot be developed independently. They are tied together by these relations, which must be satisfied as well as the experimental data.

7. *Development of a General Theory.*—The Clausius relation may be made the basis of a set of equations for a superheated vapor. Let the relation between $p$, $v$, and $T$ be given by a characteristic equation

$$v = f(p, T), \tag{9}$$

and let the specific heat $c_p$ at constant pressure, which varies with both pressure and temperature, be expressed by

$$c_p = \phi\ (p, T). \tag{10}$$

Two differentiations of the first equation give the derivative $\frac{\partial^2 v}{\partial T^2}$ and one differentiation of the second equation gives the derivative $\frac{\partial c_p}{\partial p}$. The form of the function in (9) must be so chosen that the experimental measurements of the volume are satisfied, the function $\phi$ in (10) must likewise satisfy the measurements of specific heat, and furthermore the two functions must satisfy the Clausius relation, $-AT\frac{\partial^2 v}{\partial T^2} = \frac{\partial c_p}{\partial p}$. The development of the theory may proceed along either one of two lines of attack. (1) A charcteristic equation is assumed—this amounts to fixing the function $f$ in (9)—and the constants are determined so as to make the equation represent the volume measurements. Successive differentiation gives the second derivative $\frac{\partial^2 v}{\partial T^2}$, and this is substituted in the Clausius relation. The result is an equation of the form

$$\frac{\partial c_p}{\partial p} = \phi\ (p, T),$$

whence an expression for $c_p$ is found by an integration with respect to $p$. This will involve an arbitrary function of $T$ as a constant of integration. Finally the equation for $c_p$ is tested by comparison with experimental results. (2) The preceding method may be reversed. Starting with a system of $c_p$ curves laid down from the available experimental evidence, the derivative $\frac{\partial c_p}{\partial p}$ is evaluated, then by two integrations the volume $v$ is determined as a function of $p$ and $T$.

The first method, which is employed in this investigation, has an inherent difficulty and for that reason has been condemned by Davis and Jakob. It requires the evaluation of the second derivative of an observed magnitude $v$. Now when a function is empirically fitted to a series of observed values, the errors of the function are liable to be magnified in the first derivative and the second derivative is still more uncertain. This point may be made clearer by a geometrical illustration. If the experimental points are plotted on the $v$, $T$ plane in groups, each representing a constant pressure, a series of curves may be made to pass through the groups of points with reasonable accuracy. The second derivative $\frac{\partial^2 v}{\partial T^2}$ measures the curvature of these constant-pressure lines, hence the curves must not only fit the observed values of $v$, but they must have precisely the proper curvature.

The second method has been used by Jakob in his investigation (see p. 14). Its chief disadvantage is the practical difficulty of performing the mathematical operations involved. Jakob avoided this difficulty by the use of graphical methods, and his results were thus obtained by the measurement of lines and areas. Herein lies the advantage of the first method if it can be successfully applied. The mathematical manipulation is simple and direct, the various thermal magnitudes are given by closed expressions and therefore they may be calculated without recourse to measurement or approximation.

With the expressions for $v$ and $c_p$ established by either method, an equation for the heat content $i$ is found upon substituting these expressions in the general equation (4). A similar substitution in the general equation (2) leads to an equation for the entropy. The heat content equation when applied at the saturation limit must give values that satisfy the Clapeyron relation, eq. (8). Another check on the $i$ equation is furnished by the various sets of throttling experiments. According to thermodynamic theory the heat content $i$ remains constant in a throttling process; hence the constant-$i$ lines deduced from the equation for $i$ should lie in close coincidence with the experimental points.

## III. Review of Earlier Investigations.

8. *Zeuner's Theory.*—In this section is given a brief review of the more important theoretical investigations on the thermal properties of steam. Aside from historical interest, such a review is valuable in showing the development of the analytical processes with increasing experimental evidence; and also in exhibiting the various methods of attacking the problem.

The starting point of Zeuner's investigation[1] is the assumption of a characteristic equation for superheated steam of the form

$$pv = BT - R,$$

---

[1] Zeuner's Technical Thermodynamics, Klein's Tr., Vol. II, pp. 229-243.

in which the correction term $R$ is considered to be a function of the pressure $p$ only. Following Regnault's experiments, the specific heat $c_p$ is taken as a constant, viz. $c_p = 0.4805$. Then from the fundamental equations of thermodynamics, the law governing the adiabatic expansion of superheated steam is deduced in the form

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^m$$

which is the form found for adiabatic expansion of a gas, provided

$$m = \frac{k-1}{k} .$$

By a comparison of two expressions for the entropy of saturated steam the constant $m$ is given the value 0.25. The correction term $R$ is taken proportional to the fourth root of the pressure, and the characteristic equation thereupon becomes

$$pv = BT - Cp^{\frac{1}{4}}.$$

The energy and heat content of unit weight of saturated steam are given respectively by the simple expressions

$$u = u_0 + \frac{Apv}{k-1}$$

$$i = u_0 + \frac{k}{k-1} Apv,$$

and with $k = \frac{4}{3}$, these become, in English units,

$$u = 857 + 3\,Apv$$

$$i = 857 + 4\,Apv.$$

Zeuner compared the saturation volumes deduced from the characteristic equation with those given in Regnault's tables and found excellent agreement. The volumes of the superheated vapor deduced from the equation were compared with a few experimental values given by Hirn and Battelli and fair agreement was observed. The heat content of the saturated vapor calculated from the formula for $i$ checked with the values derived from Regnault's formula.

Zeuner's theory represented excellently the experimental evidence available at the time it was presented. The Munich experiments, however, have shown conclusively (1) that the form of the characteristic equation assumed by Zeuner cannot possibly represent correctly the volume measurements; (2) that the specific heat $c_p$ is not a constant but a function of both the temperature and pressure. Furthermore, the admirable investigation of Davis has shown that Regnault's linear formula for the heat content is clearly in error. The foundations of Zeuner's theory are therefore torn away.

9. *Callendar's Theory*[1].—The theory proposed by Callendar is assumed to apply to all vapors. The starting point of the investigation is a modification of the Joule-Thomson equation

$$v = \frac{BT}{p} - \frac{a}{BT^2}$$

Callendar observes (1) that the equation of a perfect gas at high temperatures is not $pv = BT$, but $p(v - b) = BT$, where $b$ is the minimum volume or "co-volume" of Hirn and van der Waals. The value of $b$ may probably be taken as equal to the volume of the liquid when the vapor pressure is small. (2) If it be assumed that the average total kinetic energy of the molecules of a gas is directly proportional to the energy of translation, then the limiting value of the specific heat of a gas in the ideal state ($p = 0$, $v = \infty$) either at constant pressure or at constant volume must be constant. It is further assumed that the kinetic energy of the vapor is proportional to $p(v - b)$ at all stages. Then the exponent of $T$ in the small correction term $\frac{a}{BT^2}$ of the Joule-Thomson equation is not 2 but $n = \frac{(c_v)_0}{B}$, the ratio of the limiting value of the specific heat at constant volume to the limiting value of $pv/T$. Next adopting the hypothesis of Maxwell regarding the partition of energy, the limiting value of this ratio for a triatomic gas like steam or $CO_2$ should be 3.5. The equation deduced from these considerations is

$$v - b = \frac{BT}{p} - c_0\left(\frac{T_0}{T}\right)^{3.5},$$

in which $T_0$ denotes the absolute temperature corresponding to 0° C. (273.1 C. or 459.6 F.). Taking $c_0 T_0^{3.5}$ as a single constant $m$, the equation becomes

$$v - b = \frac{BT}{p} - \frac{m}{T^{3.5}}.$$

By the application of the Clausius relation, the specific heat at constant pressure is given by the expression

$$c_p = (c_p)_0 + \frac{Amn\,(n+1)}{T^{4.5}}$$

in which $(c_p)_0$ denotes the assumed constant limiting value of $c_p$ when

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[1] On the Thermodynamical Properties of Gases and Vapors. Proc. of the Royal Soc. of London, Vol. 67 (1900), pp. 266-286.

$p=0$. Expressions for heat content, energy, and entropy are readily derived. The value of the constant $(c_p)_0$ is given as 0.4966, and with this Callendar computed values of $c_p$ at the saturation limit for the range 0°–200° C. Values of the total heat, latent heat, and entropy for the same range were also computed and tabulated.

Callendar's work was published in 1900, five years before the appearance of the Munich experiments and eight years before the publication of Davis' work on the total heat of steam. It is interesting to note that Callendar, on purely theoretical grounds, and in opposition to the then existing experimental evidence, forecast the more important of the recent developments of the subject. He showed the variation of the specific heat with pressure and temperature, though the values he calculated are not in good agreement with those now generally accepted. He challenged the accuracy of Regnault's linear equation for total heat and predicted the proper form of the total heat curve, as it was afterward established by Davis. He calculated from Joly's experiments the value of the latent heat at 100° C. (212° F.) as 540.2 Cal., almost exactly the value that is now considered most probable. Upon the appearance of the Munich volume measurements it was found that Callendar's characteristic equation represented them quite accurately up to 160° C. Above 160° the agreement was not so good for the reason that the equation gives the isotherms on the $pv$–$p$ plane as straight lines, while the experimental points indicated isotherms with an appreciable curvature.

The fundamental defect in Callendar's theory is the assumption of the constancy of the specific heat $(c_p)_0$ at zero pressure. In the absence of experimental evidence the assumption was natural and the basis for it appeared plausible. The experiments of Knoblauch and Jakob and the later confirmatory experiments of Knoblauch and Mollier showed conclusively that $(c_p)_0$ cannot possibly be a constant. The same thing is shown by the specific heat measurements of Mallard and Le Chatelier and of Langen at very high temperatures.

Callendar's paper is properly regarded as one of the most important of the contributions to the literature on the properties of vapors. It contains suggestions of the highest value, and it shows what may be accomplished by the application of pure theory. However, in the light of the experimental evidence accumulated since the publication of the paper, it is clear that Callendar's equations without modification should not at present be used as a basis for a tabulation of steam properties.

Professor Mollier of Dresden, recognizing the importance of Callendar's investigation, made it the basis of a set of steam tables published in 1906.[1] The work of Mollier has been practically duplicated by Smith and Warren[2] in their new steam tables published in 1912. It is difficult to justify the appearance of the last set of tables. The authors make a claim for the consistency of the values, a claim which is readily granted since the values were calculated from a set of equations properly related through the general thermodynamic equations. Consistency is obtained, however, at the expense of accuracy. No account whatever is taken of the Munich experiments on specific heat and even Barnes' noteworthy experiments on the specific heat of water are neglected. The Mollier and the Smith and Warren tables are doubtless much better than the older tables based entirely on Regnault's data; but on the score of accuracy, they are not to be compared with the Marks and Davis tables or with Peabody's latest tables.

10. *Davis' Investigations.*—To Dr. H. N. Davis of Harvard University we are indebted for two of the most valuable and important contributions to the literature of the properties of steam.

In his first paper,[3] Dr. Davis investigated the Joule-Thomson effect in the case of steam, using for this purpose the throttling experiments of Grindley, Griessmann, Peake, and Dodge. The object of the investigation was the verification of the law of corresponding states. From the four sets of throttling experiments, values of the Joule-Thomson co-efficient $\mu$ were determined, and the "reduced" values of $\mu$ were compared with values of $\mu$ for carbon dioxide. It was found that the law was verified within the limit of error of the experiments. With respect to the properties of steam, the most useful result of the investigation was the establishment of a curve showing the variation of $\mu$ with the temperature.

The important feature of Davis' second paper[4] is the discussion of the new formula for the heat content of saturated steam. The throttling experiments were undertaken primarily for the purpose of gaining information on the specific heat of superheated steam. All attempts along this line were unsuccessful for the reason that the calculated specific heat depends upon the *rate* of variation of the heat content, and consequently errors in the formula for heat content are enormously magnified in the calculated values of the specific heat. Grindley and Griessmann used Regnault's linear formula for heat content and were unable to obtain any consistent results. When the calculation is reversed, that is, when the variation of $i$ is determined from the assumed values of $c_p$, errors in $c_p$ are reduced in the calculation of $i$. The experiments of Knoblauch and Jakob give reliable values of $c_p$. With these available, Davis conceived the possibility of reversing the method. Taking the two sets of experimental data (1) the throttling experiments, (2) the Knoblauch values of $c_p$, he deduced his well-known formula

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[1] Neue Tabellen und Diagramme für Wasserdampf. Berlin. 1906.

[2] The New Steam Tables, D. Van Nostrand Co., N. Y. 1912.

[3] On the Applicability of the Law of Corresponding States to the Joule-Thomson Effect in Water and Carbon Dioxide. Proc. Am. Acad. Arts & Sciences, Vol. 45, pp. 243-264.

[4] Notes on Thermal Properties of Steam. Proc. Am. Acad., Vol. 45, pp. 267-311, 1909. See also Proc. A. S. M. E., Vol. 30, p. 1419, 1908.

$$i'' = i_{212} + 0.3745\,(t-212) - 0.00055\,(t-212)^2$$

which is universally accepted for the temperature range 212°–400° F. That the Regnault linear formula for heat content was inexact had been apparent for some time; in fact, Callendar in 1900 had called attention to the matter, and from his theory had obtained an $i$-curve differing materially from Regnault's straight line. The achievement of Davis in definitely settling this vexed question must be regarded as one of the most noteworthy in the history of the subject.

A section of the paper is devoted to a critical discussion of the specific heat of superheated steam, and the results of the researches of the Joule-Thomson effect are used to develop important relations between the specific heat and the coefficient $\mu$. (See p. 36.)

Davis also discusses the test furnished by the Clausius relation (page 8) and points out that Linde's characteristic equation can not be reconciled with the Knoblauch specific heat measurements through this relation. He goes so far as to say that such reconciliation is impossible, taking the accepted volume and specific heat measurements.

The investigations of Davis are embodied in the Marks and Davis steam tables.

11. *Jakob's Investigation.*[1]—The Clausius relation is taken as the basis of the investigation, but the method used is the reverse of that used in the development of the present theory. Jakob first lays down a system of $c_p$-curves in accordance with the experiments of Knoblauch and Jakob and Knoblauch and Mollier. In the adjustment of the curves use is made of the thermodynamic relation suggested by Davis (see p. 36). The characteristic equation is given the form

$$v = \frac{BT}{p} - R,$$

in which the correction term $R$ is a function of $p$ and $T$. Since

$$-\frac{\partial^2 v}{\partial T^2} = \frac{\partial^2 R}{\partial T^2}$$

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[1] Zeit. des Verein. deutsch. Ing., Vol. 56, pp. 1980-1988. 1912.

the Clausius relation gives

$$\left(\frac{\partial c_p}{\partial p}\right)_T = AT\frac{\partial^2 R}{\partial T^2},$$

whence

$$R = \frac{1}{A}\int\int\frac{1}{T}\frac{\partial c_p}{\partial p}dt^2.$$

To obtain a lower integration limit, Jakob attempts to set a value of $T$ at which temperature the derivative $\frac{\partial c_p}{\partial p}$ reduces to zero. By a rather arbitrary process he arrives at the value of $T = 1200°$ C., approximately. Hence at this temperature the term $R$ changes sign. No attempt is made to deduce an equation $c_p = \phi\,(p, T)$, and consequently the integrations required in the determination of $R$ are performed graphically. The final result of the process is a set of values of the specific volume $v$, which are compared with the volumes calculated from Linde's equation.

Jakob also gives a tabulation of the heat content of superheated steam. From the assumed specific heat curves, the mean specific heat from saturation to a given temperature $t$ is obtained by measurement; the product $c_{pm}\,(t - t_s)$ then gives the heat absorbed during superheat, and the sum $i_{sat} + c_{pm}\,(t - t_s)$ is the required value of $i$.

12. *Heck's Theory.*—Professor R. C. H. Heck has published several papers on the properties of steam, and in his latest paper[1] he has developed a complete theory.

The starting point of Heck's investigation is the Joule-Thomson effect. Taking the identity

$$\left(\frac{\partial i}{\partial p}\right)_t = -\left(\frac{\partial i}{\partial t}\right)_p\left(\frac{\partial t}{\partial p}\right)_i = -c_p\,\mu,$$

the variation of $\mu$ with the temperature is known at least approximately from the work of Davis. A relation between $i$ and $p$ with $t$ constant is thus determined, and ultimately an equation for $i$ is deduced. This has the form

$$i = i_0 + y'p + z'p^{2.4}$$

in which $i_0$, $y'$ and $z'$ are rather complicated functions of the temperature. The function $i_0$ is the heat content for $p = 0$. To get this the specific heat $c_{p0}$ for $p = 0$ has to be determined, and for this purpose the Knoblauch and Mollier experiments are used. The equation for $c_{po}$ is

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[1] Journal A. S. M. E., Nov. 1913, pp. 1619-1630.

$$c_{po} = 0.302 + \frac{114.66}{t + 688} + 0.000144t.$$

The general equation (6), p. 7, now furnishes a relation from which the form of the characteristic equation may be inferred. That this relation may be satisfied it is sufficient to give the characteristic equation the form

$$pv = BT - yp - zp^{2.4}$$

in which $y$ and $z$ are functions of $t$ that are related respectively to the functions $y'$ *and* $z'$. The constants in the various functions are so adjusted that the $i$ and $v$ equations satisfy the Clausius relation, and at saturation the Clapeyron relation.

Heck's theory satisfies all the requirements imposed by thermodynamic laws. Unfortunately, the paper was published in abstract and the most valuable part—the comparison of the theory with experimental data—was omitted. It may safely be assumed, however, that the agreement is satisfactory.

13. *Other Investigations.*—Other investigations of less immediate importance may be noted.

Linde[1] in his discussion of the experiments of the Knoblauch, Linde, and Klebe experiments, after establishing his two characteristic equations, attempted to deduce from them the latent heat of steam and the specific heat of superheated steam. The results obtained were not confirmed by the later experiments on specific heats.

Dieterici's paper entitled "Energieisothermen des Wassers bei hohen Temperaturen"[2] describes an attack on the problem of the properties of steam by an original and ingenious method. In the light of our present knowledge, some of the basic assumptions are unsound, and the results therefore have no significance.

Schüle[3] has recently published a tabulation of steam properties. The values were obtained by purely empirical processes, and no attempt was made to correlate them through thermodynamic relations. Perhaps the most valuable part of Schüle's work is the attempt to determine the properties in the vicinity of the critical temperature.

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[1] Mitteil. uber Forschungsarbeiten. Vol. 21, pp. 57-92.

[2] Annalen der Physik (4), Vol. 16, pp. 907-930.

[3] Zeit. des Verein. deutsch. Ing., Vol. 55, pp. 1506-1512, 1561-1567.

## IV. Development of a General Theory of the Properties of Steam.

14. *Units and Constants Employed.*—In this section is given the development of the equations that represent the various properties of saturated and superheated steam. The order of topics is as follows:

A. Relation between pressure and temperature of saturated steam.
B. Specific volumes. Characteristic equation of superheated steam.
C. Specific heat of superheated steam.
D. Heat content, latent heat, heat of liquid, energy, and entropy.
E. The Joule-Thomson effect.
F. Thermal properties near the critical temperature.

As a preliminary step, the establishment of certain units and constants is necessary. These are

1. The thermal unit.
2. The mechanical equivalent of heat.
3. The temperature of melting ice on the absolute scale.

In this investigation the mean B.t.u. is taken as the thermal unit. This is defined as 1/180th of the heat required to raise the temperature of a pound of water from 32° to 212° F. The corresponding mean calorie is by Griffiths[1] identified with the $17\frac{1}{2}°$ calorie and by Barnes with the 16° calorie.

The various determinations of the mechanical equivalent seem to justify the value established by Griffiths[1] in 1893, namely,

$$1 \text{ mean calorie} = 4.184 \text{ joules}$$
$$1 \text{ mean B.t.u.} = 777.64 \text{ standard ft. lb.}$$

This value has been used.

Various determinations of the absolute temperature of the ice-point are discussed by Marks and Davis[2], and the value chosen by them is 491.°64. It does not seem that the evidence is sufficient to justify the degree of accuracy indicated by the fifth figure; hence in this investigation 491.°6 has been used. The relation between absolute and ordinary temperatures is thus given by

$$T = t + 459.6.$$

### A. RELATION BETWEEN TEMPERATURE AND PRESSURE OF SATURATED STEAM.

15. *Experimental Data.*—The important relation $p = f(t)$ for saturated steam has been definitely determined by three sets of experi-

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[1] The Thermal Measurement of Energy.
[2] Steam Tables, p. 88.

ments conducted at the Reichsanstalt within the last five years. Each set covers a different range of temperature; Scheel and Heuse's[1] experiments cover the lower range 0–50° C. (32°–122° F.), Holborn and Henning's[2] the range 50°–200° C. (122°–392° F.), while Holborn and Baumann's[3] experiments extend from 200° to the critical temperature. The character of the work done at the Reichsanstalt is a sufficient guaranty that every precaution was taken and that the measurements are as precise as can be obtained with modern instruments and methods.

The values of the saturation pressure as deduced from the respective sets of experiments are given in the following tables. In the third table the values are not those given by Holborn and Baumann but values deduced therefrom by Prof. Marks.[4]

### TABLE 1.

### SCHEEL AND HEUSE.

| Temp. C. | Pressure in mm. of mercury | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0 | 4.579 | 4.926 | 5.254 | 5.685 | 6.101 | 6.543 | 7.014 | 7.514 | 8.046 | 8.610 |
| 10 | 9.210 | 9.845 | 10.519 | 11.233 | 11.989 | 12.790 | 13.637 | 14.533 | 15.840 | 16.481 |
| 20 | 17.539 | 18.655 | 19.832 | 21.074 | 22.383 | 23.763 | 25.217 | 26.747 | 28.558 | 30.052 |
| 30 | 31.834 | 33.706 | 35.674 | 37.741 | 39.911 | 42.188 | 44.577 | 47.082 | 49.708 | 52.459 |
| 40 | 55.341 | 58.36 | 61.52 | 64.82 | 68.28 | 71.90 | 75.67 | 79 62 | 83.74 | 88.05 |
| 50 | 92.54 | | | | | | | | | |

### TABLE 2.

### HOLBORN AND HENNING.

| Temp. C. | Pressure in mm. of mercury | | | | |
|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 |
| 50 | 92.3 | 101.9 | 112.3 | 123.6 | 135.9 |
| 60 | 149.2 | 163.6 | 179.1 | 195.9 | 214.0 |
| 70 | 233.5 | 254.5 | 277.1 | 301.3 | 327.2 |
| 80 | 355.1 | 384.9 | 416.7 | 450.8 | 487.1 |
| 90 | 525.8 | 567.1 | 611.0 | 657.7 | 707.3 |
| 100 | 760.0 | 815.9 | 875.1 | 937.9 | 1004 |
| 110 | 1074.5 | 1149 | 1227 | 1310 | 1397 |
| 120 | 1489 | 1586 | 1687 | 1795 | 1907 |
| 130 | 2026 | 2150 | 2280 | 2416 | 2560 |
| 140 | 2709 | 2866 | 3030 | 3202 | 3381 |
| 150 | 3569 | 3764 | 3968 | 4181 | 4402 |
| 160 | 4633 | 4874 | 5124 | 5384 | 5655 |
| 170 | 5937 | 6229 | 6533 | 6848 | 7175 |
| 180 | 7514 | 7866 | 8230 | 8608 | 8999 |
| 190 | 9404 | 9823 | 10256 | 10705 | 11168 |
| 200 | 11647 | 12142 | 12653 | | |

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[1] Annalen der Physik (4), Vol. 31, pp. 715-735, 1910.

[2] Annalen der Physik (4). Vol. 25, pp. 833-883, 1908.

[3] Annalen der Physik (4), Vol. 31, pp. 945-970, 1910. See also articles by Risteen; The Locomotive, Vol. 26, pp. 85, 183, 246; Vol. 27, p. 54; Vol. 28, pp. 88, 118.

[4] Proc. A. S. M. E., Vol. 33, p. 572.

### TABLE 3.

### HOLBORN AND BAUMANN.

| Temp. F. | Pressure in lb. per sq. in. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 400 | 246.99 | 276.34 | 308.33 | 343.18 | 380.92 | 421.85 | 465.95 | 513.65 | 565.08 | 620.18 |
| 500 | 679.26 | 742.55 | 810.31 | 882.58 | 959.85 | 1042.2 | 1130.2 | 1223.7 | 1323.0 | 1428.3 |
| 600 | 1539.9 | 1657.8 | 1782.9 | 1915.3 | 2055.1 | 2203.1 | 2359.2 | 2523.4 | 2697.1 | 2882.3 |
| 700 | 3083.4 | | | | | | | | | |

16. *Pressure-Temperature Formulas.*—A large number of formulas have been proposed to represent the relation between pressure and temperature of saturated steam. The greater number of these are purely empirical, but a few have a semi-rational basis.

Of the earlier formulas suggested, two have been quite generally used, namely, Roche's and Biot's.[1] From certain theoretical considerations Roche deduced the form

$$p = a\,\alpha^{\frac{\theta}{m+n\theta}}$$

in which $a$, $\alpha$, $m$, and $n$ are constants and $\theta = t + \text{const.}$, that is, $\theta$ denotes the temperature measured from some zero arbitrarily chosen. Biot suggested the more general form.

$$\log p = a + b\alpha^{\theta} + c\beta^{\theta}$$

which has five constants. The Biot formula has been generally used to represent Regnault's experiments.

A group of formulas may be deduced from the following considerations. The Clapeyron relation

$$r = AT\,(v'' - v')\,\frac{dp}{dt}$$

may be written in the form

$$\frac{dp}{p} = \frac{1}{T}\;\frac{r}{Ap\,(v'' - v')}\,dT.$$

In the fraction of the second term the numerator is the latent heat $r$ and the denominator is the external latent heat,

$$\psi = Ap\,(v'' - v')$$

that is, the part of $r$ that is used in overcoming external resistance. If values of $r$ and of $\psi$ be plotted as ordinates against temperatures as abscissas, the resulting curves are of such character that they can be represented very closely by equations of the type

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[1] Preston's Theory of Heat, Art. 188.

$$r = a_1 + a_2T + a_3T^2 + a_4T^3 + \ldots\ldots$$
$$\psi = T\,(b_1 + b_2T + b_3T^2 + b_4T^3 + \ldots\ldots)$$

The second curve should properly pass through the origin, since the product $pv''$ should approach zero as $T$ approaches zero. By division

$$\frac{r}{\psi} = \frac{1}{T}\,(c_1 + c_2T + c_3T^2 + c_4T^3 + \ldots\ldots)$$

and the substitution of this expression in the original Clapeyron relation leads to the differential equation

$$\frac{dp}{p} = \frac{1}{T^2}\,(c_1 + c_2T + c_3T^2 + c_4T^3 + \ldots\ldots)\,dT.$$

The integration of this equation gives the following general form for the relation $p = f(t)$

$$\log p = A + \frac{B}{T} + C\log T + DT + ET^2 + FT^3 + \ldots\ldots \qquad (11)$$

The number of constants may be increased indefinitely by taking more terms involving the higher powers of $T$; hence the equation may be fitted if desired to a large number of experimental points. The signs of the coefficients $B$, $C$, $D$, $E$, etc., may, of course, be either positive or negative.

Several formulas that have been proposed are simply modifications of this general equation.

*a.* The *Dupré-Hertz formula,* which is usually written in the form

$$\log p = k - m\log T - \frac{n}{T},$$

includes the first three terms of the general equation.

*b.* *Callendar's formula.*[1]—Callendar deduced from his equations for total heat and entropy a pressure-temperature relation of complicated form, but remarked that the equation could be written in the form

$$\log p = A + \frac{B}{T} + C\log T + \text{small terms.}$$

*c.* *Bertrand's formulas.*[2]—If the expressions for $r$ and $Ap\,(v'' - v')$ are taken as linear in $T$, the integrand assumes a form that permits integration in finite terms. Bertrand assumed that for water vapor the following relations are sufficiently well satisfied.

$$p\,(v'' - v') = B\,(T + a), \quad r = M - nT$$

The Clapeyron relation then becomes

$$\frac{dp}{p} = \frac{m - nT}{ABT\,(T + a)}\,dT$$

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[1] Proc. Royal Soc. of London, Vol. 67, p. 285, 1900.
[2] Chwolson, Lehrbuch der Physik, Vol. 3, p. 736.

from which
$$p = k \frac{T^a}{(T+a)^\beta}$$
or
$$\log p = \log k + a \log T - \beta \log (T + a)$$

In deducing Bertrand's second formula, it is assumed that the relation

$$\frac{Ap\,(v'' - v')}{r} = aT - b$$

holds good for steam. From this relation the equation

$$p = k \left( \frac{T - m}{T} \right)^n$$
or
$$\log p = \log k - n \log \frac{T}{T - m}$$

is easily obtained. While Bertrand's equations are convenient for the purposes of calculation, they cannot be extended over any considerable temperature range without change of constants. The same statement applies to the Dupré-Hertz formula.

*d. Marks' formula.*—If in the general equation (11) the constant $C$ is made equal to zero and terms containing powers of $T$ above the second are suppressed, the resulting equation has the form deduced from quite different considerations by Prof. Marks.[1]

One other formula should be mentioned. Thiesen's formula involves the critical temperature $t_k$ and atmospheric pressure. In the metric units ($p$ in mm. of mercury and $t$ in deg. C.) it may be written in the form

$$T \log \frac{p}{760} = A\ (t - 100) - B\ (t_k - t)^4 - (t_k - 100)^4$$

Thiesen's formula is used as a standard of reference by Henning[2] in an elaborate discussion of various pressure-temperature measurements.

It is an easy task to fit any of the proposed equations to the experimental values through a limited temperature range; but to obtain a single equation that will satisfactorily represent the experiments over the entire range from 32° to the critical temperature is a problem of some difficulty.

Marks' equation,

$$\log p = A - \frac{B}{T} - CT + DT^2,$$

represents the experimental values above 400° F. with remarkable accuracy and may be extended to 300° F. with satisfactory results. Below 300° the Marks curve begins to run below the experimental points, as shown in Fig. 1, and the Scheel and Heuse points are missed entirely. Heck[1] has slightly changed the constants of the Marks equation with the avowed purpose of getting the proper value of $p$ at 212° F. At this temperature the Marks equation gives $p = 14.672$, while the exact value is 14.697. The two sets of constants are as follows:

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[1] Proc. A. S. M. E., Vol. 33, p. 573.

[2] Annalen der Physik (4), Vol. 22, pp. 609-630, 1907.

| | Marks | Heck |
|---|---|---|
| $A =$ | 10.515345 | 10.606400 |
| $\log B =$ | 3.6878597 | 3.6897500 |
| $\log C =$ | $\bar{3}.6075880$ | $\bar{3}.6205462$ |
| $\log D =$ | $\bar{6}.1439400$ | $\bar{6}.1601803$ |

$$T = t + 459.64$$

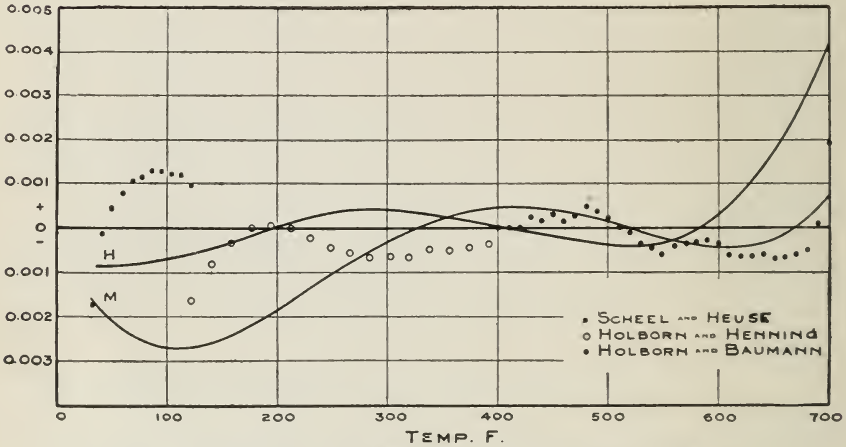

Fig. 1. Pressure and Temperature of Saturated Steam.

The curve H, Fig. 1, shows the agreement between Heck's equation and the experimental points. It is apparent that Heck's modification improves the agreement throughout the lower range of temperature, which is the most important. Above 600°, however, Heck's curve begins to deviate widely from the Holborn and Baumann points.

The equation worked out in the present investigation may in a sense be regarded as a modification of Marks' equation. In the first place a term involving log $T$ was added as is suggested by the general form (11). The equation thus took the form

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[1] Journal A. S. M. E., Vol. 35, p. 1627, 1913.

$$\log p = A - \frac{B}{T} - C \log T - DT + ET^2,$$

and the following constants were determined:

$$A = 10.5688080 \qquad \log D = \bar{3}.6088020$$
$$\log B = 3.6881209 \qquad \log E = \bar{6}.1463000$$
$$C = 0.0155 \qquad T = t + 459.6$$

With these constants, the equation represents the Holborn and Baumann points above 400° with substantially the same accuracy as the Marks equation, and it represents the experiments throughout the range 32°–300° very much better than the Marks equation. It is, however, open to two objections: (1) At 212° it gives $p = 14.694$, which is still too low; (2) at 32° it gives a value of $p$ considerably higher than is indicated by the Scheel and Heuse experiments.

It was found that these objections could be removed and the agreement improved throughout by the inclusion of a small correction term. The final equation is, therefore,

$$\log p = A - \frac{B}{T} - C \log T - DT + ET^2 - \Delta \qquad \text{(A)}$$

where

$$\Delta = 0.0002\,(10 - 10\,\theta^2 + \theta^4),$$

and

$$\theta = \frac{t - 370}{100}.$$

The addition of the term $\Delta$ amounts to the inclusion of terms involving $T^3$ and $T^4$ in the general formula (11). The values of $C \log T$ and $\Delta$ are easily calculated, and since $\Delta$ is an even function of $t$ the values below 370° are duplicated above 370°. The labor of calculation is therefore not materially increased by the inclusion of these terms.

17. *Comparison of Formulas.*—In Fig. 1, equation (A) is used as a standard of reference. The points plotted are taken from the preceding tables, and the curves $M$ and $H$ represent respectively the equations of Marks and Heck. Ordinates represent the relative deviation from the value of $p$ calculated from the formula. It is seen that the proposed equation (A) represents the experimental values with a high degree of accuracy. The deviations except at two or three isolated points are well within 1 part of 1000, which is probably within the limit of accuracy of the experiments. For the lower range 32° to 400°, Heck's formula is superior to Marks', and the proposed formula (A) is superior to both. Above 450° Marks' formula gives slightly better results than the new formula, and Heck's equation shows considerable deviation from the experimental points.

It will be seen that none of the three curves follows the general course of the Scheel and Heuse points. Both the $M$ and $H$ curves tend to favor the low Holborn and Henning point at 122° F. rather than the high Scheel and Heuse point at the same temperature. The curve of equation (A) on the other hand, lies nearer the Scheel and Heuse points throughout except at the extreme end of the range near 32°. The discrepancy between the two points at 122° is greatly magnified in the figure; the actual pressure difference is 0.24 mm. of mercury. Scheel and Heuse have noted this discrepancy and they defend the higher point. Equation (A) is probably more accurate throughout the range 40°–200° than either of the others. At 32° the value calculated from the equation is 4.587 mm. while the accepted value is 4.579 mm. of mercury. The discrepancy is unimportant so far as pressures are concerned. The significant fact is that, accepting the Scheel and Heuse points as authentic, the derivative $\frac{dp}{dT}$ calculated from any one of the three of the equations must be too small in the range 32°–80°. The effect of this error will be shown in another section.

## B. VOLUME OF SUPERHEATED AND SATURATED STEAM. CHARACTERISTIC EQUATIONS.

18. *Experimental Data.*—Direct experiments on the specific volume of saturated and superheated steam have been made by Ramsay and Young[1], by Battelli[2], and by Knoblauch, Linde, and Klebe[3]. The experiments in the Munich laboratory conducted by Knoblauch, Linde, and Klebe were so superior in all respects to those of the other investigators, that the results are generally accepted as decisive.

The apparatus used in the Munich experiments was so arranged that three sets of observations were made. 1. The pressure of the saturated vapor corresponding to the temperature. 2. Simultaneous values of pressure and temperature corresponding to a predetermined constant volume of superheated steam. 3. The corresponding saturation values of $p$ and $t$ for the given volume. In conducting the experiment the volume of a predetermined weight of steam was kept constant and corresponding temperatures and pressures were observed. These observed values of $p$ and $t$ when plotted give a constant volume curve, or "isochor" on the $pt$-plane. It was found that the curves, within the limits of accuracy of the experiments, were straight lines. These lines were prolonged to intersect the saturation curve $p = f(t)$, and the points of intersection gave, therefore, simultaneous values of $p$, $v$, and $t$, at the saturation limit.

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[1] Phil. Trans. Roy. Soc. of London, Vol. 183-A, p. 107 (1892).
[2] Annales de Chimie et de Physique (7), Vol. 3, p. 408 (1894).
[3] Mitteilungen über Forschungsarbeit., Vol. 21, pp. 33-72 (1905).

For convenience in establishing a characteristic equation, Linde made use of the scheme of representation devised by Amagat. Values of the product $pv$ were plotted as ordinates against values of $p$ as abscissas. The experimental points were not taken for this purpose but rather the points determined by the intersection of the successive isochors by lines of constant temperature. In this way the points on the $pv$-$p$ plane are separated into groups, each of which is associated with a particular temperature. In other words, curves through the successive sets of points are lines of constant temperature, or isotherms. Fig. 2 shows the points as thus determined. These were not copied from Linde's chart, but were calculated independently from the experimental data by Mr. Simmering.

19. *Characteristic Equations.*—A large number of equations have been proposed to represent the relation between the $p$, $v$, and $t$ of superheated vapors. In general these equations have the form

$$pv = BT - R$$

or the form

$$p(v - b) = BT - R,$$

in which $R$ is the so-called *correction term*. In the second form account is taken of the co-volume (see p. 11). The term $R$ is taken as a function of one or more of the variables, $p$, $v$, $t$. Zeuner makes $R = Cp^{1/4}$, Tumlirz assumes simply $R = Cp$. In Callendar's equation, namely

$$p\,(v - b) = BT - pc_0\left(\frac{T_0}{T}\right)^{-3.5}$$

$R$ is proportional to the product $pT^{-3.5}$. It may be noted that if $R$ is taken as a function of $p$ and $T$, or as a function of $p$ alone, and $p$ appears in the first power only, then the isothermals on the $pv$-$p$ plane are straight lines. The Tumlirz equation $pv = BT - cp$ gives a group of parallel straight lines, while the Callendar formula gives straight lines inclined at different angles. In one group of equations the term $R$ is made a function of $v$ alone or a function of $v$ and $t$. In the well known van der Waals equation

$$R = C\,\frac{v - b}{v^2},$$

and in the Clausius equation

$$R = \frac{C}{T}\,\frac{v - a}{(v + B)^2}.$$

In the first empirical formula proposed by Linde

$$R = \frac{1}{v}\left[C\left(\frac{373}{T^2}\right) - D\right].$$

Since $p$ and $T$ are always taken as the independent variables and $v$ is the magnitude calculated, it is extremely inconvenient to have an equation with a power of $v$ higher than the first; hence equations having $R$ a function of $p$ and $T$ have a practical advantage. With this point in view, Linde constructed his second equation, namely

$$pv = BT - p\,(1 + ap)\left[C\left(\frac{373}{T}\right)^3 - D\right].$$

This equation has been generally accepted, and from it have been calculated the values of $v$ that appear in the Marks and Davis and the Peabody steam tables.[1]

While Linde's second equation represents the experiments within the limits of accuracy it is open to two serious objections. (1) At 402° C. the correction term $R$ vanishes and for higher temperatures it changes sign. In the language of Linde, the vapor bcomes a more than perfect (übervollkomenes) gas at temperatures above 402°. (2) Taking the specific heat measurements of Knoblauch and Mollier as decisive, it is impossible to satisfy the Clausius thermodynamic relation with the values of the derivative $\frac{\partial^2 v}{\partial T^2}$ obtained from Linde's equation. This means that while the constant pressure curves deduced from Linde's equation pass through the experimental points with sufficient accuracy, they have not the proper curvature. Dr. Davis in his paper on the properties of steam[2] has pointed out this defect, and has expressed the opinion that no reliable $c_p$ values can be obtained through the Clausius equation from any volume measurements as yet available. That this view is not justified will be shown in the following section.

In the course of the present investigation a number of equations have been developed. These differ slightly in form, and each may be considered a modification of Linde's second equation. The first equation[3] was given the form

$$v + c = \frac{BT}{p} - (1 + ap)\,\frac{m}{T^n}.$$

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[1] Marks and Davis, Steam Tables and Diagrams, p. 98.

[2] Am. Acad. of Arts and Sciences, Vol. 45, p. 288 and p. 303.

[3] Goodenough's Principles of Thermodynamics, p. 203.

The equation resembles Linde's equation in retaining the expression $(1+ap)$ which serves to give the parabolic form to the isothermals when drawn in the *pv-p* plane. See Fig. 2. The constant $D$ in Linde's equation is dropped, but another constant is added to $v$. The values assigned to the constants were as follows:

| Metric units<br>($p$ in kg. per sq. m.) | English units<br>($p$ in lb. per sq. in.) |
|---|---|
| $B = 47.113$ | $B = 0.5963$ |
| $\log m = 11.19839$ | $\log m = 13.67938$ |
| $n = 5$ | $n = 5$ |
| $c = 0.0055$ | $c = 0.088$ |
| $a = 0.00000085$ | $a = 0.0006$ |

With these constants the equation represents the experimental results with substantially the same accuracy as Linde's equation; and by means of the Clausius relation the specific heat measurements of Knoblauch and Mollier are fairly well verified.

A careful study of conditions to be satisfied led to further modifications. It was found that more consistent results could be obtained by taking a fractional power of $p$ in the correction term. Further, it was found that by taking 4 instead of 5 for the exponent $n$ the constant $c$ could be omitted. Hence the second equation was given the form

$$v = \frac{BT}{p} - (1 + 3ap^{\frac{1}{2}})\frac{m}{T^n}$$

The constants for this equation are

| Metric ($p$ in kg. per sq. m.) | English ($p$ in lb. per sq. in.) |
|---|---|
| $\log B = 1.67274$ | $\log B = \bar{1}.77508$ |
| $\log m = 8.65429$ | $\log m = 10.88000$ |
| $3a = 0.001131$ | $3a = 0.03$ |
| $n = 4$ | $n = 4$ |

In all respects this equation is an improvement over the first equation.

Further consideration of the question led to another slight modification with a corresponding change of constants. Accepting Callendar's suggestion that the first member should contain a term to represent the co-volume, the equation was given the form

$$v - c = \frac{BT}{p} - (1 + 3ap^{\frac{1}{2}})\frac{m}{T^n} \qquad \text{(B)}$$

and the value of $c$ was taken as the specific volume of water. Hence, when the equation is used to determine the specific volume of saturated steam, the first member becomes simply $v'' - v'$. Since this difference, rather than the steam volume $v''$, occurs in the Clapeyron relation and

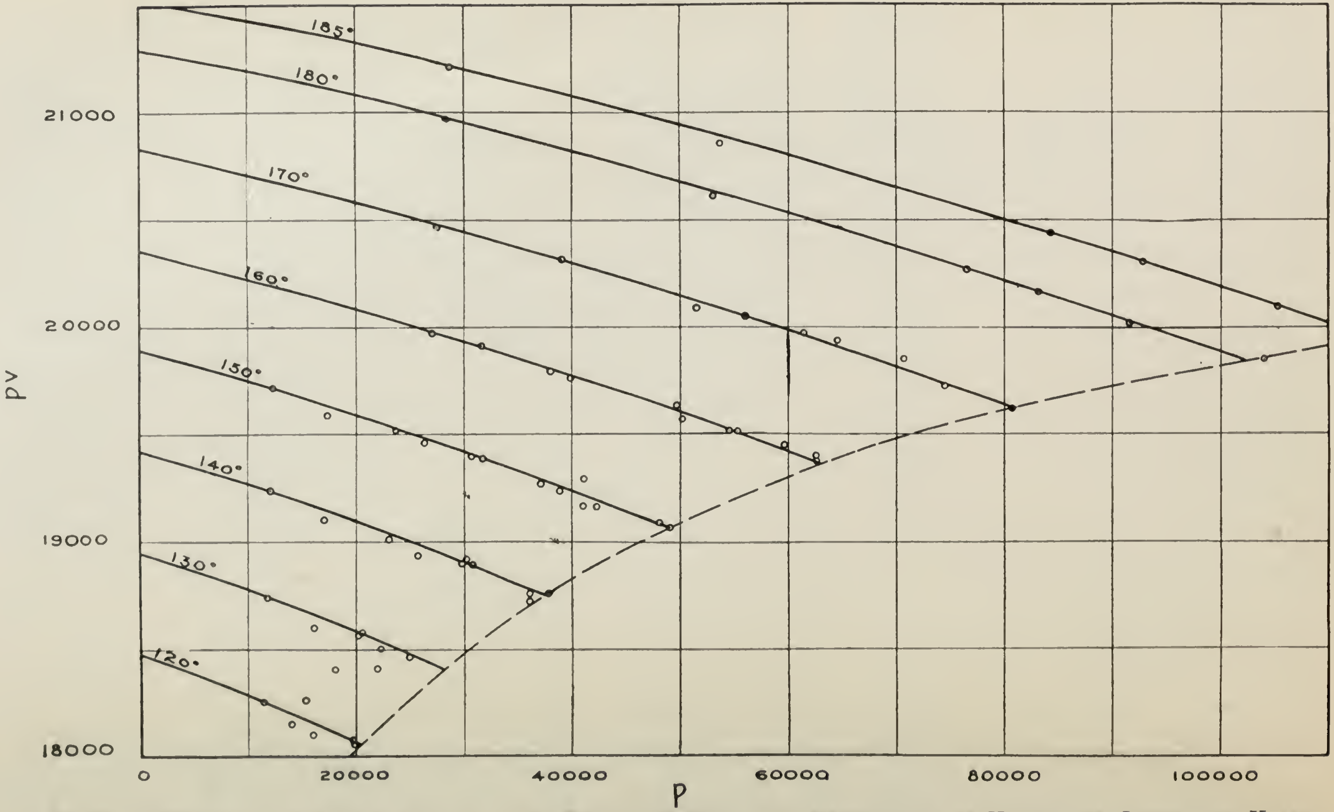

Fig. 2. Isothermal Curves from Eq. (B). The Points Represent the Experiments of Knoblauch, Linde, and Klebe.

in other thermodynamic formulas, the inclusion of the constant $c$ really simplifies the calculation of the thermal magnitudes that involve specific volumes. The constants of the final equation are as follows:

| Metric | English |
|---|---|
| $\log B = 1.67206$ | $\log B = \bar{1}.77441$ |
| $\log m = 8.59929$ | $\log m = 10.82500$ |
| $\log 3a = \bar{3}.28644$ | $\log 3a = \bar{2}.71000$ |
| $n = 4$ | $n = 4$ |

With the satisfaction of the Clausius relation in view, Heck[1] has developed an equation which is in form considerably different from any heretofore suggested. It is

$$v = \frac{BT}{p} - \frac{E}{(t+130)^2} - \frac{F}{(t+40)^8}\,p^{1.4}$$

The constants are: $B = 0.5956$, $\log E = 4.66365$, $\log F = 11.02244$. Pressures are to be taken in lb. per sq. *inch.*

It may safely be assumed that Heck's equation represents the volume measurements sufficiently well and that it satisfies the Clausius relation.

20. *Tests of the Characteristic Equation.*—The following comparisons are made to establish the validity of equation (B) as far as specific volumes are concerned. The question of the satisfaction of the Clausius relation is discussed in the following section.

1. Isothermal curves are calculated from (B) and drawn on the *pv-p* plane along with the points plotted from the measurements of Knoblauch, Linde and Klebe. The result is shown in Fig. 2, which may be compared with Fig. 13 in Linde's paper.[2] The agreement between the curves and points is thoroughly satisfactory.

2. As has been stated, the constant volume lines of Knoblauch, Linde, and Klebe when drawn on the *pt*-plane appeared to be straight lines. Linde[3] has compared the slopes of these lines as observed with the slopes calculated from his first equation. The slopes *at the saturation limit* have been calculated from (B), and the following table gives a comparison

TABLE 4. VALUES OF $\left(\frac{\partial p}{\partial T}\right)_v$

| $t =$ | 101.4 | 112.3 | 126.5 | 139.2 | 150.3 | 163.2 | 170 | 180.6 | 183 |
|---|---|---|---|---|---|---|---|---|---|
| From (B) | 30.7 | 44.0 | 68.4 | 99.0 | 134.7 | 189.2 | 224.8 | 291.5 | 309.7 |
| Linde | 30.4 | 43.9 | 67.4 | 97.9 | 132.9 | 188.9 | 221.7 | 287.5 | 303.7 |
| Observed | 30.4 | 44.3 | 68.0 | 99.3 | 134.0 | 188.6 | 225.0 | 293.0 | 313.0 |
| Heck | 30.5 | | | 96.3 | | 178.7 | | | 285.2 |

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[1] Journal Am. Soc. Mech. Eng'nrs, Nov., 1913, p. 1619.
[2] Mitteil. über Forschungsarbeit., Vol. 21, p. 58.
[3] Loc. cit., p. 67.

The comparison is somewhat misleading, because both Linde's second equation and equation (B) give values of the slope that vary slightly with the temperature. This variation is shown for four of the experiments.

TABLE 5.

VARIATION OF $\left(\frac{\partial p}{\partial T}\right)_v$ WITH TEMPERATURE.

| | | | | | |
|---|---|---|---|---|---|
| Exper. No. 1 | $t =$ | 101.4 | 120.0 | 140.0 | 160.0 |
| | $\left(\frac{\partial p}{\partial T}\right)_v =$ | 30.7 | 30.2 | 30.1 | 29.9 |
| No. 12 | $t =$ | 126.5 | 140.0 | 160.0 | 180.0 |
| | $\left(\frac{\partial p}{\partial T}\right)_v =$ | 68.4 | 67.7 | 67.0 | 66.9 |
| No. 22 | $t =$ | 150.3 | 160.0 | 180.0 | |
| | $\left(\frac{\partial p}{\partial T}\right)_v =$ | 134.7 | 133.2 | 131.2 | |
| No. 29 | $t =$ | 170.0 | 180.0 | | |
| | $\left(\frac{\partial p}{\partial T}\right)_v =$ | 224.8 | 219.9 | | |

The change in slope introduces a curvature in the constant volume lines, so slight however, that it can scarcely be detected in the figure. It is clear that the slopes deduced from (B) agree sufficiently well with the observed slopes.

3. Values of $v''$, the specific volume of the saturated vapor, as calculated from (B) are compared with values calculated from Linde's equation. The following table shows the comparison:

TABLE 6.

COMPARISON OF SPECIFIC VOLUMES.

| Temp. C. | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 |
|---|---|---|---|---|---|---|---|---|---|
| $v''$ from (B)..... | 1.6738 | 1.211 | 0.8926 | 0.6692 | 0.5095 | 0.3923 | 0.3075 | 0.2431 | 0.1943 |
| $v''$, Linde........ | 1.6740 | 1.211 | 0.8922 | 0.6690 | 0.5091 | 0.3921 | 0.3073 | 0.2430 | 0.1943 |

At the saturation limit the two formulas give practically identical values throughout the range of temperature covered by the experiments.

4. For the range 0°–100° C. and for temperatures above 180° C. there are no reliable experimental values to serve as a check of the formula. However, the specific volumes of the saturated vapor given in the Marks and Davis tables may be accepted as fairly accurate as they

were calculated by means of the Clapeyron relation from values of the latent heat that are at least approximately correct. The following table shows a comparison of volumes obtained from (B) with the Marks and Davis values.

### TABLE 7.
### COMPARISON OF SPECIFIC VOLUMES.

| Temp. F. | 32 | 40 | 80 | 120 | 160 | 200 | 212 | 240 | 280 | 320 | 360 | 400 | 440 | 480 | 520 | 560 | 600 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $v''$ from (B) | 3296 | 2440 | 632.8 | 203.2 | 77.28 | 33.65 | 26.81 | 16.33 | 8.66 | 4.92 | 2.964 | 1.868 | 1.220 | 0.820 | 0.562 | 0.391 | 0.274 |
| $v''$, M. & D. | 3294 | 2438 | 632.8 | 203.1 | 77.20 | 33.60 | 26.79 | 16.32 | 8.64 | 4.91 | 2.957 | 1.872 | 1.229 | 0.81 | 0.55 | 0.39 | 0.27 |

From the comparison it appears that formula (B) may be accepted, so far as saturation volumes are concerned, for the entire range 32°–600° F.

5. For the volumes in the region of high superheat there is at present no check. The measurements of Knoblauch, Linde, and Klebe reach only to about 50° C. (120° F.) of superheat. The extension of any formula to 400° or 500° superheat is therefore an extrapolation, the validity of which is uncertain. On account of the character of the correction term in Linde's formula (see p. 26), it is practically certain that at high superheat Linde's values are too high. Jakob[1] in the course of this investigation has deduced values that run consistently lower than those of Linde at the higher superheats. As a matter of interest a comparison has been made between three sets of values for three different pressures, 1, 9 and 19 kg. per sq. cm. This is shown in the following table:

### TABLE 8.
### SPECIFIC VOLUMES OF SUPERHEATED STEAM.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ = 10000 kg. per sq. m. | | | | | | | |
| $t$ = | 99.1 | 130 | 160 | 190 | 220 | 250 | 300 |
| $v$ from (B) | 1.7254 | 1.8777 | 2.0233 | 2.1675 | 2.3109 | 2.4536 | 2.6904 |
| Linde | 1.7260 | 1.8783 | 2.0245 | 2.1695 | 2.3137 | 2.4571 | 2.6954 |
| Jakob | 1.7281 | 1.8789 | 2.0237 | 2.1674 | 2.3107 | 2.4535 | 2.6909 |
| $p$ = 90000 | | | | | | | |
| $t$ = | | | 174.6 | 190 | 220 | 250 | 300 |
| $v$ from (B) | | | 0.2193 | 0.2293 | 0.2480 | 0.2659 | 0.2946 |
| Linde | | | 0.2190 | 0.2293 | 0.2483 | 0.2665 | 0.2959 |
| Jakob | | | 0.2195 | 0.2296 | 0.2479 | 0.2655 | 0.2939 |
| $p$=190000 | | | | | | | |
| $t$= | | | | 208.9 | 220 | 250 | 300 |
| $v$ from (B) | | | | 0.1068 | 0.1107 | 0.1208 | 0.1362 |
| Linde | | | | 0.1068 | 0.1108 | 0.1213 | 0.1374 |
| Jakob | | | | 0.1071 | 0.1113 | 0.1210 | 0.1359 |

The comparison is shown graphically in Fig. 3. The ordinates of curve L represent the difference between Linde's values and the values given

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[1] Zeit. des Verein. deutsch. Ing., Vol. 56, p. 1987.

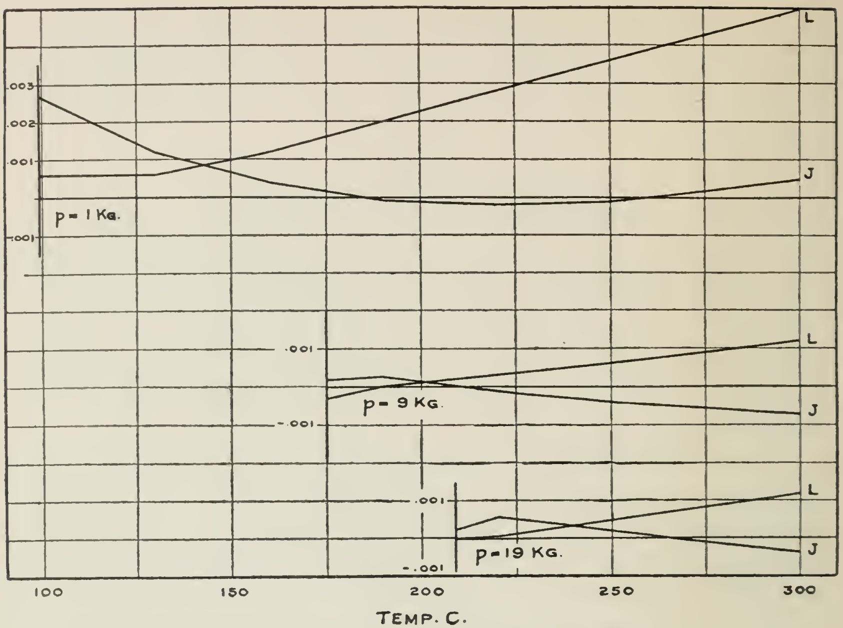

Fig. 3. Comparison of Specific Volumes.

by eq. (B). Similarly, curve J applies to Jakob's values. It will be seen that, in general, the volumes calculated from (B) lie near Linde's volumes at the saturation limit, but approach more nearly Jakob's values with increasing superheat. It will be observed that the curve J cuts the base line (which represents formula (B)) in two points for the pressure of 1 kg. If the superheat is carried sufficiently high at the other pressures, a second intersection will likewise be obtained. The correction term $R$ in Linde's equation changes sign at 402° C., and in Jakob's system the change of sign occurs at 920° C. Hence before reaching 920° Jakob's values must rise above those calculated from (B).

The correction term in eq. (B) does not change sign at any temperature. It approaches zero as $T$ is indefinitely increased, and it becomes negligible compared with the term $\frac{BT}{p}$ when the pressure becomes very small. In these two limiting cases, therefore, equation (B) merges into the equation

$$p\,(v-c)=BT.$$

Evidently the *form* of equation (B) is such as to justify extrapolation beyond the region of the experiments; and it is probable that the values of $v$ calculated from this equation are worthy of confidence both at the saturation limit and in the region of superheat.

### C. SPECIFIC HEAT OF SUPERHEATED STEAM.

21. *Experimental Data.*—The experiments on specific heat may be divided into groups as follows:

1. The early experiments of Regnault with steam at atmospheric pressure and at temperatures relatively close to saturation.
2. The experiments of Mallard and Le Chatelier, Langen, and Pier at very high temperatures.
3. The experiments of Holborn and Henning with steam at atmospheric pressure and a temperature range of 110°–1400° C.
4. The experiments of Grindley and Griessmann using the throttling method.
5. Recent direct experiments with steam at various pressures. Of these, the experiments of Knoblauch and Jakob and of Knoblauch and Mollier performed in the Munich laboratory are specially noteworthy. Similar experiments have been made by Thomas.

Regnault's experiments made in 1862[1] indicated a constant value of $c_p = 0.4805$. Davis[2] has recomputed Regnault's values and has deduced a somewhat smaller value, namely, $c_p = 0.4762$. For the pressure and range of temperature covered in the experiment, Regnault's value agrees well with the results of recent experiments.

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[1] Mem. Inst. de France, Vol. 26, p. 167 (1862).
[2] Proc. Am. Acad., Vol. 45, p. 286 (1910).

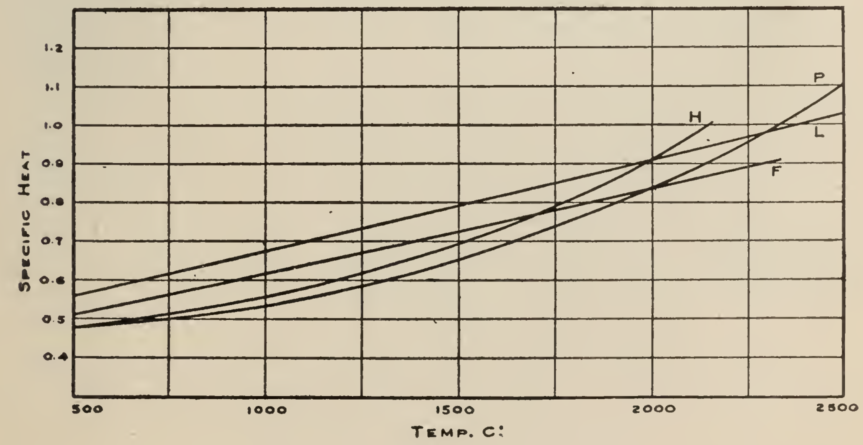

FIG. 4. CURVES OF SPECIFIC HEAT.

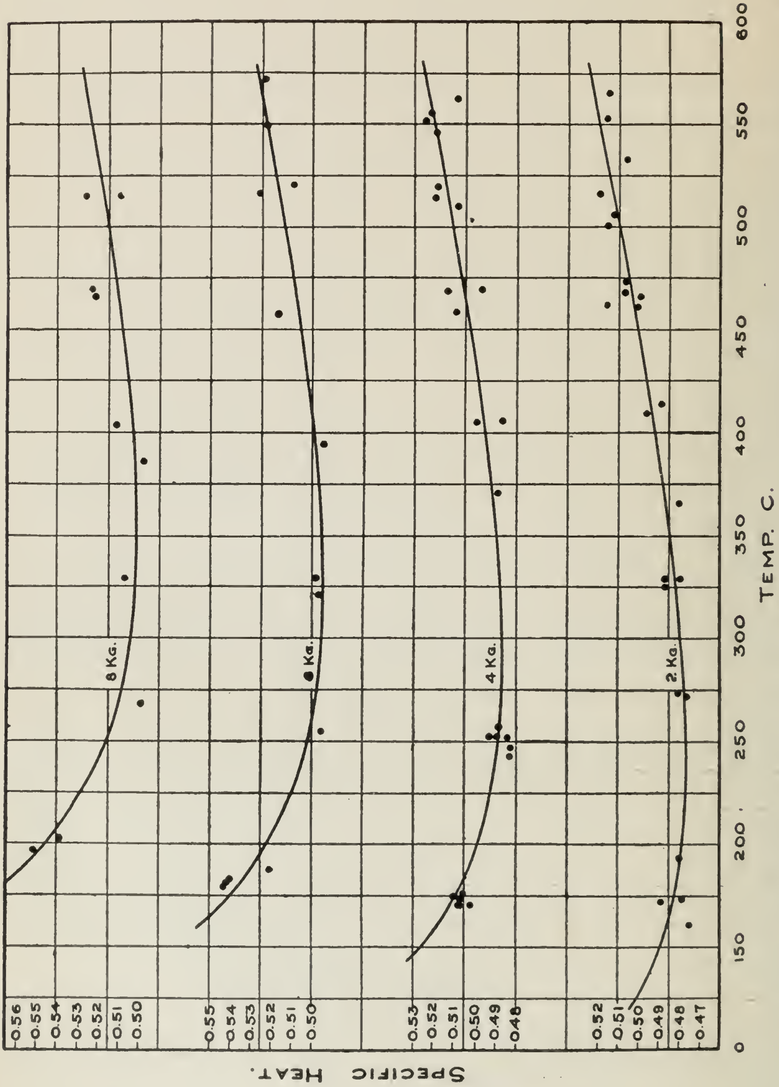

Fig. 5. Curves of Specific Heat Deduced from Eq. (C). The Points Represent the Experiments of Knoblauch and Mollier.

The high temperature experiments noted in group 2 have only an indirect bearing on the present investigation. The results obtained by the different investigators are discordant, but they all agree in showing a marked increase of specific heat with rising temperature. In Fig. 4 the straight line L represents the linear relation

$$c_p = 0.439 + 0.000239t$$

established by Langen, curve P represents Pier's equation, and curve H the equation given by Holborn and Henning.

The experiments of Holborn and Henning[1] form a link between the high temperature experiments of group 2 and the experiments of group 5. These measurements indicate values of $c_p$ consistently lower than those obtained in the Munich experiments. While considerable weight must be attached to the Holborn and Henning experiments, it seems probable that preference must be given the Knoblauch and Mollier measurements. Callendar[2] has expressed the opinion that the Holborn and Henning values are too low by as much as 10 per cent.

The efforts of Grindley and Griessmann to determine $c_p$ by the method of throttling were futile, and the results obtained by them are without value.

Of the direct experiments, preference is justly given to those of Knoblauch and Jakob[3] and Knoblauch and Mollier.[4] The latter experiments supplement and extend the range of the former. A third set of experiments is now being conducted, and preliminary reports indicate that the earlier results will be sustained. The results reported by Thomas[5] are of value indirectly as in some degree corroborating the Munich experiments. As conclusively shown by Davis,[6] the Thomas experiments are not to be compared with the Knoblauch experiments in point of accuracy.

After reviewing all the experimental evidence one must be convinced that for the range of temperature covered, the Knoblauch and Mollier measurements should be accepted without modification. They are therefore used in the present investigation. The points plotted in Fig. 5 are those determined by Knoblauch and Mollier. For convenience in the identification of the measurements associated with the four pressures employed, the points have been separated into four groups.

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[1] Annalen der Physik, Vol. 18, p. 739 (1905); Vol. 23, p. 809 (1907).
[2] Report of British Assoc. Committee on Gaseous Explosions, pp. 31, 32 (1908).
[3] Mitteil. über Forschungsarbeit, Vol. 35, p. 109.
[4] Zeit. des Ver. deutsch. Ing., Vol. 55, p. 665 (1911).
[5] Proc. Am. Soc. Mech. Engrs., Vol. 29, p. 633 (1907).
[6] Proc. Am. Acad. of Arts and Sciences, Vol. 45, pp. 269-272.

22. *Systems of Specific Heat Curves.*—The Knoblauch and Jakob experiments showed unmistakably the variation of specific heat with both pressure and temperature. At constant temperature, an increase of pressure resulted in an increase of specific heat; and at constant pressure the value of $c_p$ first decreased from the saturation limit, attained a minimum and then increased. Knoblauch and Jakob exhibited the variation of $c_p$ by means of constant pressure curves drawn on a plane with $c_p$ as ordinate and $t$ as abscissa. The curves were so drawn as to represent as closely as possible the experimental points and by a doubtful process of extrapolation the pressures were carried up to 20 kg. per sq. cm.

Taking the Knoblauch and Jakob curves as a basis, Davis developed a system of specific heat curves (Marks and Davis, Steam Tables and Diagrams, p. 97) from which the properties of superheated steam were deduced. Davis modified the Knoblauch curves in two respects. 1. Accepting the Holborn and Henning measurements, he lowered the $c_p$ curve for atmospheric pressure so that it would join the Holborn and Henning curve. In the light of the Knoblauch and Mollier results, this modification was doubtless a mistake. 2. The Knoblauch values at low pressures near the saturation limit were changed so as to bring them more nearly in accord with Regnault's value. Davis also developed a thermodynamic relation by which the spacing of the curves could be tested. Denoting by $\mu$ the Joule-Thomson coefficient, the relation is

$$\frac{c_p}{c_{po}} = e^{-\int_{p_o}^{p} \left(\frac{\partial \mu}{\partial T}\right)_p dp} \tag{12}$$

Davis had already investigated the Joule-Thomson effect for steam[1] and had obtained an approximate relation between $\mu$ and $t$. By means of the relations $\mu = f(t)$ he was able to evaluate the integral in (12) and thus to adjust the spacing of the $c_p$-curves.

Another system of $c_p$-curves has been worked out by Jakob.[2] The curves were adjusted to the Knoblauch and Mollier points, from Davis' relation the curve for $c_{po}$ (*i. e.*, $p = 0$) was determined, and then by the same relation $c_p$-curves for 10, 12, 14, 16, 18, and 20 kg. per sq. cm. were established. Near the saturation limit the Jakob curves agree closely with the Davis curves; at the higher superheats the increase of $c_p$ with the temperature is more marked in the Jakob curves, as it doubtless should be.

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[1] Proc. Amer. Acad. of Arts and Sciences, Vol. 45, p. 261.

[2] Zeit. des Verein. deutsch. Ing., Vol. 56, pp. 1981-3 (1912).

23. *Equation for Specific Heat.*—From the characteristic equation

$$v - c = \frac{BT}{p} - (1 + 3ap^{1/2})\,\frac{m}{T^n}$$

the second derivative required in the Clausius relation is obtained. It is

$$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = -\,(1 + 3ap^{1/2})\,\frac{mn\,(n+1)}{T^{n+2}}.$$

Hence

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\,\frac{\partial^2 v}{\partial T^2} = \frac{Amn\,(n+1)}{T^{n+1}}\,(1 + 3ap^{1/2}).$$

An integration with $T$ constant gives an expression for $c_p$, namely

$$c_p = F(T) + \frac{Amn\,(n+1)}{T^{n+1}}\,p\,(1 + 2ap^{1/2}).$$

The arbitrary function $F(T)$ is evidently $c_{po}$, that is, the specific heat at zero pressure. This was taken as a constant by Callendar. The experiments of Knoblauch and Mollier show that $c_{po}$ cannot be constant, and this conclusion is confirmed by the high-temperature experiments of Langen and others. It has been suggested that a simple linear relation

$$c_{po} = \alpha + \beta T$$

may be assumed, and this assumption was made in the writer's earlier paper. It is found, however, that better agreement is obtained by a relation of the form

$$c_{po} = \alpha + \beta T + \frac{\gamma}{T^2}.$$

Writing the equation for $c_p$ in the form

$$c_p = F(T) + f(p, T)$$

values of the term $f(p, T)$ may be calculated for each of the Knoblauch and Mollier experiments and by subtraction the corresponding values of $c_{po} = F(T)$ are found. From the curve through these points the constants $\alpha$, $\beta$, and $\gamma$ are obtained. The equation for $c_p$ finally takes the form

$$c_p = \alpha + \beta T + \frac{\gamma}{T^2} + \frac{Amn\,(n+1)}{T^{n+1}}\,p\,(1 + 2ap^{1/2}) \qquad \text{(C)}$$

and the constants are

| | Metric | English |
|---|---|---|
| $\alpha =$ | 0.320 | 0.320 |
| $\beta =$ | 0.0002268 | 0.000126 |
| $\gamma =$ | 7371 | 23583 |

The constants $a$, $m$, and $n$ are those of the characteristic equation. (See p. 29.)

In Fig. 5 the curves calculated from equation (C) are shown. The agreement between the curves and experimental points is sufficiently evident. It would be difficult to obtain a family of curves connected by a relation that would represent more accurately the groups of points.

The perfect correlation of the Munich experiments is shown by a comparison of Figs. 2 and 5. Equation (B) gives the curves of Fig. 2, equation (C) the curves of Fig. 5. The equations certainly represent the experimental data within the limits of accuracy of the experiments, and they are properly connected by the Clausius relation. The difficulty of making the correlation lies not in the measurements themselves,—they are sufficiently accurate—but in the choice of a proper form for the characteristic equation. The results here shown amply justify the method used in attacking the problem, and they strongly confirm the validity of equation (B).

In the following table values for $c_p$ for various temperatures and pressures are given. The numbers in parentheses are the corresponding values deduced by Jakob from his $c_p$-curves:

TABLE 9.

SPECIFIC HEAT OF SUPERHEATED STEAM.

| Pressure kg. per sq. cm. | Temperature C | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 |
| 0 | 0.457<br>(0.462) | 0.460<br>(0.465) | 0.466<br>(0.469) | 0.473<br>(0.474) | 0.480<br>(0.481) | 0.489<br>(0.489) | 0.498<br>(0.497) | 0.508<br>(0.505) | 0.518<br>(0.514) |
| 2 | 0.490<br>(0.486) | 0.479<br>(0.477) | 0.477<br>(0.477) | 0.480<br>(0.480) | 0.485<br>(0.485) | 0.492<br>(0.492) | 0.500<br>(0.500) | 0.509<br>(0.508) | 0.519<br>(0.516) |
| 4 | 0.527<br>(0.524) | 0.498<br>(0.494) | 0.489<br>(0.486) | 0.487<br>(0.486) | 0.490<br>(0.490) | 0.496<br>(0.496) | 0.503<br>(0.503) | 0.511<br>(0.510) | 0.520<br>(0.517) |
| 6 | | 0.522<br>(0.514) | 0.503<br>(0.495) | 0.496<br>(0.492) | 0.496<br>(0.494) | 0.500<br>(0.500) | 0.505<br>(0.506) | 0.513<br>(0.512) | 0.521<br>(0.518) |
| 8 | | 0.545<br>(0.538) | 0.517<br>(0.505) | 0.504<br>(0.498) | 0.502<br>(0.499) | 0.504<br>(0.503) | 0.508<br>(0.508) | 0.515<br>(0.514) | 0.523<br>(0.520) |

The agreement between the two sets of values is remarkably close; except for three or four points the difference is well within one per cent. From 350° to 500° the values are practically identical, but above 500° the specific heats calculated from the equation begin to rise above the values found by Jakob, while these in turn are higher than the Davis values. The question of the proper course of the $c_p$-curves at 550° is important in connection with the $c_p$ values at high superheat. The only guide in settling this question is the experimental evidence furnished by the high-temperature experiments. Referring to Fig. 4, curve $F$ represents the function $F(T) = \alpha + \beta T + \frac{\gamma}{T^2}$ and at high temperatures this curve is practically identical with the specific heat curve, since the term $f(p, T)$ in the specific heat equation becomes vanishingly small. It will be seen that curve $F$ is in good agreement with the other curves between 500° and 2000° C. If Jakob's curves were prolonged in the same way, they would merge into a single curve lying somewhat below curve $F$. This is more clearly shown in Fig. 6, in which various determinations of the function $c_{p0} = F(T)$ are plotted. Curve $H$ represents the variation of $c_{p0}$ assumed by Heck, curve $A$ represents the preceding equation, and the points are plotted from the values of $c_{p0}$ given

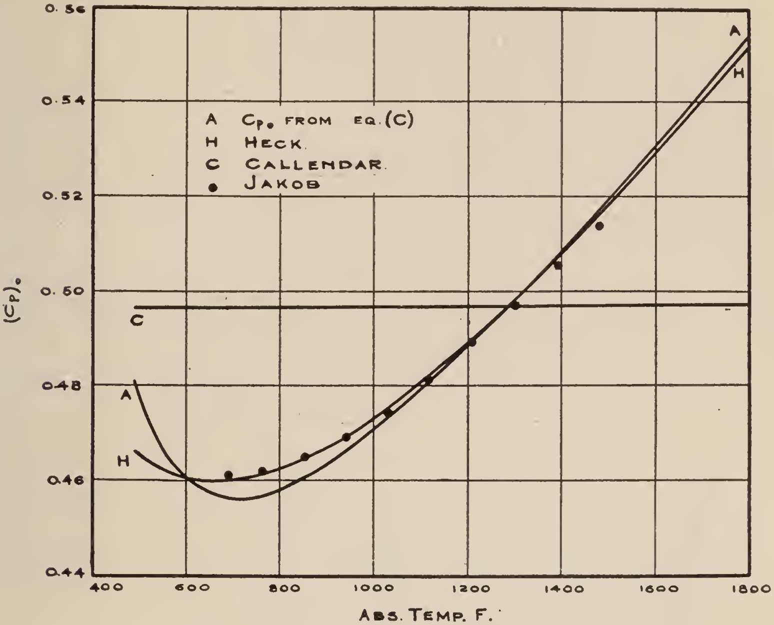

FIG. 6. CURVES OF SPECIFIC HEAT $(C_p)_0$.

by Jakob. As a matter of interest Callendar's constant value is shown by the straight line $C$. It appears that the three curves agree quite well. At the higher temperatures curves $A$ and $H$ are almost coincident, but the Jakob points show a tendency to run low. It is probable that equation (C) gives fairly good values of $c_p$ up to about 2000° C., that Heck's equation may safely be used through the same range, but that Jakob's curves if prolonged lie somewhat low and are not valid above 1000° C.

At the lower temperatures Heck and Jakob are in close agreement, while curve $A$ shows quite a different course. Experimental evidence in

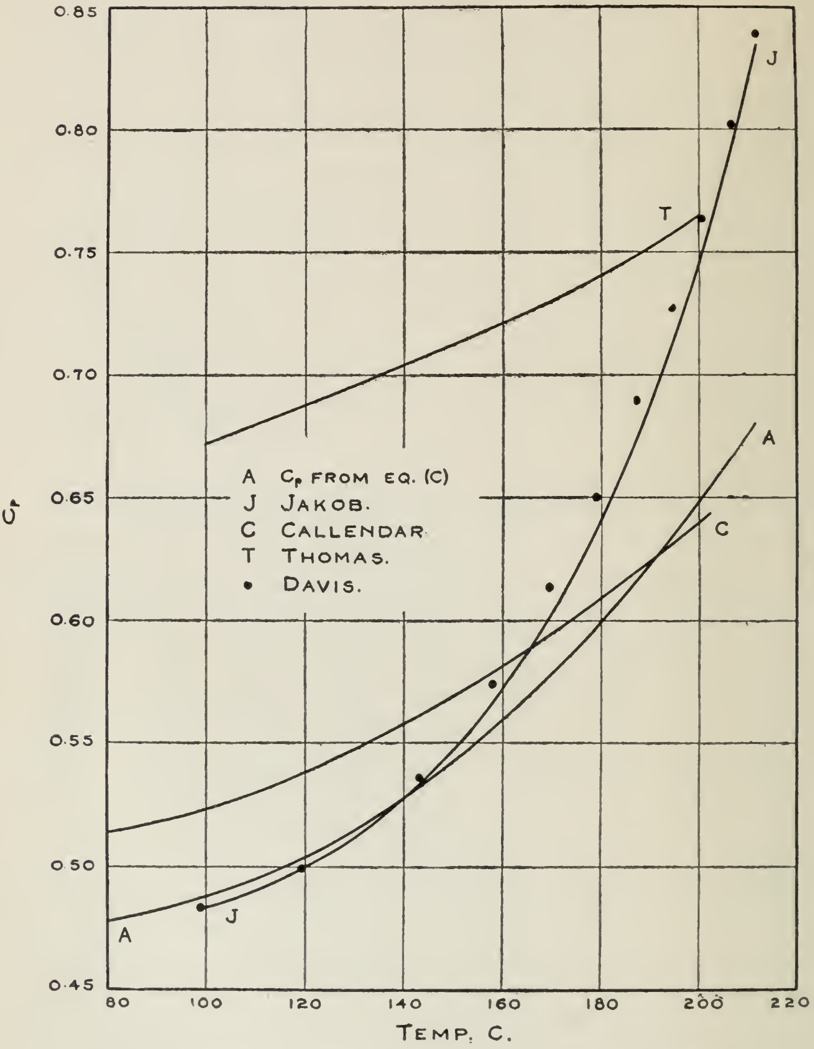

FIG. 7. CURVES OF SPECIFIC HEAT AT THE SATURATION LIMIT.

this region is entirely lacking, and consequently no statement can be made as to the probable relative accuracy of the two curves.

In all discussions of specific heat much emphasis has been laid on the values of $c_p$ at the saturation limit. These give a "saturation curve." From the specific heat equation (C) values of $(c_p)_{sat}$ are readily calculated by taking for $p$ and $t$ corresponding saturation values. Curve *A*, Fig. 7, is the curve thus derived. Davis[1] has given a very full discussion of this subject. The empirical curves of Knoblauch and Jakob were tested by Planck's thermodynamic relation

$$(c_p)_{sat} = \frac{dH}{dt} - \frac{r}{T} + AT\left(\frac{\partial v}{\partial t}\right)\left(\frac{dp}{dt}\right)_{sat}$$

and fair agreement was shown. Davis therefore accepted the Knoblauch values of $(c_p)_{sat}$ with a reservation. Regarding the tremendous rise of Knoblauch's saturation curve at even moderately high temperatures he says: "It is probable that this feature of Knoblauch's curves, although near enough the truth to satisfy the present needs of engineering practice, will have to be revised later." The points in Fig. 7 represent the values of $(c_p)_{sat}$ given by Davis.

Jakob arbitrarily established the values of $(c_p)_{sat}$ for four pressures: 2, 4, 6, and 8 kg. per sq. cm., and from these determined the constants in the assumed formula.

$$(c_p)_{sat} = 0.455 + 2 \cdot 10^{-20} \frac{T_s^8}{T_k - T_s}$$

in which $T_s$ denotes the saturation temperature and $T_k$ the critical temperature. The curve thus found is curve *J*, Fig. 7. It agrees very closely with the Davis curve but runs slightly lower. According to Jakob's formula $(c_p)_{sat}$ becomes infinite at the critical temperature.

Curve *T*, Fig. 7, represents the experiments of Thomas, and curve *C*, the saturation values of $c_p$ calculated by Callendar from purely theoretical considerations.

It is a safe conclusion that curve *A* is nearer the truth than curve *J* or the Davis curve. In the first place, the values represented by curve *A* are obtained from an equation, and as shown in Fig. 5, this equation represents very accurately the best experimental data. The equation automatically extends the curves from the region of the experiments to the saturation curve, thus obviating a doubtful extrapolation that was necessary in laying down the empirical curves. Again, the relation of the curves to the experimental points exhibited in Fig. 5 indicates accurate spacing. The curves show no tendency to run high or low with increasing pressure. Hence it appears probable that equation (C) may be used with confidence for pressures considerably higher than those used in the experiments. At any rate, there is no reasonable doubt that the $c_p$-curve for a pressure of 20 at. calculated from eq. (C) is likely to be nearer the truth than the corresponding curve obtained by the system of extrapolation employed by Knoblauch and Jakob.

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[1] Proc. Am. Acad., Vol. 45, pp. 295-303.

It should be observed that the Davis equation for spacing the curves (see p. 36) and Planck's thermodynamic relation are both automatically satisfied, as both reduce to identities when applied to the equations developed in this investigation.

While Regnault's measurements of $c_p$ at atmospheric pressure are not to be considered as possessing any great degree of precision, some importance may be attached to a comparison of Regnault's results with the corresponding values of $c_p$ calculated from equation (C). The four series of experiments covered the temperature range 122.8°–231.1° C. The mean value of $c_p$ given by Regnault was 0.4805, but this value is lowered to 0.4762 by Davis. All experiments were conducted at atmospheric pressure. The following table gives values of $c_p$ at atmospheric pressure calculated from the equation, also the values assigned by Jakob for the slightly lower pressure, 1 kg. per sq. cm.

TABLE 10.

SPECIFIC HEAT AT ATMOSPHERIC PRESSURE.

| Temp. C. | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
|---|---|---|---|---|---|---|---|
| From Eq. (C)..... | 0.489 | 0.474 | 0.470 | 0.472 | 0.476 | 0.483 | 0.491 |
| Jakob........... | 0.482 | 0.473 | 0.471 | 0.473 | 0.477 | 0.483 | 0.490 |

The mean $c_p$ deduced from the equation agrees very well with the recomputed value 0.4762, and the Regnault experiments therefore strengthen the evidence in favor of the specific heat equations.

## D. TOTAL HEAT, LATENT HEAT, HEAT OF LIQUID.

24. *Equation for Heat Content.*—From the characteristic equation and the equation for specific heat an expression for the heat content is readily deduced. In the general equation

$$di = c_p\,dT - A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]dp$$

we introduce the expression for $c_p$ given by eq. (C) and the expression for the derivative $\left(\frac{\partial v}{\partial T}\right)_p$ obtained from the characteristic equation, namely

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{B}{p} + \frac{mn}{T^{n+1}}\left(1 + 3ap^{\frac{1}{2}}\right).$$

The result of the substitutions is the equation

$$di = \left[\alpha + \beta T + \frac{\gamma}{T^2} + \frac{Amn\ (n+1)}{T^{n+1}}\, p\ (1 + 2ap^{\frac{1}{2}})\right] dT$$
$$- A\left[\frac{m\ (n+1)}{T^n}\ (1 + 3ap^{\frac{1}{2}}) - c\right] dp,$$

which upon integration gives the following equation for the heat content

$$i = \alpha T + \tfrac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Am\ (n+1)}{T^n}\ p\ (1 + 2ap^{\frac{1}{2}}) + Acp + i_0. \quad \text{(D)}$$

The constant $i_0$ is determined as follows. Corresponding saturation values of $p$ and $t$ at some definite temperature, say 212°, are substituted in the equation, which for this purpose may be written

$$i_{\text{sat}} = \phi\ (p, T) + i_0.$$

The function $\phi\ (p, T)$ is thus calculated, and $i_{\text{sat}}$ being known, $i_0$ is found by subtraction.

Since the constant $c$ is taken as the liquid volume $v'$, the term $Acp$ is $Apv'$, which (see p. 6) is the small difference between the heat content $i''$ and the total heat $q''$. Hence, when applied at the saturation limit, equation (D) gives $i''$ and the same equation with the term $Acp$ omitted gives $q''$.

25. *Tests of the i-equation.*—Three tests may be applied to the equation (D) for heat content. Two of these apply at the saturation limit, the third in the region of superheat.

1. Values of $i_{\text{sat}}$ calculated from equation (D) are compared with values obtained independently by other means. The following table shows a comparison with the Marks and Davis values for $i$:

TABLE 11.

HEAT CONTENT OF SATURATED STEAM.

| Temp. F. | $i''$ | | Temp. F. | $i''$ | | Temp. F. | $i''$ | |
|---|---|---|---|---|---|---|---|---|
| | Eq. (D) | M. & D. | | Eq. (D) | M. & D. | | Eq. (D) | M. & D. |
| 32 | 1072.83 | 1073.4 | 212 | 1151.58 | 1150.4 | 440 | 1202.33 | 1207.1 |
| 40 | 1076.64 | 1076.9 | 240 | 1161.90 | 1160.4 | 480 | 1200.43 | 1210 |
| 80 | 1095.30 | 1094.8 | 280 | 1175.15 | 1173.3 | 520 | 1194.02 | 1207 |
| 120 | 1113.34 | 1112.3 | 320 | 1186.30 | 1184.4 | 560 | 1182.76 | 1196 |
| 160 | 1130.64 | 1129.5 | 360 | 1194.86 | 1193.7 | 600 | 1166. | 1176 |
| 200 | 1146.94 | 1145.8 | 400 | 1200.37 | 1201.3 | | | |

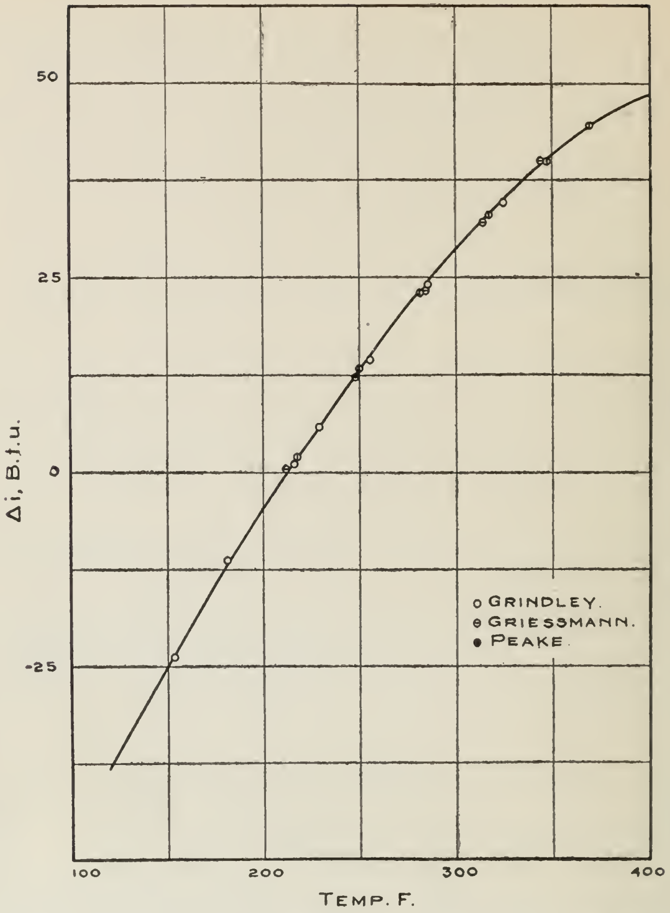

FIG. 8. COMPARISON OF i″-CURVE FROM EQ. (D) WITH POINTS DEDUCED FROM THE THROTTLING EXPERIMENTS.

The considerable discrepancy above 400° has no significance, for in this range Davis makes no claim for the accuracy of the M. and D. values. The effective test is furnished by a comparison of the two sets of values within the range 212° — 400°, where the Davis formula for heat content is surely valid. The comparison is shown graphically in Fig. 8. The points are those determined by Davis from the throttling experiments of Grindley, Greissmann and Peake, and they are plotted from the data given in Table 1 of Davis' paper.[1] The ordinates represent the difference between the $i$ at the given temperature $t$ and the $i$ at 212°. The curve therefore represents the equation

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[1] Proc. Am. Acad., Vol. 45, p. 276.

$$i'' - i''_{212} = f(t),$$

where $i''$ is calculated from the formula and $i''_{212} = 1151.58$. The curve does not fit the points quite as well as the Davis second-degree curve, but the agreement is satisfactory, and is probably well within the limits of accuracy of the throttling experiments. Beyond the last point the curve begins to bend downward rather sharply and thus diverge from the prolonged Davis curve. The maximum value of $i''$ is reached at about 440°, the Davis equation gives the maximum at about 550°, and in the Marks and Davis tables the maximum occurs at 480°.

2. The Clapeyron-Clausius relation furnishes a valuable test within the range 32° — 212° (see p. 7). The discussion of this point is given in another section.

3. In the region of superheat the heat content formula may be checked by the throttling experiments of Grindley, Griessmann and Peake. According to the principles of thermodynamics a throttling process is also a constant-$i$ process. That is, the points obtained in any particular throttling experiment when plotted on the $p$, $t$ plane should lie on a curve $i = \text{const.}$

In Fig. 9 the points are plotted from the experiments of Peake and Grindley, and constant-$i$ curves calculated from eq. (D) are superposed. The general agreement is satisfactory, especially at the lower pressures. At the highest pressures Peake's highest curve has a smaller slope than the curve for $i = 1200$ and intersects it. It will be observed, however, that the curve has the slope indicated by Peake's points near saturation and the slope indicated by Grindley's points at some distance from the saturation limit.

The divergence of the curves from Peake's points naturally suggests the possibility that the curves run too low at the higher superheats. If this were the case, values of $i$ calculated from equation (D) would be somewhat greater than the true values. Some light is thrown on this point by the series of throttling experiments made by Dodge.[1] In these experiments the steam was initially highly superheated, and the initial pressure was about 300 lbs. per sq. in. gauge. The points of 14 tests are shown in Fig. 10; these were taken directly from Davis'[2] discussion of the throttling experiments. Through each group of high-side points

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[1] Jour. A. S. M. E., Vol. 28, p. 1265 (1907); Vol. 30, p. 1227 (1908).
[2] Proc. Am. Acad., Vol. 45, p. 253-257.

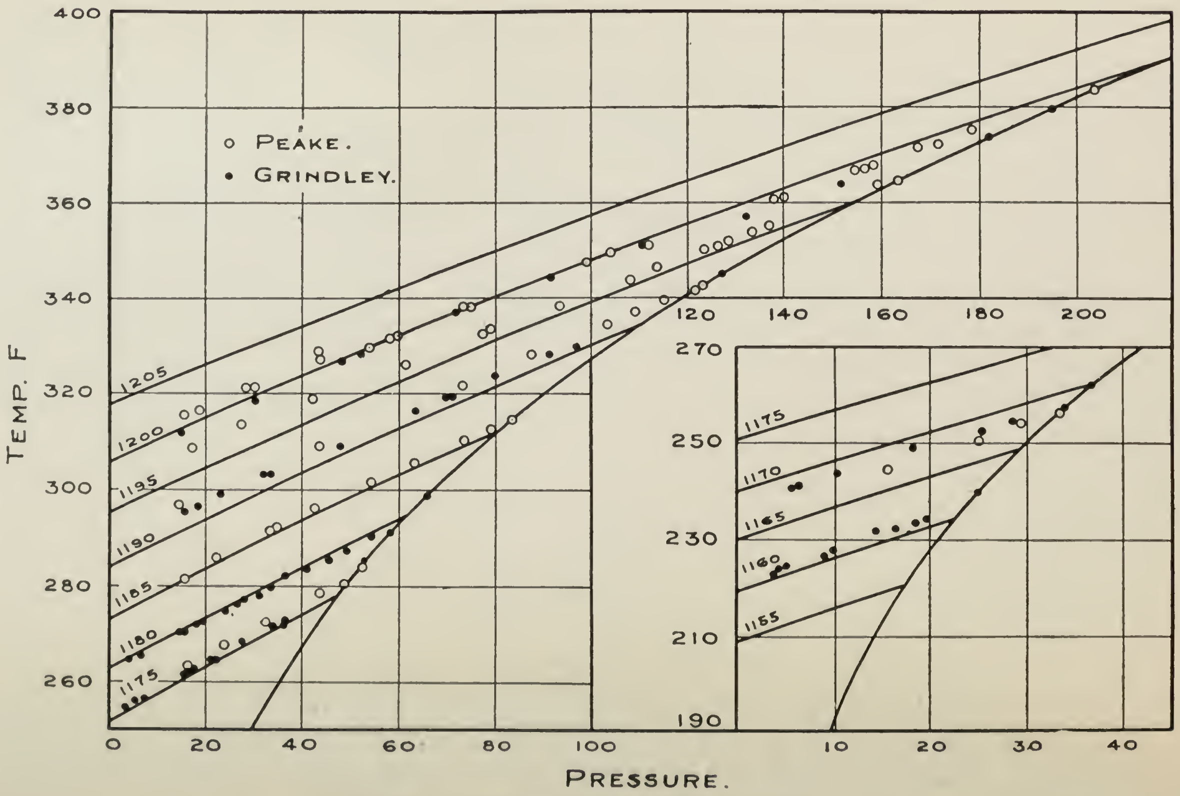

Fig. 9. Curves of Constant Heat Content.

a constant-*i* curve has been drawn. It will be seen that the curves agree fairly well with the points and at the higher superheats have the slopes indicated by general trend of the points. The marked drop from the initial point to the first of the low-side points is a phenomenon that is difficult to explain. In this connection Davis says: "It will be noticed that in every case a smooth curve through the low side points runs considerably below the corresponding high side points, just as did Grindley's curves. In Grindley's case this was because the entering steam carried water in suspension, the presence of which made the true total heat of the incoming mixture less than its apparent total heat regarded as homogeneous saturated steam, and dropped all the low side points onto throttling curves lower than those on which they apparently belonged. A similar phenomenon may be in evidence in Dodge's case, for although the incoming steam was superheated, it may still have been carrying in suspension a part of the water which had been sprayed into it for temperature regulation just before it reached the high-side chamber. It must, however, be admitted that if this explanation is to account for the whole of the discrepancy in Dodge's results, an extraordinarily large

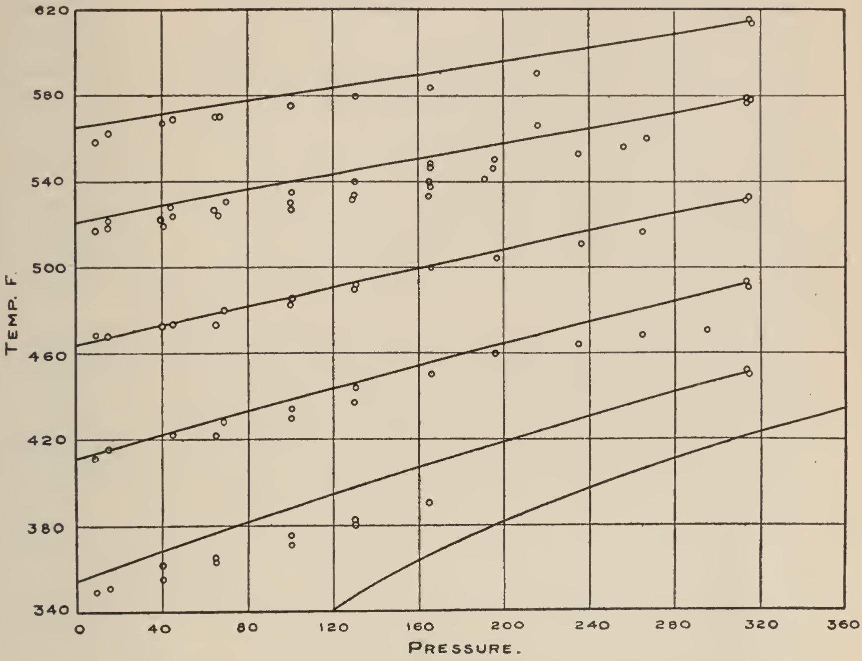

FIG. 10. COMPARISON OF CONSTANT-i CURVES WITH POINTS FROM DODGE'S THROTTLING EXPERIMENTS.

amount of water in suspension must have reached the high-side chamber—from one to one and a half per cent of the whole weight present. It is therefore probable that there is another source of error not yet discovered."

At the high superheats employed by Dodge it appears unlikely that such an amount of water could have been present, and it is probable that all the throttling experiments were vitiated to some extent by systematic errors. A probable source of error was the use of mercury thermometers. It has been shown quite conclusively that a mercury thermometer is not a good instrument for measuring temperatures in a current of superheated steam. If the throttling experiments could be repeated with the same precautions as have been observed in the other experiments on thermal properties, the results would have great value.

Bearing in mind the probable lack of precision in the sets of throttling experiments, it may be concluded that these experiments, on the whole, sustain the validity of the proposed heat content equation in the region of superheat.

A comparison of various values of the heat content $i$ in the region of superheat is shown in Fig. 11, which was suggested by Fig. 21 of Heck's paper. The full lines represent equation (D), the dash lines represent Heck's new equation, and the points represent values given in the Marks and Davis tables. It is significant that the two equations deduced from entirely different bases should agree so closely.

26. *Specific Heat of Water. Heat of Liquid.*—For the temperature range 32°–212° F. (0°–100° C.) there are available five sets of experiments on the variation of the specific heat of water with the temperature. The curves that represent the results of these experiments are separable into two groups having quite different characteristics. Ludin[1] working with the method of mixtures obtained a curve which shows a minimum value of $c'$ at about 20° C., then a rapid rise to a maximum at 87° C. (See Marks and Davis Steam Tables, Fig. 1, p. 88). The curves obtained by Dieterici[2] and Barnes[3] are similar in character; each shows a decrease of $c'$ to a well defined minimum then a steady rise without any suggestion of a maximum. The experiments of Regnault and Dieterici above 100° C. show a steady rise of the specific heat with the temperature; hence if Ludin's curve be accepted, the specific heat after reaching its maximum at 87° must diminish and then increase

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[1] Inaug. Diss. Zurich, 1895.

[2] Annalen der Physik (4), Vol. 16, pp. 593-620 (1905).

[3] Phil. Trans., Vol. 199-A, pp. 55-148, 149-263 (1902).

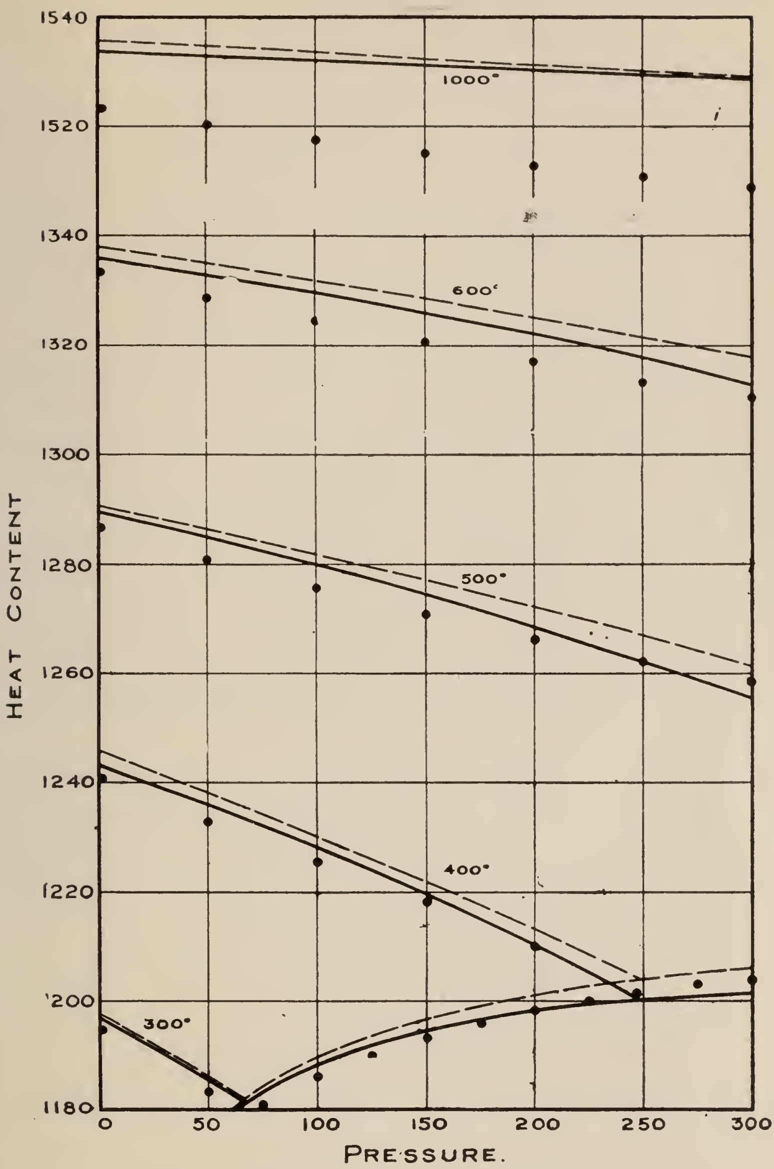

FIG. 11. ISOTHERMAL CURVES ON THE ip-PLANE.

again. It is difficult to account for such a variation on any rational basis, and on the score of intrinsic probability, the curves of Barnes and Dieterici should be preferred to Ludin's curve. Davis[1] attached no weight whatever to Ludin's values and adopted a curve lying between those of Barnes and Dieterici, with Barnes' values given double weight. However, the question is again complicated by the experiments of W. R. and W. E. Bousfield[2] which reproduce Ludin's results, although the method employed (electric heating with a vacuum-jacket calorimeter) was entirely different from Ludin's method of mixtures. Callendar[3] has undertaken to throw light on the subject by a set of experiments in which a new and very accurate method is employed. Callendar's paper contains an exhaustive and valuable discussion of the whole subject.

The methods used by Barnes and Callendar, respectively, have the marked advantage of being continuous. In the Barnes experiments a steady current of water was heated through a small range of temperature by an electric current, and the result obtained was therefore the actual specific heat at a pre-determined temperature rather than the mean specific heat over a considerable range. Callendar used a continuous-mixture method in which two steady currents of water at different temperatures are passed through a system of concentric tubes which constitute a heat exchanger. The continuous flow methods have obvious advantages over other methods. The water equivalent of the calorimeter is not required, and various corrections that involve uncertain measurements are eliminated.

The results of Callendar's experiments by the continuous-mixture method completely verify the earlier experiments of Barnes by the continuous-electric method. As these two independent methods are much superior to the other methods used and give identical results, there can be no question that these results should be accepted.

Taking the specific heat of water at 20° C. as unity, Callendar gives the following equation for the variation of the specific heat with temperature

$$c' = 0.98536 + \frac{0.504}{t+20} + 0.0084\frac{t}{100} + 0.0090\left(\frac{t}{100}\right)^2.$$

From the specific heat $c'$ the heat content $i'$ of the liquid is derived by the relation

$$i' = \int c'dt.$$

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[1] Steam Tables and Diagrams, p. 89.

[2] Phil. Trans., Vol. 211-A, pp. 199-251 (1911).

[3] Phil. Trans., Vol. 212-A, pp. 1-32 (1913).

After changing from C. to F. temperatures and applying a factor to reduce from the 20°-calorie to the mean calorie, the equation for $i'$ becomes

$$i' = 0.9838t + 2.0856 \log (t+4) + 0.233 \left(\frac{t-32}{100}\right)^2 + 0.09245\left(\frac{t-32}{100}\right)^3 - 34.73.$$

Between 32° and 212° this equation gives values that agree very closely with the Marks and Davis values, as shown by the following table.

TABLE 12.

HEAT CONTENT OF LIQUID, 32°–212° F.

| Temp. F. | 40 | 80 | 120 | 160 | 200 | 212 | 240 |
|---|---|---|---|---|---|---|---|
| Callendar................. | 8.05 | 48.05 | 87.94 | 127.87 | 167.94 | 180 | 208.2 |
| M. & D.................... | 8.05 | 48.03 | 87.91 | 127.86 | 167.94 | 180 | 208.3 |

For temperatures above 212°, two sets of experiments are available, Regnault's and Dieterici's, neither of which can be accepted as thoroughly reliable. Regnault's results have been recomputed by various investigators. In Fig. 12, six mean values deduced from Callendar's computation are shown. The ordinates in this figure represent values of

$$\Delta i' = i' - (t-32),$$

that is, the excess of the heat content over $t-32$, the temperature range.

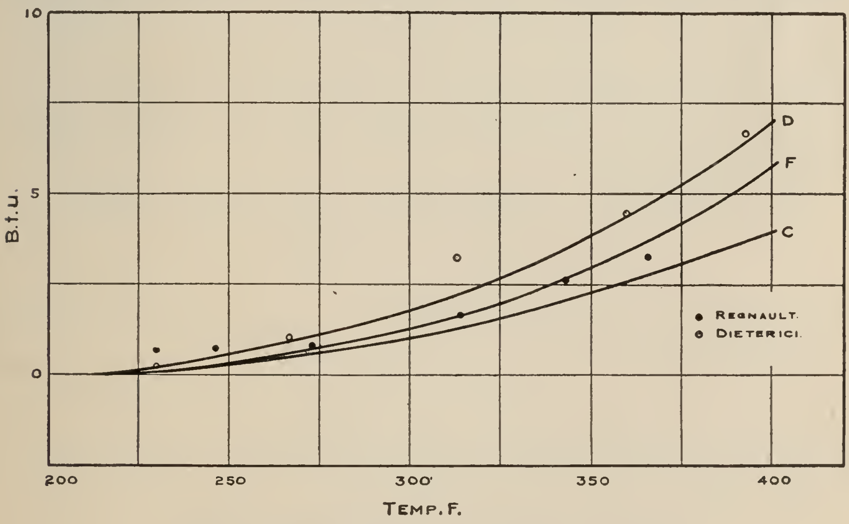

FIG. 12. HEAT CONTENT OF WATER, 212°-400° F.

Abscissas are temperatures F. In the same figure are shown five points obtained by Dieterici, and in Fig. 13, the temperature range is extended and all of Dieterici's points are plotted. The curve *D*, Fig. 12, represents the equation adopted by Dieterici and curve *C* represents Callendar's equation extended beyond 212°.

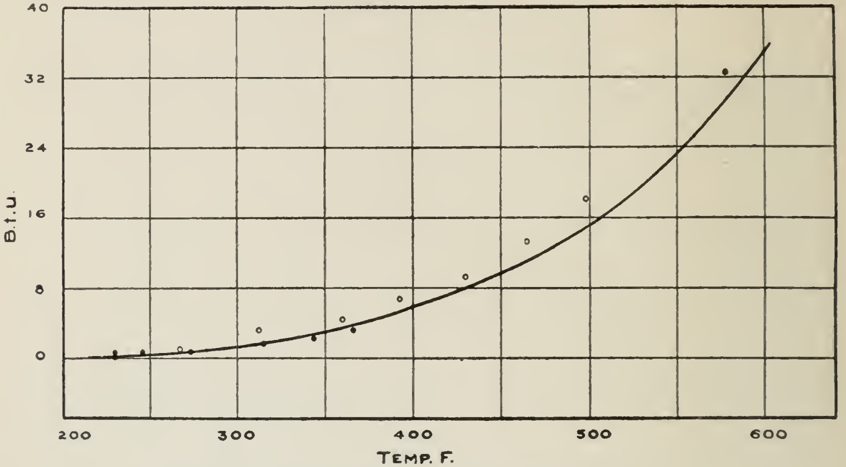

FIG. 13. HEAT CONTENT OF WATER, 212°-600° F.

Callendar questions the accuracy of Dieterici's experiments and gives preference to his equation extrapolated through the range 100° — 200° C. It is probable that Dieterici's points are considerably in error, as the method of the experiments involved large corrections, and it is also probable that Regnault's points are no more reliable. However, there seems to be no valid reason for choosing a curve, like curve *C*, lying below both sets of points.

27. *Latent Heat of Saturated Steam.*—The Clapeyron relation

$$r = A\,(v'' - v')\,T\,\frac{dp}{dT}$$

gives a means of calculating the latent heat. It is convenient to write the equation in the form

$$r = Ap\,(v'' - v')\,\frac{T}{p}\,\frac{dp}{dT}$$

in which the second member is made up of two factors. From the characteristic equation, the first is expressed by

$$Ap\,(v'' - v') = ABT - Ap\,(1 + 3ap^{1/4})\,\frac{m}{T^n}\,.$$

Upon differentiating equation (A) connecting the pressure and temperature of saturated steam, namely

$$\log p = A - \frac{B}{T} - C \log T - DT + ET^2 - \Delta$$

the second factor is obtained in the form

$$\frac{T\,dp}{p\,dT} = 2.3026 \left[ \frac{B}{T} - DT + 2ET^2 - T\frac{d\Delta}{dt} \right] - C$$

For the range 32° — 212°, within which the heat of the liquid is given accurately by the experiments of Barnes and Callendar, a second independent method of calculating the latent heat is available. Saturation values of $i$ are calculated from the formula for heat content and from these are subtracted the corresponding known values of the heat of the liquid. The difference gives, of course, the latent heat. The following table gives values of $r$ obtained by the two methods, and for comparison the Marks and Davis values are included.

TABLE 13.

LATENT HEAT, 32°–212° F.

| Temp. F. | 32 | 40 | 80 | 120 | 160 | 200 | 212 |
|---|---|---|---|---|---|---|---|
| $i''$ from Eq. (D) ............ | 1072.83 | 1076.64 | 1095.30 | 1113.34 | 1130.64 | 1146.94 | 1151.58 |
| $i'$ Barnes & Callendar......... | 0 | 8.05 | 48.05 | 87.94 | 127.87 | 167.94 | 180.00 |
| $r$ by subtraction............. | 1072.83 | 1068.59 | 1047.25 | 1025.40 | 1002.77 | 979.00 | 971.58 |
| $r$ by Clapeyron relation ...... | 1072.19 | 1067.96 | 1046.81 | 1025.11 | 1002.62 | 978.97 | 971.58 |
| $r$ Marks & Davis ............ | 1073.4 | 1068.9 | 1046.7 | 1024.4 | 1001.6 | 977.8 | 970.4 |

Above 212° the heat of the liquid is so uncertain that the method of determining $r$ by subtraction is hardly justified. Hence values of $r$ are calculated from the Clapeyron relation, and subtracted from corresponding values of $i''$. The result is a set of values of $i'$ that may be compared with the Regnault and Dieterici experimental values. The following table exhibits the details of the calculation.

TABLE 14.

LATENT HEAT AND HEAT OF LIQUID, 212°–600° F.

| Temp. F. | 212 | 240 | 280 | 320 | 360 | 400 | 440 | 480 | 520 | 560 | 600 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $i''$ from Eq. (D)... . | 1151.58 | 1161.90 | 1175.15 | 1186.30 | 1194.86 | 1200.37 | 1202.33 | 1200.43 | 1194.02 | 1182.76 | 1166.00 |
| $r$ from Clap. rel. ... | 971.58 | 953.73 | 926.42 | 896.51 | 863.45 | 826.66 | 785.60 | 739.64 | 687.96 | 629.55 | 563.06 |
| $i'$ by subtraction.... | 180 | 208.17 | 248.73 | 289.79 | 331.41 | 373.71 | 416.73 | 460.79 | 506.06 | 553.21 | 602.93 |
| $r$ Marks & Davis ... | 970.4 | 952.1 | 924.3 | 894.2 | 861.8 | 827.2 | 790.1 | 748 | 700 | 642 | 572 |
| $i'$ " " " ... | 180 | 208.3 | 249.0 | 290.2 | 331.9 | 374.1 | 417 | 462 | 507 | 554 | 604 |

Referring to the first of the preceding tables, the close agreement of the two sets of values of $r$ may be noted. The greatest difference, which occurs at 32°–40° is about 6 in 10 000. This agreement is a decisive test of the validity of the analysis. The two sets of numbers are obtained independently, one from the characteristic equation, the other from the heat-content equation, and the agreement between the two shows the satisfaction of the Clapeyron relation. Of the two sets the one obtained from the heat-content equation should be chosen, rather than the set derived by means of the Clapeyron relation. The reason for this lies in the slight uncertainty in the exact value of the derivative $\frac{dp}{dt}$ at low temperatures. It was shown in connection with Fig. 1 that the course of the Scheel and Heuse points indicates that the true value of this derivative at 32° is probably slightly greater than the value obtained from either of the three formulas. The slightly lower values of $r$ calculated from the Clapeyron relation in the range of 32° — 80° may be ascribed, therefore, to a small error in the derivative.

For the range 212°–600° the important result is the set of values of $i'$, heat of the liquid. The fairly close agreement of these values with the Marks and Davis values will be observed. The latter numbers were calculated from the Dieterici formula. In Fig. 12 curve $F$ represents the new set of values for the range 212°–400° F., and in Fig. 13 the curve is extended to 600° F. The curve lies between Dieterici's curve and Callendar's extrapolated curve and represents very well the Regnault experiments as interpreted by Callendar. Above 400° the curve runs from 1 to 3 B. t. u. lower than the Dieterici points, a deviation of 0.2 to 0.6 per cent. Dieterici admits a possible error of 0.3 to 0.5 per cent in the experiments to determine the mean specific heat $c_m$ and a further error in the reduction of $c_m$ to the actual specific heat. It is likely that a possible error of at least 1 per cent may be attached to Dieterici's points; hence if the points are too high, as is indicated by Regnault's experiments and Callendar's extrapolated formula, the curve probably represents the true values fairly well.

28. *Experiments on Latent Heat.*—The values of the latent heat $r$ given in Table 13 may be compared with direct experiments within the range 32°–212°. For this purpose four sets of experiments are available, those of Dieterici,[1] Griffiths,[2] Smith,[3] and Henning.[4] The

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[1] Annalen der Physik, Vol. 37, pp. 494-508 (1889).
[2] Phil. Trans., Vol 186-A, pp. 261-341 (1895).
[3] Phys. Review, Vol. 25, pp. 145-170 (1907).
[4] Annalen der Physik (4), Vol. 21, pp. 849-878 (1906).

following table gives the results of these experiments expressed in a common unit, the mean B. t. u.

TABLE 15.

EXPERIMENTAL DETERMINATIONS OF LATENT HEAT.

| | Temp. F. | Latent Heat B. t. u. |
|---|---|---|
| **Dieterici** | 32 | 1072.9 |
| **Griffiths** | 86 | 1045.1 |
| | 104.3 | 1034.1 |
| Smith | 57.1 | 1061.6 |
| | 70.1 | 1054.5 |
| | 82.5 | 1047.6 |
| | 103.6 | 1035.0 |
| Henning | 86.2 | 1043.2 |
| | 120.5 | 1026.2 |
| | 148.7 | 1008.0 |
| | 171.2 | 995.4 |
| | 192.7 | 983.3 |
| | 213.1 | 969.8 |

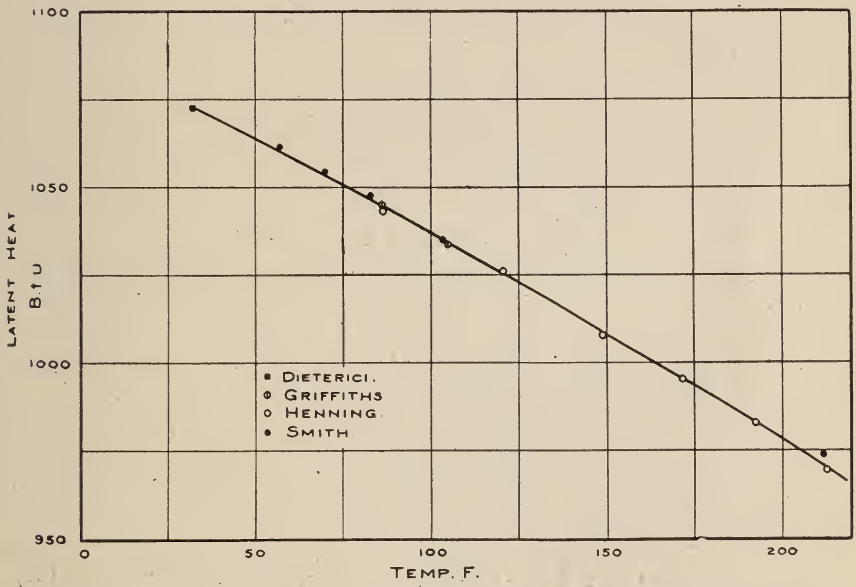

FIG. 14. COMPARISON OF CURVE FOR LATENT HEAT WITH EXPERIMENTAL VALUES.

In Fig. 14 these results are shown by the plotted points and the curve represents the variation of $r$ according to the heat-content equation.

The agreement is satisfactory, though Smith's points would indicate that the calculated values may be slightly low.

Special interest attaches to the value of $r$ at 212° F. For years Regnault's number 966 B. t. u. was universally accepted. Callendar in his 1900 paper had the courage to raise this value to 972, which is almost precisely the value that is now considered most probable. Davis made use of the experiments of Henning and Joly at 212° and set the value of $r$ at 970.4 B. t. u. Smith's recent experiments[1] on slow vaporization of water under atmospheric pressure indicate a value higher than any yet assumed, and Dr. Davis in a communication to the writer expresses the opinion that the number should be at least 972. Heck uses the value 971.2, Mollier uses 971.4. The present investigation leads to the value 971.6, which is probably quite close to the truth, though if anything slightly low.

29. *Entropy.*—An expression for the entropy of superheated steam is readily obtained from the fundamental equation,

$$dq = c_p dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp.$$

Dividing by $T$,

$$ds = \frac{dq}{T} = c_p \frac{dT}{T} - A\left(\frac{\partial v}{\partial T}\right)_p dp.$$

From the characteristic equation

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{B}{p} + \frac{mn}{T^{n+1}}(1 + 3ap^{\frac{1}{2}}).$$

Introducing this and the expression for $c_p$ in the preceding equation, the result is

$$ds = \left[\frac{\alpha}{T} + \beta + \frac{\gamma}{T^3} + \frac{Amn\ (n+1)}{T^{n+2}}\,p\,(1 + 2ap^{\frac{1}{2}})\right]dT - \frac{AB}{p}dp$$
$$- \frac{Amn}{T^{n+1}}(1 + 3ap^{\frac{1}{2}})\ dp.$$

The integration of this exact differential equation gives the following equation for the entropy

$$s = \alpha \log_e T + \beta T - \tfrac{1}{2}\frac{\gamma}{T^2} - AB \log_e p - \frac{Amn}{T^{n+1}}\,p\,(1 + 2ap^{\frac{1}{2}}) + s_0. \quad \text{(E)}$$

The constant $s_0$ is found by applying the equation at the saturation limit. The value thus determined is $s_0 = 0.08085$.

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[1] Physical Review, Vol. 33, p. 183 (1911).

For the range 32°–212°, within which Callendar's formula for the heat of the liquid is surely applicable, there are available two independent methods of calculating the entropy of saturated steam. 1. The entropy of the liquid $s'$ is determined by the integration of Callendar's equation for $i'$ and the entropy of vaporization $\frac{r}{T}$ is added. 2. Corresponding saturation values of $p$ and $T$ are substituted directly in the preceding formula for $s$. The two methods should give substantially identical results.

Above 212° the entropy $s''$ of saturated steam is calculated from formula (E) and the entropy of the liquid $s'$ is obtained by the relation

$$s' = s'' - \frac{r}{T}.$$

Integration of Callendar's $i'$-equation gives the following formula for $s'$:

$$s' = 2.3623 \log T + 0.0045775 \log (t + 4) - 0.00022609\, T$$
$$+ 0.000\,000\,13867\, T^2 - 6.28787.$$

The following table shows the results of the calculation, the Marks and Davis values being included for comparison.

TABLE 16.

ENTROPY OF LIQUID AND SATURATED VAPOR, 32°–212° F.

| Temp. F. | 32 | 40 | 80 | 120 | 160 | 200 | 212 |
|---|---|---|---|---|---|---|---|
| $s'$ from Callendar Eq. | 0. | 0.0163 | 0.0933 | 0.1646 | 0.2312 | 0.2939 | 0.3120 |
| $\frac{r}{T}$ | 2.1823 | 2.1389 | 1.9408 | 1.7692 | 1.6184 | 1.4842 | 1.4467 |
| $s''=s'+\frac{r}{T}$ | 2.1823 | 2.1552 | 2.0341 | 1.9338 | 1.8496 | 1.7781 | 1.7587 |
| $s''$ from Eq. (E) | 2.1822 | 2.1550 | 2.0340 | 1.9338 | 1.8497 | 1.7782 | 1.7587 |
| Marks & Davis | | | | | | | |
| $s'$ | 0. | 0.0162 | 0.0932 | 0.1645 | 0.2311 | 0.2937 | 0.3118 |
| $\frac{r}{T}$ | 2.1832 | 2.1394 | 1.9398 | 1.7674 | 1.6165 | 1.4824 | 1.4447 |
| $s''$ | 2.1832 | 2.1556 | 2.0330 | 1.9319 | 1.8476 | 1.7761 | 1.7565 |

30. *Intrinsic Energy.*—From the defining equation

$$i = A\,(u + pv)$$

the energy $u$ in thermal units is readily obtained by subtraction; thus

$$u = i - Apv$$

Combination of equations (B) and (D) gives therefore the following explicit expression

$$u = (a - AB)\, T + \tfrac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Amnp}{T^n}\left(1 + \frac{2n-1}{n} a p^{\frac{1}{2}}\right) + i_0. \quad \text{(F)}$$

## E. JOULE-THOMSON EFFECT.

31. *The Joule-Thomson Coefficient.*—The ratio of the drop in temperature to the decrease of pressure when a fluid is subjected to a throttling process is the Joule-Thomson coefficient $\mu$. Since in such a process the heat content $i$ is constant, the value of $\mu$ is given by the derivative $\frac{dT}{dp}$ evaluated under the condition that $i$ is constant; that is

$$\mu = \left(\frac{dT}{dp}\right)_{i\,=\,\text{const.}}$$

It follows, therefore, that the slope of a throttling curve (See Fig. 9) at any point is the value of $\mu$ at that point.

An expression for $\mu$ is readily derived from the equation for heat content,

$$i = \alpha T + \frac{1}{2}\beta T^2 - \frac{\gamma}{T} - \frac{Am(n+1)}{T^n}\,p\,(1 + 2ap^{\frac{1}{2}}) + Acp + i_0.$$

With $i$ constant

$$\left(\frac{dT}{dp}\right)_{\text{i}} = -\left(\frac{\partial i}{\partial p}\right)_{\text{T}} \div \left(\frac{\partial i}{\partial T}\right)_{\text{p}}.$$

But

$$\left(\frac{\partial i}{\partial T}\right)_{\text{p}} = c_{\text{p}},$$

hence

$$\mu = \frac{A}{c_{\text{p}}}\left[\frac{m\,(n+1)}{T^n}(1 + 3ap^{\frac{1}{2}}) - c\right] \qquad \text{(G)}$$

32. *Comparison with the Throttling Experiments.*—From the four available sets of throttling experiments, Davis[1] has computed values of $\mu$ covering a wide range of pressure and temperature. As might be expected, the results when plotted showed a broad band of points (See Fig. 6 of Davis' paper) and individual measurements showed considerabl discrepancy. By passing a smooth curve through the band of points Davis obtained the relation between $\mu$ and $t$ that was afterwards used by himself in spacing the specific heat curves, and by Heck in the development of his theory. Davis therefore assumes that $\mu$ is a function of the temperature only, while according to the preceding equation (G) $\mu$ must vary with the pressure also.

A comparison of equation (G) with the values of $\mu$ computed by Davis is shown in Fig. 15. Four different pressures are used, 36, 120, 200, and 300 lb. per sq. in., and the points are plotted from the data

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[1] Proc. Am. Acad., Vol. 45, pp. 243-264.

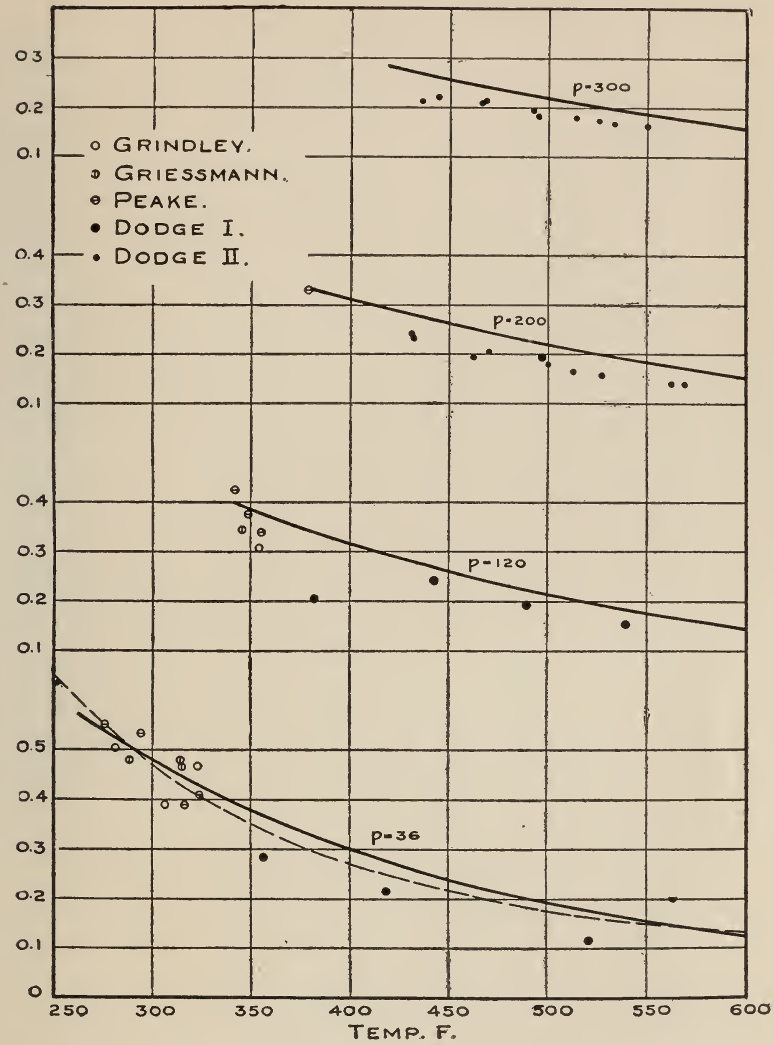

FIG. 15. JOULE-THOMSON COEFFICIENT.

given in the tables of Davis' paper. The points in each group are so chosen that the pressure in the experiments represented by them is very nearly the pressure attached to the corresponding curve. The dash curve through the lowest group is the Davis curve for $\mu$.

It will be seen that the curves agree sufficiently well with the points obtained from the first three sets of throttling experiments, but that they lie somewhat above the points deduced from Dodge's experiments. Reference to Fig. 10 will afford an explanation of the discrepancy in the case of Dodge's first series. The peculiar discontinuity between the high-side point and the first low-side point has already received comment (See p. 47). Davis rejected the high-side point entirely and obtained values of $\mu$ by a comparison of low-side points only. This was equivalent to taking the slopes of smooth curves running through the low-side points, and it is evident from the figure that the values of $\mu$ thus obtained are smaller than the values given by the slopes of the theoretical curves. Dodge's second series of 33 experiments was of such a character that the high-side observations had to be used. The values of $\mu$ obtained directly from the observations were, according to Davis, abnormally high, and certain corrections were made to reduce them to the level required by the law of corresponding states. The chief correction was a reduction of the high-side temperature by an amount ranging from 11° to 14°. If the high-side points are incorrect, as assumed by Davis, because of water carried in suspension in the highly superheated steam, the plotted points representing Dodge's experiments must be accepted, and the curves obtained from eq. (G) run somewhat high. If, on the other hand, the error lies in the low-side points, the values of $\mu$ calculated by Davis are too low, and the curves probably are nearer the truth than the points. The question can only be settled by a new series of experiments with suitable precautions to insure the dryness of the steam and with improved methods of temperature measurement.

The other question that presents itself relates to the variation of $\mu$ with the pressure. According to equation (G) this variation must exist, though it is small. Davis states that he was unable to detect such variation from an examination of the observations. Heck assumed at the start of his investigation that $\mu$ depended on $t$ alone, but the further development of the theory led him to the conclusion that the assumption could not be rigorously correct. It is apparently impossible to construct a general theory that will give satisfactory agreement in other parts of the field and at the same time leave the coefficient $\mu$ unaffected by the pressure. We conclude, therefore, that $\mu$ *is* a function of the pressure; and so far as our present knowledge extends, the value of $\mu$ is given satisfactorily by equation (G).

### F. THERMAL PROPERTIES NEAR THE CRITICAL TEMPERATURE.

33. *Previous Investigations.*—That the equation (B) to (F) (at the saturation limit) should hold good up to the critical temperature is not to be expected. In the vicinity of the critical point the variation of some of the thermal properties is so rapid that to represent these properties, equations of a different form must be used. As there are no trustworthy experiments in the range 600° F. to the critical temperature, any formulation covering this region must be regarded as a guess. However, even a conjecture may have a qualitative value, and in the present case the investigation at least serves to establish quite closely the upper limit of temperature for the equations developed in the preceding sections.

Several attacks have been made on this problem. Davis[1] by the use of Thiesen's exponential formula determined the latent heat $r$, and by "the law of the straight diameter" established tentatively the specific volume $v''$. Schüle[2] has likewise estimated certain of the thermal properties near the critical temperature, and his work appears to have a reasonable basis. Another tabulation was published in *Engineering* (Jan. 4, 1907); this is palpably incorrect and has no value whatever.

34. *Thiesen's Exponential Formula.*—At the critical temperature, the latent heat $r$ becomes zero, and furthermore $\frac{dr}{dt} = -\infty$. An equation connecting $r$ and $t$ may be given a form such that these conditions will be satisfied. The simplest form is that suggested by Thiesen, namely.

$$r = C\,(t_k - t)^n,$$

in which $t_k$ denotes the critical temperature, and $n < 1$. If now the constants can be so determined as to give correct values of $r$ at ordinary temperatures, then it may be inferred that the formula will give reasonably good values for higher temperatures. The formula may be written

$$\log r = \log C + n \log\,(t_k - t)$$

a form more convenient for purposes of calculation.

Davis[3] found that with $\log C = 2.14302$, $n = 0.315$, and $t_k = 689$, the formula represented the available experimental values of $r$ with remarkable accuracy. The number 689 is however the older value of

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[1] Marks and Davis Steam Tables, pp. 102-105.

[2] Zeit. des Verein. deutsch. Ing., Vol. 55, pp. 1561-1567 (1911).

[3] Proc. Am. Acad., Vol. 45, p. 284.

$t_k$ due to Cailletet and Colardeau. The recent experiments of Holborn and Baumann[1] raise the value of $t_k$ to 706.3° F., and with this value it appears to be impossible to make Thiesen's formula fit any series of probable values of $r$ through a wide range of temperature. It is found, however, that the equation

$$\log r = 2.00117 + 0.3685\ (706.3 - t)$$

fits reasonably well the values of $r$ from 360° to 560°. The following table shows the agreement.

TABLE 17.

COMPARISON OF VALUES OF LATENT HEAT.

| Temp. F. | 360 | 400 | 440 | 480 | 520 | 560 | 600 |
|---|---|---|---|---|---|---|---|
| $r$ deduced from Eq. (D) ....... | 863.5 | 826.7 | 785.5 | 739.6 | 688.0 | 629.6 | 563.1 |
| $r$ from Thiesen's Eq. ........... | 864.9 | 826.6 | 785.1 | 739.4 | 688.2 | 629.6 | 559.7 |

Below 360° and above 560° the two sets of values diverge rapidly. It is assumed that the formula will give approximate values of $r$ between 560° and 706.3°, but too much confidence should not be placed in the accuracy of the results thus obtained.

35. *Heat Content of Liquid and Vapor.*—The law of the mean diameter has frequently been applied in determining the specific volume of the vapor. A somewhat similar application is here made in connection with the heat contents $i'$ and $i''$. Taking the sum $i' + i''$ for the temperature range 400°–600° F., it is found that this sum is given very accurately by the equation

$$i' + i'' = 956.94 + 1.923t - 0.00095t^2.$$

It is assumed that this relation also holds good from 600° to the critical temperature. Hence with the sum $i' + i''$ determined by this equation and the difference $i'' - i' = r$ determined from the Thiesen exponential equation, values of $i''$ and $i'$ are readily calculated. The variation of these thermal magnitudes near the critical temperature is shown in Fig. 16. Curve $CD$ is the mean diameter, its ordinate being $\frac{1}{2}\ (i' + i'')$, curve $AD$ represents the heat content $i''$ of the vapor, and curve $BD$ the heat content $i'$ of the water. These are two branches of a continuous curve, and the common value of $i'$ and $i''$ at the critical temperature is apparently about 921 B. t. u.

36. *Specific Volume of Steam.*—The method of Davis[2] has been followed, except that 706.3 is used for the critical temperature. Values of $v''$ the specific volume of steam were calculated from the characteristic

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[1] Annalen der Physik, Vol. 31, pp. 945-970 (1910).

[2] Steam Tables and Diagrams, pp. 103-105.

equation (B). For the values of $v'$, the specific volume of water, the formula of Avernarius

$$v' = a - b \log (t_k - t)$$

was found to give exact results for the temperature range 280° — 600° F., with the following constants: $a = 0.044825$, $b = 0.0105$. Denoting by $2s$ the sum of the densities of the liquid and vapor, that is

$$2s = \frac{1}{v'} + \frac{1}{v''}$$

the variation of this sum with the temperature is given by the equation

$$2s = 67.1 - 0.029131\, t - 0.000009342\, t^2.$$

The Clapeyron relation gives another relation connecting $v'$ and $v''$, namely

$$r = A\, (v'' - v')\, T \frac{dp}{dT}$$

in which the derivative $\frac{dp}{dT}$ is found from eq. (A) and $r$ from the Thiesen formula. The values of $v''$ and $v'$ are therefore readily obtained by the solution of a quadratic equation.

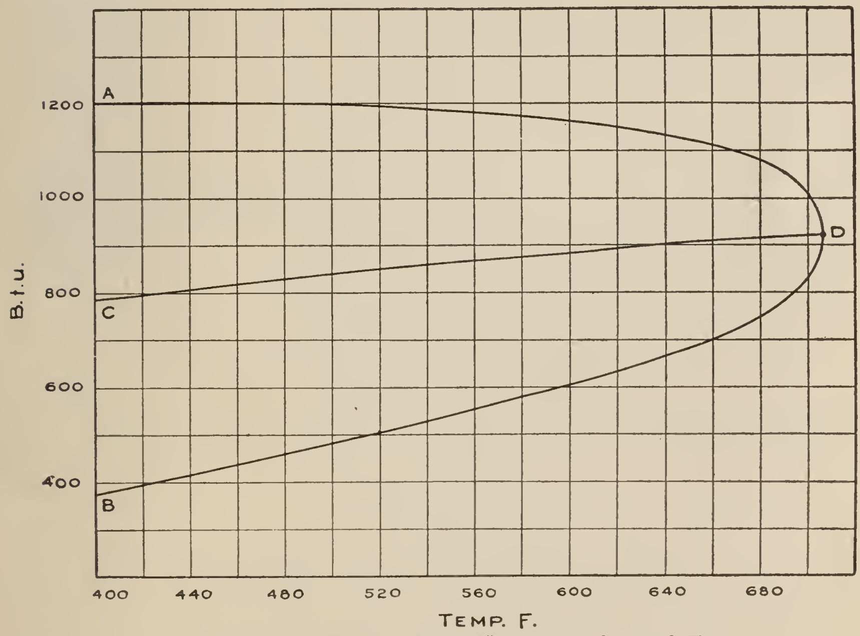

FIG. 16. CURVES FOR i′ AND i″ FROM 400°-706.3° F.

37. *Table of Properties at High Temperatures.*—The following table gives the thermal properties of saturated steam between 600° and the critical temperature, as calculated by the methods just described. It is to be understood that the tabulation is merely a conjecture and that no claim of extreme accuracy is asserted.

TABLE 18.

PROPERTIES OF SATURATED STEAM FROM 600° TO 706.3° F.

| Temp. F. | Pressure: lb. per sq. in. | Volume of 1 lb. | Weight of 1 cu. ft. | Heat Content | | Latent heat |
|---|---|---|---|---|---|---|
| | | | | of liquid | of vapor | |
| 600 | 1540.4 | 0.272 | 3.68 | 604.5 | 1164 | 560 |
| 620 | 1658.7 | 0.241 | 4.15 | 633 | 1151 | 518 |
| 640 | 2056.6 | 0.187 | 5.35 | 663 | 1136 | 473 |
| 660 | 2360.8 | 0.151 | 6.63 | 700 | 1112 | 412 |
| 680 | 2699.1 | 0.118 | 9.86 | 745 | 1080 | 335 |
| 700 | 3074.5 | 0.080 | 12.46 | 823 | 1016 | 193 |
| 706.3 | 3200 | 0.048 | 20.92 | 921 | 921 | 0 |

Comparing the equations used in deducing the values in this table with the original equations (B) and (D), it is found that the former merge into the latter at a temperature lying between 560° and 600° F. The discrepancy at 600° is small. It may be reasonably assumed, therefore, that the general equations developed in this investigation hold good up to 600° F. when applied to the saturated vapor. The corresponding pressure limit is above 1500 lb. per sq. in.

## V. RESUMÉ OF FORMULAS.

For convenience of reference the principal formulas developed in the preceding section are here collected. The constants are for English units with pressures expressed in pounds per square *inch*. The logarithms involved are common logarithms.

1. Relation between pressure and temperature:

$$\log p = 10.5688080 - \frac{4876.643}{T} - 0.0155 \log T - 0.00406258\, T$$

$$+ 0.00000400555T^2 - 0.00002\left[10 - 10 \left(\frac{t-370}{100}\right)^2 + \left(\frac{t-370}{100}\right)^4\right] \quad \text{(A)}$$

$$T = t + 459.6$$

2. For the specific volume:

$$v - c = 0.59465 \frac{T}{p} - (1 + 0.05129p^{½}) \frac{C_1}{T^4} \qquad \text{(B)}$$

in which $\log C_1 = 10.82500$, and $c$ may be given a mean value 0.017.

3. For the specific heat:

$$c_p = 0.320 + 0.000126T + \frac{23583}{T^2} + \frac{C_2 p\,(1 + 0.0342p^{1/2})}{T^5} \qquad \text{(C)}$$

$$\log C_2 = 11.39361.$$

4. For the heat content:

$$i = 0.320T + 0.000063T^2 - \frac{23583}{T} - \frac{C_3 p\,(1 + 0.0342p^{1/2})}{T^4}$$

$$+ 0.00333p + 948.7 \qquad \text{(D)}$$

$$\log C_3 = 10.79155.$$

5. For the entropy of superheated steam

$$s = 0.73683 \log T + 0.000126T - \frac{11791.5}{T^2} - 0.25355 \log p$$

$$- \frac{C_4 p\,(1 + 0.0342p)}{T^5} - 0.08085 \qquad \text{(E)}$$

$$\log C_4 = 10.69464$$

6. For the intrinsic energy of superheated steam, in B. t. u.:

$$u = 0.2099T + 0.000063T^2 - \frac{23583}{T} - \frac{C_5 p\,(1 + 0.02992p^{1/2})}{T^4} + 948.7 \qquad \text{(F)}$$

$$\log C_5 = 10.69464$$

7. Heat of the liquid between 32° and 212° from Callendar's formula:

$$i' = 0.9838t + 2.0856 \log (t + 4) + 0.233 \left(\frac{t - 32}{100}\right)^2 +$$

$$0.09245 \left(\frac{t - 32}{100}\right)^3 - 34.73.$$

8. Entropy of the liquid between 32° and 212°, from Callendar's formula

$$s' = 2.3623 \log T + 0.0045775 \log (t + 4) - 0.00022609\,T$$

$$+ 0.0000012867T^2 - 6.2879.$$

9. External latent heat of saturated steam:

$$\psi = 0.1101\,T - \frac{C_6 p\,(1 + 0.05129p)}{T^4}$$

$$\log C_6 = 10.09258.$$

10. Latent heat of saturated steam:

$$r = \psi\, \frac{T}{p}\, \frac{dp}{dT}$$

in which $\frac{T}{p}\frac{dp}{dT}$ is derived from formula (A).

The preceding equations are sufficient for the calculation of all the thermal properties of saturated and superheated steam that are usually found in steam tables.

## VI. CONCLUSION.

38. *Comparison of Tables and Formulations.*—A table of the properties of steam, or a formulation from which a table may be calculated, should possess two characteristics, *consistency* and *accuracy.* A table or formulation is consistent when the values are obtained from equations that are properly connected by thermodynamic relations, such as the Clausius and Clapeyron relations. It may be considered accurate if the calculated values show close agreement with the most reliable experimental data. In this sense, accuracy is only relative. A table that would have been considered accurate ten years ago would be regarded today as extremely inaccurate in the light of our present knowledge of the subject.

Taking these two characteristics as a basis of classification, the existing tables and formulations may be divided into four groups:

1. Those that possess neither consistency nor accuracy.
2. Those that are consistent but inaccurate.
3. Those that are reasonably accurate but lack consistency.
4. Those that are both consistent and accurate.

In the first group may be placed all the older tables, such as Rankine's, Buel's, Porter's, and Dery's. The tabular values were calculated from empirical formulas based chiefly on Regnault's data, and the necessary connection through thermodynamic relations was not recognized.

In the second group are the tables based on Callendar's theory. As regards consistency these tables are entirely satisfactory, as the thermodynamic relations are satisfied. Mollier's tables appeared before the publication of the decisive Munich experiments, which invalidated some of the fundamental assumptions of Callendar's theory. Smith and Warren, however, have apparently ignored completely the mass of experimental evidence accumulated since 1900, and have deliberately sacrificed accuracy for the sake of consistency. While these two sets of tables are probably more accurate than those in the first group, they are nevertheless grossly inaccurate according to present standards.

The noteworthy researches of Davis have been embodied in the Marks and Davis steam tables and in the last edition of Peabody's tables. The establishment of the new formula for the heat content of saturated steam was the principal achievement of Dr. Davis, and the incorporation of this formula in the tables marks an important advance in accuracy. The Marks and Davis tables are not rigorously consistent, and in some particulars they also exhibit a lack of extreme accuracy. The system of specific heat curves used as the basis of the values in the region of superheat is probably somewhat in error, as noted in a preceding section. The values of the specific volume of superheated steam are probably too high, and the values of the heat content in the region of superheat are almost certainly too low (See Fig. 11). The value 970.4 B. t. u. assumed for the latent heat at 212° F. is doubtless slightly low. Notwithstanding minor defects in accuracy and consistency, the Marks and Davis tables are justly held in high regard, and they have been very generally accepted as standard in engineering practice.

In the fourth group may be included the formulations of Jakob and Heck, and the formulation developed in the preceding sections. In each of these, consistency is secured by a strict observance of thermodynamic relations; at the same time the available experimental data are used to check the equations, and thus a high degree of accuracy is insured. It is probable that a table derived from any one of these formulations would surpass in accuracy the best of existing tables.

With respect to the relative advantages of the three formulations, the following statements may be made. Jakob's scheme is open to the objection that it is graphical. Numerical results must be obtained by measurement of geometrical magnitudes. Furthermore, Jakob's system of specific heat curves, like Davis' system, is probably erroneous near the saturation limit. For these reasons Jakob's formulation may be considered the least satisfactory of the three. Heck's theory doubtless shows good agreement with experimental data, and a table constructed from it should be thoroughly satisfactory as regards accuracy and consistency. In the few cases in which comparisons have been made, the theory described in this bulletin appears to show somewhat closer agreement with experimental data than Heck's theory. Equation (A) for the pressure-temperature relation is evidently superior to Heck's modification of Marks' formula; the characteristic equation (B) represents the Knoblauch measurements more accurately than Heck's equation; and equation (D) for the heat content gives values of $i$ at saturation that agree very closely with the points deduced by Davis from the throttling experiments, while Heck's curve for $i$ runs appreciably higher.

As regards accuracy, the theory under discussion does not suffer in comparison with Heck's theory. In other respects it presents certain obvious advantages. (1) The range within which the formulation may be regarded as valid is somewhat greater. At saturation Heck's equations hold good up to 500° F., or perhaps a little higher; the equations (B) to (F) are valid up to nearly 600° F. (2) Heck's equations are excessively complicated and ill-adapted for computation. Equations (B) to (F) are in comparison relatively simple, and because of the repetition of certain terms in the several formulas the labor of computation is greatly abridged, and numerical results may be obtained with comparative ease.

39. *Effect of Future Experiments.*—The theory under discussion has been correlated with the most reliable experimental data available at the present time. It is, of course, probable that subsequent experiments of a higher order of precision will modify these data to some extent and possibly render necessary a revision of the formulation. It may be profitable in this connection to discuss briefly the sufficiency of existing data and indicate the possible effect of future experiments.

The pressure-temperature determinations may be regarded as final. It is not likely that further experiments will modify these to any extent. Further experiments on specific volumes are desirable; these should include a higher superheat and much greater pressure range than the Munich experiments. Such experiments would probably confirm the results of Knoblauch, Linde, and Klebe. The results of the experiments of Lanz and Schmidt on the specific heat of steam at the higher pressures are awaited with interest. A preliminary report indicates that the experiments of Knoblauch and Mollier are substantially confirmed. The experiments at higher pressures should settle the question of the proper course of the specific heat curves near the saturation limit. With reference to the specific heat of water, the results of Barnes and Callendar for the range 32°–212° should be accepted. For the range above 212° the experimental evidence is at present unsatisfactory and further experiments in this field are urgently needed. Finally, the questions raised by the various discrepancies in the throttling experiments should be settled by a new set of experiments conducted with proper precautions to insure the dryness of the steam and the accuracy of the measurements.

While the theory may require modification as additional experimental data become available, it is believed that such modifications as may be necessary can be made without essential changes in structure. The formulation is extremely sensitive to changes in the constants and any modifications that are likely to be made in the fundamental data, volumes, specific heats, etc., can be accommodated by slight changes in the constants.

40. *Summary.*—In this bulletin is described the development of a general theory of the properties of superheated and saturated steam. For this theory the following claims are urged.

1. *Consistency.*—In the deduction of the fundamental equations the thermodynamic relations have been recognized, and the satisfaction of these relations assures consistency.

2. *Accuracy.*—The equations have been checked with the most reliable experimental data available, and in every respect the agreement is extremely close. It may be asserted that the formulation is at least as accurate as any other that has thus far appeared.

3. *Simplicity.*—The equations are relatively simple and well adapted for computation of numerical results.

4. *Flexibility.*—The structure of the formulation is such that modifications necessitated by further experimental data can probably be made by slight changes of the constants.

5. *Range of Validity.*—At the saturation limit the equations are valid throughout the range 32°–600° F., a range far wider than is at present employed in engineering practice.

A.

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# THE ANALYSIS OF COAL WITH PHENOL AS A SOLVENT

BY

S. W. PARR
AND
H. F. HADLEY

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 76 NOVEMBER, 1914

# THE ANALYSIS OF COAL WITH PHENOL AS A SOLVENT
## (WITH A STUDY OF THE RESULTING COMPOUNDS)

BY S. W. PARR, PROFESSOR OF APPLIED CHEMISTRY, AND
H. F. HADLEY, RESEARCH FELLOW, DEPARTMENT OF CHEMISTRY

## CONTENTS

PAGE
I. INTRODUCTION ..... 3
1. Present Methods of Coal Analysis ..... 3
2. Purpose of the Proposed Study ..... 4
3. Outline of the Investigation ..... 5
4. Summary of Results ..... 5
II. EXPERIMENTAL ..... 6
A. Apparatus and Method of Procedure ..... 6
5. Preliminary ..... 6
6. Phenol and the Conditions which Influence the Amount and Character of Extract ..... 7
7. Apparatus ..... 11
8. Process of Extraction ..... 11
9. Method of Procedure ..... 13
B. Application of the Method to Different Types of Coal ..... 15
C. General Properties of Extract and Residue ..... 18
10. Relation of Extract and Residue to the Coking Properties ..... 18
11. Effect of Heating the Residue and Extract in the Air ..... 19
12. Avidity of Residue and Extract for Water and Oxygen at Ordinary Temperature ..... 21
13. Volatile Matter Determination of Coal, Residue, and Extract ..... 24
14. Ultimate Analysis of Coal, Residue, and Extract ..... 25
15. Thermal Decomposition of Residue and Extract ..... 27
16. The Effect of Oxidation on the Volatile Matter Determination ..... 29
17. The Effect of Oxidation on the Amount of Material Extracted from Coal by Phenol ..... 32
18. The Effect of Oxidation on the Composition of the Gases Produced by Destructive Distillation ..... 33
III. SUMMARY AND CONCLUSIONS ..... 36
IV. HISTORICAL ..... 36

# THE ANALYSIS OF COAL WITH PHENOL AS A SOLVENT

## I. Introduction.

1. *Present Methods of Coal Analysis.*—There are two processes in vogue at the present time for the chemical examination of coal; one is the ultimate, and the other is the proximate method of analysis. In the first the organic or combustible part of the coal is separated into its elemental constituents, carbon, hydrogen, oxygen, and nitrogen. The mineral or non-combustible portion is separately determined under two items as ash and moisture.

In the proximate method the organic material is separated into two divisions, one being that portion which under high temperature and out of contact with the air passes off in the gaseous form, and the other that part which remains behind as the non-volatile or coke-forming carbon.

Each procedure has doubtless come into use as the result of a specific demand. For example, the engineer needed the data from which he could calculate the total heat of the coal and, in arriving at a heat balance, he must also have at hand any negative factors chargeable to the fuel, such as the quantity and character of the gaseous products of combustion. These items, therefore, would call for the data furnished by the ultimate methods of analysis.

The proximate method was developed as a natural accompaniment of the gas and coke industries, since it furnished in either case an index of the yield which might be expected from a given coal. Formerly, also, the quantity of volatile matter was made to serve as an index of the grade or quality of a coal. Thus the data from proximate analyses have been put into the form of "fuel ratios" or the ratio of the non-volatile to the volatile part of the coal, such ratios supposedly serving as an indication of the general class or type to which the coal belonged.

It is evident from this brief statement that the results obtained from ordinary methods of analysis are of rather narrow application and that the data from neither the ultimate nor the proximate methods convey much information which can be applied to the every-day problems connected with the handling and use of coal such as coking, storage, deterioration, oxidation, weathering, and spontaneous combustion.

It is to be noted in this connection that the ordinary methods of analysis give the percentage amounts of certain elements or compounds which result from destructive processes and do not even remotely take account of the actual constituents of the organic or active portions of the coal as they occur in unaltered form. But it is these unaltered constituents which have to do with such matters as coal storage and spontaneous combustion. Especially do they have to do with coking properties, with smoke formation, and the type of distillation products which they may be made to yield.

2. *Purpose of the Proposed Study.*—The purpose of this investigation has been to examine and enlarge upon certain processes of analysis which proceed in such a manner as to separate in unaltered form the fundamental substances of which coal is composed, and to study their characteristic properties in detail.

Efforts along this line have multiplied in recent years. Much suggestive information has come from the study of the types of vegetable matter which constituted the source of the initial coal substance. The microscopical studies of White,[1] of the United States Geological Survey, and Professor Jeffrey,[2] of Harvard, have developed the general proposition that the original organic constituents would fall into two general classes: those of the cellulose and those of the resinic type. A similar line of evidence has been developed by numerous investigators, who have made use of various solvents, which, in the main, have differentiated the compounds along the same general lines. The work of Frazer and Hoffman,[3] Clark and Wheeler,[4] and Professor Lewes[5] may be cited in this connection.

In conformity, therefore, to this general proposition as to the coal constituents this investigation proceeded upon the general hypothesis that in coal two types of compounds are to be met with: (1) the degradation products of the original cellulose substances, and (2) the less altered and perhaps more characteristic compounds of the resinic type. The first line of investigation, therefore, from the chemical standpoint, would seem to consist of a study of various solvents which might lend themselves to such a separation into the two main subdivisions of the coal constituent, and, second, a study of the general properties of each branch.

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[1] Professional Paper 85-E, United States Geological Survey (1914).
[2] Proc. Am. Acad. of Arts and Sciences, 46, 273 (1910).
[3] Technical Paper 5, Bureau of Mines (1912).
[4] Journal of Chemical Society, 103, 1704 (1913).
[5] Progressive Age, 29, 1030 (1911).

3. *Outline of the Investigation.*—(1). Various extractive agents were used to determine the extent or completeness of their solvent properties. Solvents were selected which would seem to have the greatest ability to remove those bodies which were of the resinic type. The substances tried are arranged in the order of their solvent properties in so far as that would seem to be indicated in preliminary studies directly concerned with the extent of the solvent action of each. The list is as follows: phenol, ortho-cresol, low boiling tar distillate, para-cresol, pyridine, phenol-toluene mixture, aniline, methyl aniline, acetone, toluene, benzene, carbon disulphide, and turpentine. While some of the above solvents might warrant further study, there are evident advantages over the other compounds in the use of phenol and it was made the subject of extended study.

(2). A general survey of the various types of coal was made to determine the relative amounts of phenol-soluble extract contained in each. The average amount of material extracted from Pocahontas coal was 1.8 per cent and the results varied in the case of Illinois coals from 22 per cent to 39 per cent. These results were calculated on an ash and moisture free basis.

(3). By effecting a separation as indicated under (1), it was possible to study separately the insoluble residue and the extracted material, to determine which had the greater avidity for oxygen, also to which division should be credited the coking constituents, and what products of decomposition, such as gas, illuminants, tar, etc., could be assigned to each.

4. *Summary of Results.*—1. Phenol at 100°C. will dissolve certain constituents of bituminous coals in their natural state. The two subdivisions, designated as *insoluble residue* and *extraction material,* together make substantially 100 per cent or the amount of the original substance.

2. The amount of extractive material is definite and susceptible of quantitative determination.

3. Studies upon these two type substances indicate that the extract is the vital constituent concerned in the coking of coal. It conforms to the principle already enumerated[1] in that it has a sufficiently definite melting point and a decomposition temperature which is above that of the melting point.

4. Each subdivision is capable of absorbing oxygen. The effect upon the extract is to modify its coking properties by lowering or

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[1] Bulletin No. 60, U. of Ill. Eng. Exp. Sta., p. 20.

greatly reducing its power to form a firm and coherent mass.

5. The oxygen taken up in either case is found to be chemically combined. The oxygen taken up by the fresh coal is similarly held. The insoluble portion has the greater avidity for oxygen. This relation to oxygen has a bearing upon other topics aside from that of coking, as the heating of coal in storage, spontaneous combustion, weathering, and deterioration.

## II. Experimental.

### A. APPARATUS AND METHOD OF PROCEDURE.

5. *Preliminary.*—In the preliminary part of the work, conducted to determine which solvent would be the best suited to the purpose, the adaptability of the solvent was determined by the amount of material which it would remove, and also, by the absence of chemical activity involved in its use. That is to say, the action must be that of a true solvent and must not develop chemical changes either in its own structure or that of the components of the coal. These features could be most readily indicated by noting (1) the amount of material dissolved, and (2) the sum of the extract and residue; this sum should correspond substantially to the weight of the original material, provided no chemical action had taken place. The refinements in analytical methods in the preliminary work were not developed beyond the point necessary to determine the applicability of the extractive agent to the work in hand. The solvents used are given in the order of their effectiveness, the most active being placed first: phenol, ortho-cresol, low boiling tar distillate, para-cresol, pyridine, phenol-toluene mixture, aniline, methyl aniline, acetone, toluene, benzene, carbon disulphide, and turpentine.

Para-cresol, ortho-cresol, and low boiling tar distillate decomposed during the extraction, thus apparently increasing the amount of extracted material. Ortho-cresol gave but little decomposition product and as a solvent it ranked next to phenol. Methyl aniline and aniline presumably underwent decomposition.[1] Pyridine and phenol-toluene mixture gave a certain amount of extract but it was not as large as that obtained by use of phenol alone. Furthermore, the increase in the amount of nitrogen which is found in both the extract and residue after treatment with pyridine must be taken as an index of the formation of substitution products not originally present in the coal substances.[2] Acetone, benzene, carbon disulphide and turpentine extracted only small amounts of material and were therefore discarded.

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[1] Lewes, Progressive Age, 29, 1033.

[2] Lewes, Progressive Age, 29, 1033.
Clark and Wheeler, Journal Chem. Soc., 103, 1709.

6. *Phenol and the Conditions which Influence the Amount and Character of Extract.*—As a result of the preliminary work, it was evident that phenol was the chemical best suited for use as an extractive agent. A study, therefore, of the ultimate requirements necessary in its use, was the next step to be undertaken. The fundamental conditions to be observed were found to be (a) time for the solvent act, (b) temperature, (c) presence or absence of air in any part of the process. The effect of these various conditions may be shown as follows:

(a) Time of Extraction.

In order to determine the amount of material extracted for various lengths of time, five grams of coal were treated for successive periods of five hours each, and the amount of extract determined. At the end of twenty hours the extraction was discontinued and the weight of the extract determined. The results of the extraction were as follows:

TABLE 1.

WEIGHT OF MATERIAL DISSOLVED FROM COAL USING PHENOL FOR FIVE HOUR PERIODS.

| | |
|---|---|
| Weight of the extract after 1st five hours | 1.188 grams |
| " " " " " 2nd " " | .110 " |
| " " " " " 3rd " " | .040 " |
| " " " " " 4th " " | .027 " |

It will be seen from Table 1 that during the last five hours of extraction only .027 gram of material was obtained. To determine whether this last increment was true extract or decomposition products of the solvent, the phenol was distilled alone for ten hours as a blank without any coal in the extraction tube, and the phenol distilled off at the end of that time. The weight of the residue in the flask was .03 gram. This weight of decomposition product obtained after ten hours of distillation would represent an equivalent of .015 gram for five hours of distillation. When we consider that the amount of apparent extract during the last five hours was only .027 gram, if .015 gram of this were derived from the phenol, then only .012 gram would be the weight of the substance actually dissolved during that period. It was decided, therefore, to stop the extraction at the end of twenty hours and this length of time for the extraction process was used in all of the work.

(b) Temperature of the extraction.

In the earlier part of the work it was found that there was a larger amount of material extracted from the coal if the solvent were applied hot, as at 110°C,[1] instead of at 20° to 25°. The use of the higher temperature was accomplished by condensing the phenol at the temperature of boiling toluene, i. e., 110°. By causing the condensate at this temperature to flow upon the coal, which in turn was held in the vapors of the phenol, the temperature of the solvent was thereby always kept above 110°. The weight of extract and residue from two coals, when extracted with phenol at the two different temperatures, is given in the following table:

TABLE 2.

WEIGHT OF EXTRACT AND RESIDUE FROM COAL EXTRACTED AT DIFFERENT TEMPERATURES.

| Coal | Divisions | Extracted at 25° C. | Extracted at 110° C. | Difference |
|---|---|---|---|---|
| A | Extract | 1.564 | 1.691 | + 0.127 = + 2.56% |
| | Residue | 3.543 | 3.451 | — .092 = — 1.84% |
| B | Extract | 1.326 | 1.458 | + 0.132 = + 2.64% |
| | Residue | 3.788 | 3.674 | — 0.114 = — 2.28% |

It will be noticed that with both coals the weight of residue decreased and that the weight of extract increased when the extraction was carried out at the higher temperature. The discrepancies are to be referred to the decomposition resulting from the continuous boiling of the phenol. This topic is discussed more fully later. Although the additional amount of extract was not large, it was thought advisable to carry out the extraction at the higher temperature even though the apparatus which was necessary for doing it was slightly complicated.

(c) Presence of Air in the Extraction Apparatus.

In some of the preliminary experiments it was found that the weight of extract and residue combined was equal to more than the original weight of the coal. This would doubtless be due to the oxidation of some of the substances involved, i. e., extract, residue, or phenol. The extraction was, therefore, carried out in the absence of air, carbon dioxide being conducted through the apparatus. The following results show very plainly that there was an increase of weight in both the extract and residue when the distillation was carried out in the presence of air. The test was made on five grams of two different coals. These results are given in Table 3.

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[1] All temperature readings in this discussion are Centigrade.

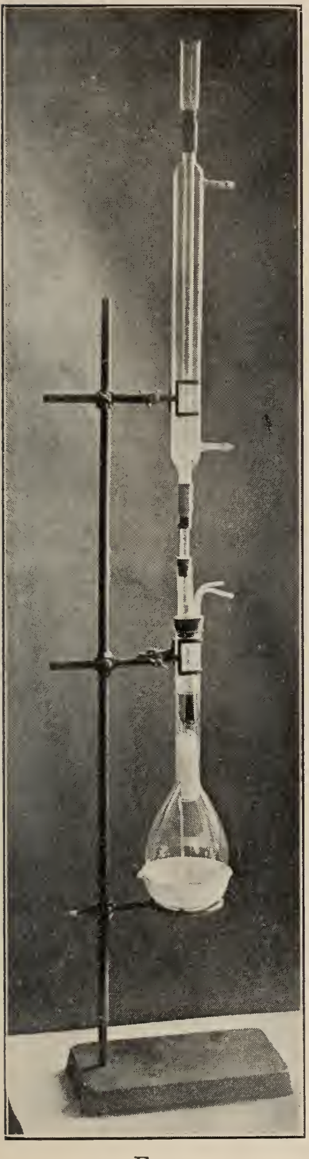

Fig. 1.
Extraction Apparatus
Ready for Operating

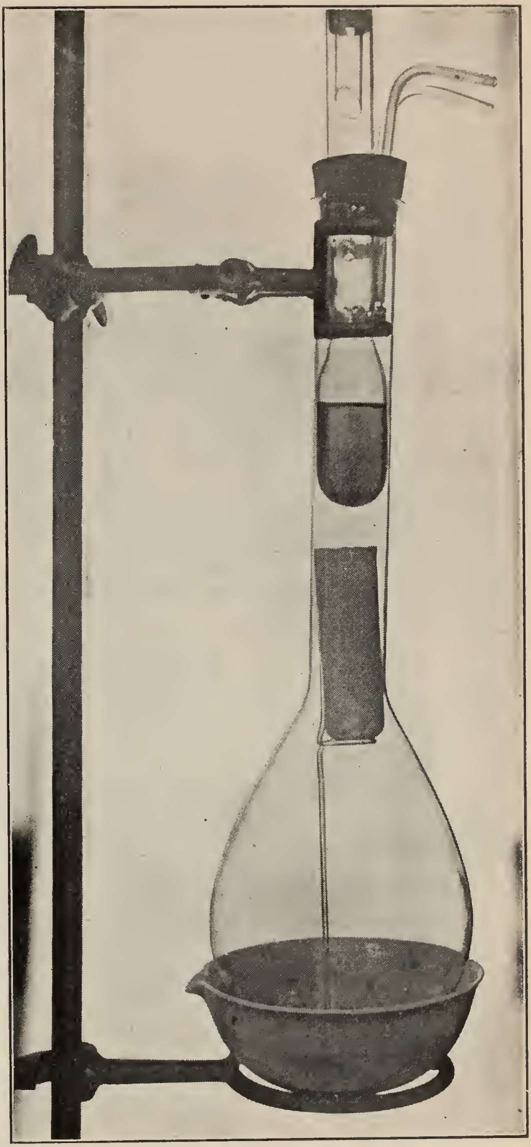

Fig. 2.
Condenser Device for Increasing the Temperature of the Solvent

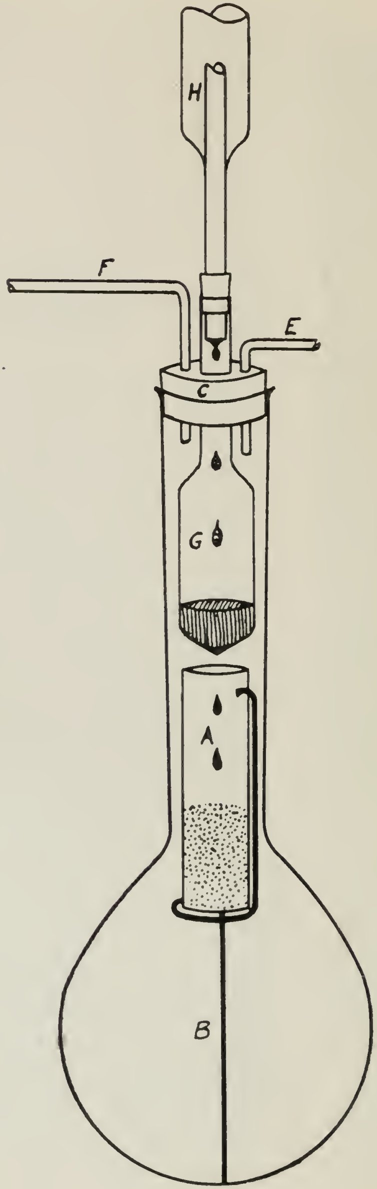

FIG. 3.
CONTINUOUS EXTRACTION
APPARATUS USING PHENOL AT 110°.

TABLE 3.

COAL EXTRACTED WITH PHENOL IN THE PRESENCE OF AIR AND CARBON DIOXIDE.

| Coal | Residual Substance | In $CO_2$ | In Air | Increase |
|---|---|---|---|---|
| A | Extract | 1.564 | 1.614 | .050 = 1.00 % |
| | Residue | 3.543 | 3.691 | .148 = 2.96 % |
| | Total | 5.107 | 5.305 | .198 = 3.96 % |
| B | Extract | 1.326 | 1.389 | .063 = 1.26 % |
| | Residue | 3.788 | 3.921 | .133 = 2.66 % |
| | Total | 5.114 | 5.310 | .196 = 3.92 % |

7. *Apparatus.*—From the preliminary work the apparatus shown in Figs. 1, 2, and 3 was decided upon as being the best adapted for carrying out the process. Figs. 1, 2, and 3 show different views of the apparatus. An extraction cone (A), 25 by 80 cm., was supported in the neck of a 750 cc. Kjeldahl flask by means of a bent glass rod (B). A No. 7 rubber stopper (C) which contained three holes was placed in the neck of the flask. Two of the holes were for small glass tubes (E and F) through which carbon dioxide could be conducted. The one large hole in the center was for the upper end of a glass bulb (G) which acted as a condenser for the phenol. This condenser-bulb was almost as large as the neck of the flask and was drawn out at the upper end to fit a water condenser. It was necessary that this bulb should be large enough to hold about 25 cc. of toluene. The toluene itself was condensed by an ordinary 70 cm. water condenser. A Woods' metal bath was used for boiling the phenol. The whole flask was protected from draughts by asbestos paper.

8. *Process of Extraction.*—One important problem which presented itself in the extraction of coal with phenol was the matter of freeing the residue and the extract from the solvent. Obviously, the extract and the residue must be treated differently because of the nature of each. In spite of the fact that all precautions as to oxidation, decomposition, etc., were observed during the extraction, it was found that the sum of the weight of extract and residue was larger than the original weight of coal. This fact seemed to indicate that phenol was being retained in either the extract or the residue by a physical or chemical union. If the phenol was being held chemically, it could not be used as a solvent. Therefore, it was necessary to determine whether such was the case or not.

After the extraction, the residue was washed first with alcohol and then with ether in a Soxhlet extractor. The residue was subsequently dried in the air for one hour at 100°. It was found that this method of freeing the residue from phenol could not be used for two reasons; even after the washing process and drying at 100° traces of phenol remained in the residue, and there was also the possibility of oxidation of the finely divided residue during the heating. In order to remove the adhering phenol the method would seem to be that of heating to a temperature above the boiling point of the solvent, i. e., to approximately 200°. Moreover by heating the residue which had been treated as above described to 200° traces of phenol were shown to be given off. The possibility of heating to this temperature raises the questions (1) of oxidation and (2) of decomposition of the material. The first point could easily be guarded by use of an inert atmosphere such as carbon dioxide. In the presence of such an atmosphere, heating to 200° could be carried on without decomposition as is shown by the following experiments.

TABLE 4.

SHOWING THE EFFECT OF HEATING COAL SUBSTANCE TO 200° IN AN ATMOSPHERE OF $CO_2$.

| Residue | Weight of Residue | Time of Heating | Weight after Heating | Difference |
|---|---|---|---|---|
| A | 4.125 | 3 hours | 4.121 | — .004 |
| B | 4.120 | 2 " | 4.123 | + .003 |
| C | 4.200 | 1 " | 4.200 | .000 |
| D | 3.994 | 2 " | 3.981 | — .013 |

As a result of these experiments the following procedure was adopted. After being washed with ether, the residue was dried for a short time in the air to allow the ether to evaporate and was then transferred to a flask. It was kept at 200° for one hour with a slow stream of dry carbon dioxide passing through the apparatus. At the end of one hour the flask was allowed to cool and the carbon dioxide displaced by dry air. The flask was then weighed. Because of the very hygroscopic nature of the material, it was necessary to protect it from all possible sources of moisture during the drying process.

To free the extract from phenol the first method used was to distil at reduced pressure. After practically all of the phenol had been driven off under this condition the temperature was raised to 200° while a current of carbon dioxide was conducted through the the flask. It was thought that by simply bringing the mass to that temperature all of the phenol would be driven off. However, it was soon found that this was not the case. The heating of the extract was then continued for one hour in a stream of carbon dioxide, in a manner exactly similar to that used with the residue. It was found that this method of treatment gave a lower weight for the extract. Although phenol was given off during the heating, it was not known what effect heating to 200° would have upon the composition of the material. To test the matter, the extract after being dried at 200° for one hour was again heated for a certain length of time under the same conditions. The results from some of the extracts as given in Table 5 show for the length of time indicated, that the decomposition, if there was any, was not large.

### TABLE 5.

### SHOWING THE EFFECT OF HEATING THE EXTRACT TO 200° IN AN ATMOSPHERE OF $CO_2$.

| Extract | Weight of Extract | Time of Heating | Weight after Heating | Difference |
|---|---|---|---|---|
| A | .895 | 1 hour | .893 | —.002 |
| B | 1.027 | 1 " | .997 | —.030 |
| C | .932 | 2 " | .932 | —.000 |

9. *Method of Procedure.*—As a result of the above experiments the following process was uniformly followed. Five grams of coal ground to 100 mesh, which had been dried at 105° for one hour, were extracted with 50 to 60 cc. of redistilled phenol. The extraction cones, previous to use, were dried and weighed with the exclusion of moisture. After extraction with phenol, the cone with the coal was placed in a Soxhlet extractor and washed for two hours with alcohol and then for two hours with ether. The ether was allowed to evaporate from the cone for a short time and the residue, when almost dry, was shaken out into a 100 cc. Erlenmeyer flask. The residue was then dried for one hour at 200° in a slow stream of carbon dioxide. The flask was allowed to cool and the carbon dioxide displaced by dry air. The flask was then weighed and the residue sealed in a small sample tube. The weight of the small amount of residue adhering to the cone was obtained by drying the cone at 105° and weighing. The phenol containing the extract was poured into a 200 cc. round-bottomed flask. The last trace of the extract was washed out of the Kjeldahl flask with pure phenol. The phenol was then distilled at reduced pressure (15 to 25 mm.) in the presence of carbon dioxide. The temperature of the Woods' metal bath was then raised to 200°, and carbon dioxide conducted through the flask to remove the last traces of phenol vapor. The flask was then transferred to an oven and maintained for one hour at 200°, while a slow current of carbon dioxide

was continued through the flask. After cooling, the carbon dioxide was displaced with dry air and the flask weighed. The extract was pulverized and preserved in sealed tubes.

After all the details of the apparatus had been worked out and the sources of error in the method corrected as far as possible, a number of coals were extracted. The method just outlined was followed in detail. The extractions were carried out to test the constancy of the results and to find the limits of accuracy by comparing the sum of extract and residue with the amount of coal taken. The results are shown in Table 6.

TABLE 6.

WEIGHT OF EXTRACT AND RESIDUE FROM FIVE GRAMS OF COAL.

| Table No. | Sample | Weight of Extract | Weight of Residue | Total Weight |
|---|---|---|---|---|
| 1 | Williamson Co., Ill. | 1.120 | 4.006 | 5.126 |
| 2 | Macoupin Co., Ill. | 1.458 | 3.674 | 5.132 |
| 3 | Madison Co., Ill. | 1.691 | 3.451 | 5.142 |
| 4 | Vermilion Co., Ill. | 1.826 | 3.236 | 5.062 |
| 5 | Madison Co., Ill. | 1.446 | 3.642 | 5.088 |
| 6 | Logan Co., Ill. | 1.887 | 3.219 | 5.106 |
| 7 | Jackson Co., Ill. | 1.246 | 3.878 | 5.125 |
| 8 | Sangamon Co., Ill. | 1.959 | 3.139 | 5.098 |
| 9 | Montgomery Co., Ill. | 1.559 | 3.498 | 5.057 |
| 10 | Williamson Co., Ill. | 1.327 | 3.749 | 5.076 |
| 11 | Vermilion Co., Ill. | 1.937 | 3.104 | 5.041 |
| 12 | Vermilion Co., Ill. | 1.689 | 3.372 | 5.061 |
| 13 | Williamson Co., Ill. | 1.616 | 3.491 | 5.107 |
| 14 | Pocahontas, W. Va. | .090 | 4.923 | 5.013 |
| 15 | Lignite, Wyo. | .630 | 4.559 | 5.189 |

It will be noted in the above table that the weight of extract combined with the weight of residue was in all cases above the weight of coal taken. This is to be accounted for in the following manner. Experiments already given (page 7) showed that when 50 cc. of phenol were distilled for five hours that the amount of decomposition product was .015 gram. For twenty hours the amount of decomposition product would be .06 gram provided the decomposition was regular and that the presence of coal did not in any way tend to increase the rate of decomposition. It must be admitted that the alteration of phenol during all of the extractions would not be the same for the reason that the rate of distillation could not be controlled accurately and as a result the decomposition would vary somewhat in amount. On examining the results above it will be seen that the sum of the weight of extract and residue was from .013 to .189 gram in excess of the original weight of the coal. These two results are extreme cases. Now granting that the amount of decomposition of phenol would vary from one-half to two times the amount determined in the blank test, then the excess of weight from the process would be substantially accounted for by referring it to the phenol residue. Furthermore, oxidization or absorption of water by either the extract or residue would cause an increase in weight and would tend to augment the plus error observed.

It might be well to note in this connection that since the decomposition product from the phenol will appear in the extract only, a corrected weight of the extract would therefore be obtained by subtracting the weight of residue from the original weight of the coal. Thus the weight of the coal minus the insoluble residue is the corrected factor showing the degree of extraction.

## B. Application of the Method to Different Types of Coals.

While the preliminary work was done on two coals and though the methods were not as accurate as those finally devised, it could be easily seen that the two coals contained markedly different amounts of material soluble in phenol. One coal was from the central and the other from the southern part of the state. As a means of comparison of coals from different localities it was decided to make an extraction of coals from the same and from different districts. Samples of Illinois coals were, therefore, obtained from local dealers. The most recent shipments and large lumps were selected as best suited to the purpose. Samples of Pocahontas coal and lignite were also obtained. The name, location, volatile matter, fixed carbon, and ash of the coals are given in the following table.

Table 7.

Showing Analysis of Coals. Dry Basis.

| Table No. | Source of Sample | Ash per cent | Volatile Matter per cent | Fixed Carbon per cent |
|---|---|---|---|---|
| 1 | Williamson Co., Ill. | 10.41 | 36.86 | 52.73 |
| 2 | Macoupin Co., Ill. | 13.54 | 40.12 | 46.34 |
| 3 | Madison Co., Ill. | 11.65 | 43.06 | 45.29 |
| 4 | Vermilion Co., Ill. | 8.68 | 44.36 | 46.96 |
| 5 | Madison Co., Ill. | 11.73 | 37.95 | 50.32 |
| 6 | Logan Co., Ill. | 8.74 | 41.99 | 49.27 |
| 7 | Jackson Co., Ill. | 6.01 | 34.93 | 59.06 |
| 8 | Sangamon Co., Ill. | 5.18 | 44.48 | 50.34 |
| 9 | Montgomery Co., Ill. | 11.84 | 42.12 | 46.02 |
| 10 | Williamson Co., Ill. | 7.88 | 36.78 | 55.34 |
| 11 | Vermilion Co., Ill. | 3.30 | 47.70 | 49.00 |
| 12 | Vermilion Co., Ill. | 6.74 | | |
| 13 | Williamson Co., Ill. | 7.85 | 37.85 | 54.30 |
| 14 | W. Va. Pocahontas | 13.38 | 16.80 | 69.82 |

The above coals were extracted with phenol. From the amounts of extract and residue as given in Table 6 the percentage amounts of each substance may be calculated, but on account of the variations in the ash it will be better to make such calculations on the ash free basis. It will be noted in Table 7 that the amounts of ash in these coals varied widely, from 3.30 to 13.54 per cent. Since the percentage of ash in the extract is seldom over 0.2 per cent it is quite evident that the phenol does not exert an appreciable solvent action on the mineral constituents in the coal.[1] Therefore, it is present simply as an inert material and serves to dilute the coal substance. If the amounts of residue and extract in one coal are to be compared with those same constituents in another coal, it can be done only after the residue and extract in each have been calculated to an ash free basis.

The following formula was used in this calculation:

$$\frac{\text{Per cent Residue as weighed} - \text{per cent ash in coal}}{100 - \text{per cent ash in coal}} = \text{Ash free Residue}$$

The corrected ash free extract was found as indicated on page 15 by subtracting the per cent of ash free residue from 100. As stated above the extract is not entirely free from ash, but as this was small in amount it was not considered in these calculations. The weight of residue and extract for Illinois coals were taken from Table 6 and calculated to an ash free basis. The results in the order of decreasing residue and increasing extract are presented in Table 8.

TABLE 8.

SHOWING PERCENTAGE OF INSOLUBLE RESIDUE AND EXTRACT REFERRED TO THE ASH FREE SUBSTANCE.

| Table No. | Description of Sample | Percentage of Ash free Residue | Percentage of Ash free Extract |
|---|---|---|---|
| 1 | Williamson Co., Ill. | 77.81 | 22.19 |
| 7 | Jackson Co., Ill. | 76.14 | 23.86 |
| 10 | Williamson Co., Ill. | 72.85 | 27.15 |
| 2 | Macoupin Co., Ill. | 69.33 | 30.67 |
| 5 | Madison Co., Ill. | 69.24 | 30.76 |
| 13 | Williamson Co., Ill. | 67.26 | 32.74 |
| 9 | Montgomery Co., Ill. | 65.93 | 34.07 |
| 12 | Vermilion Co., Ill. | 65.09 | 34.91 |
| 3 | Madison Co., Ill. | 64.94 | 35.06 |
| 4 | Vermilion Co., Ill. | 61.37 | 38.63 |
| 6 | Logan Co., Ill. | 60.97 | 39.03 |
| 8 | Sangamon Co., Ill. | 60.76 | 29.24 |
| 11 | Vermilion Co., Ill. | 60.80 | 39.20 |

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[1] Frazer and Hoffman, Bureau of Mines, Technical Paper 5, p. 8.

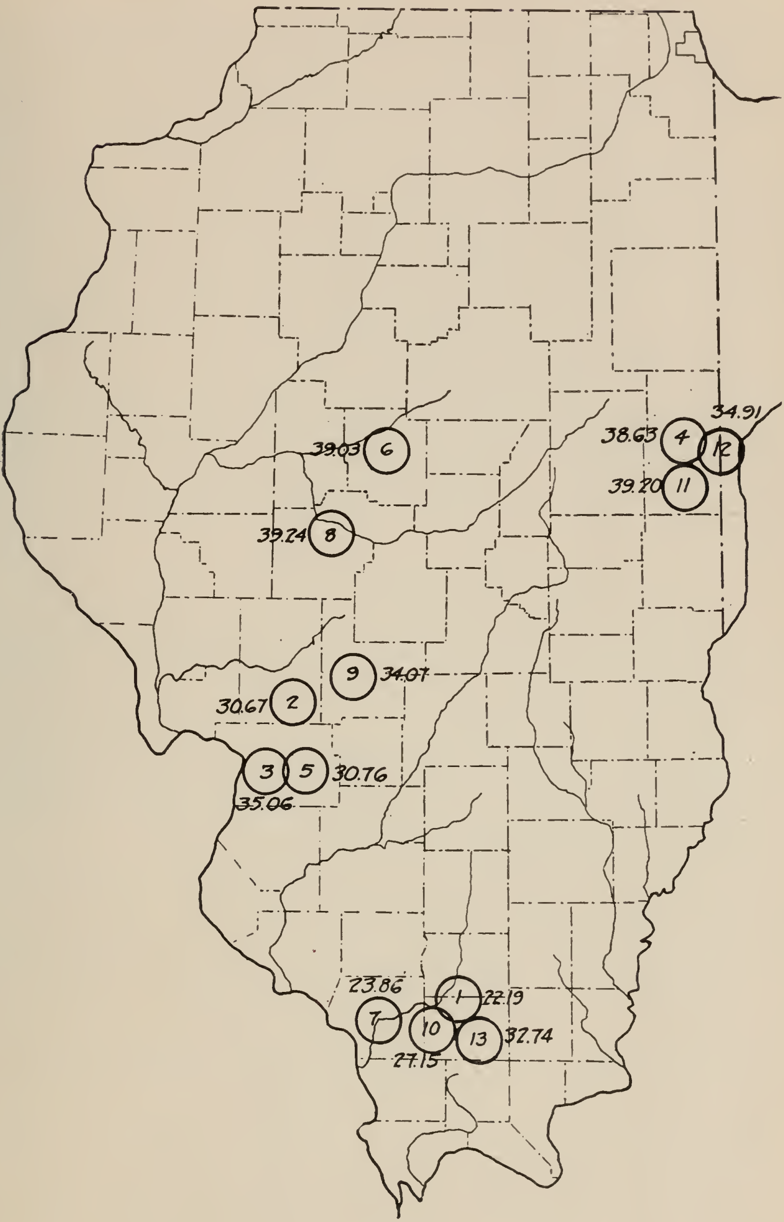

FIG. 4.
LOCATION OF THE ILLINOIS COALS.

For purposes of geographical comparison the location of the Illinois coals together with the percentage of ash free extract has been shown on a map (page 17). The coals may be divided into three classes; (1) those low in extract; (2) those medium in extract; and (3) those high in extract. The class low in extract, i. e., from 20-30 per cent, is located in the extreme southern end of the State and is confined as far as the tests extend to Jackson and Williamson counties. The class of medium extract, i. e., 30-35 per cent., is located in Madison, Macoupin, and Montgomery counties. The class high in extract, i. e., above 35 per cent, extends through the middle of the State. The coals which gave the high amount of extract were from Sangamon, Logan, and Vermilion counties.

From this preliminary survey of the coals it seems that this method of analysis may lead to a much better comparison of coals or a closer subdivision of varieties than can be shown by the ordinary methods of analysis. Of course, the study must be much wider in scope before a fixed and definite conclusion can be drawn as to its applicability in this connection.

### C. GENERAL PROPERTIES OF EXTRACT AND RESIDUE.

10. *Relation of Extract and Residue to Coking Properties.*—The relation of extract and residue to the coking properties was studied by making volatile matter determinations according to a specified method.[1] The volatile matter determination was useful in two ways: (1) the percentage of volatile matter was indicated, and (2) the nature of the coke produced could be roughly determined by examining the material remaining in the crucible. The percentage of volatile matter will be given in another connection.

The "coke" produced from the volatile matter determination of the insoluble residue showed very little coherence. The so-called "coke" was in fact not a coke at all. Its tendency to hold together depended to a certain extent upon the coal from which the residue had been obtained, for in some cases the material fell to pieces on being dumped out of the crucible and in all cases it could be disintegrated between the fingers. It was much duller in appearance than ordinary coke. The residue, therefore, could properly be designated as non-caking and in consequence non-coking.

The extract and residue differed widely in character. When heated to 300° to 350°, the extract softened but it did not produce a sticky material at first. Later, as the temperature was raised, it became sticky. On cooling it again assumed the compact brittle state, which it originally possessed. The extract when subjected to the volatile matter test swelled to three or four times its original size and gave a large amount of volatile matter. The material remaining after the test possessed a bright, shiny appearance. It was very friable. However, it was a material entirely different from that produced from the residue.

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[1] Stanton and Fieldner, "Methods of Analyzing Coal and Coke", Technical Paper 8, Bureau of Mines (1912).

If the residue and extract are mixed in the ratio in which they occurred in the coal and a volatile matter determination made, the material remaining is a coke about as strong in character and similar in appearance as that produced by the coal itself. From this it is evident that the coking constituent is the soluble extract and that its function is that of a cementing substance.

11. *Effect of Heating the Residue and Extract in the Air.*—It is well known that coal oxidizes readily in the air at 200°. It was desired to learn the behavior of the residue and extract when heated under the same conditions. The residue was weighed and heated in contact with air in an oven at 200° for successive one hour periods. The results are given in the following table.

TABLE 9.
SHOWING THE INCREASE IN WEIGHT OF RESIDUE WHEN HEATED AT 200°.

| | Weight | Increase |
|---|---|---|
| Weight of residue | 0.6782 | 0.0000 |
| after 1st one hour period | 0.6868 | 0.0086 |
| after 2nd one hour period | 0.6903 | 0.0121 |
| after 3rd one hour period | 0.6926 | 0.0144 |
| after 4th one hour period | 0.6952 | 0.0170 |
| after 5th one hour period | 0.6960 | 0.0178 |
| after 6th one hour period | 0.6973 | 0.0191 |
| after 7th one hour period | 0.6982 | 0.0200 |

The extract was also heated to 200° in the air. The results for the extract are given in Table 10. The time of the first heating was 30 minutes and after that each successive period was one hour.

TABLE 10.
SHOWING THE INCREASE IN WEIGHT OF THE EXTRACT WHEN HEATED TO 200°.

| | Weight | Increase |
|---|---|---|
| Weight of extract | 0.5718 | 0.0000 |
| after heating 30 min. | 0.5720 | 0.0002 |
| after 1st one hour period | 0.5745 | 0.0027 |
| after 2nd one hour period | 0.5770 | 0.0052 |

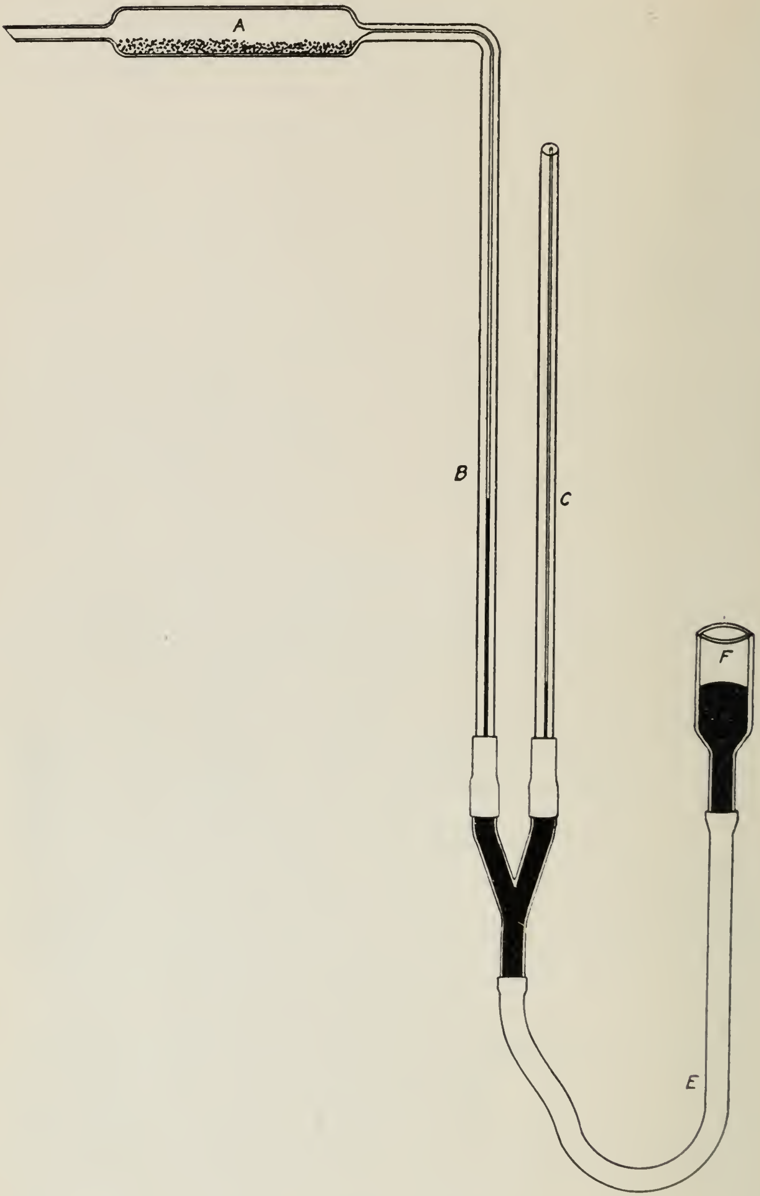

Fig. 5.
Absorption of Oxygen by Coal Residues.

It was thus shown that both the residue and extract increased in weight when heated and that the rate of increase was faster in the case of the insoluble residue. These experiments are taken to mean that the increase in weight is an indication of oxidation of residue and extract.

12. *Avidity of Residue and Extract for Water and Oxygen at Ordinary Temperature.*—The hygroscopic nature of the residue was one of the properties to be noticed first. On exposure to air it increased in weight rapidly, and on drying at 105° for half an hour most of the water was driven off. However, it did not return to its original weight on drying, so that the excess in weight was probably caused by oxidation of the material.

The hygroscopic nature of the extract was also noticeable but it was not as marked as that of the residue. The extract increased in weight when exposed to the air, and the moisture could be driven out again by drying at 105° for thirty minutes.

Table 11 shows the increase in weight of extract and residue when exposed to the moisture of the air and the decrease in weight when they were dried at 105° for 30 minutes.

TABLE 11.

EFFECT OF OXIDATION ON RESIDUE AND EXTRACT.

| Substance | Weight | Weight after Exposure | Increase | Weight after Drying | Variation in Weight from Initial Material |
|---|---|---|---|---|---|
| Residue | .3029 | .3129 | .0100 | .3037 | + .0008 |
| Extract | .2467 | .2485 | .0018 | .2462 | — .0005 |

Oxidation of the residue and extract at ordinary temperature was also studied. Because of the hygroscopic nature of the residue and extract, it was difficult to determine whether oxidation was actually taking place or not. The rate of oxidation, if it occurred, would necessarily be slow at room temperature. Several experiments were performed for the purpose of determining whether or not the material did oxidize. Obviously, the experiment must be so carried out that moisture increases may be obviated in order to know definitely that the material had oxidized and had not taken up moisture.

The apparatus for showing the absorption of oxygen consisted of a barometer for measuring diminished pressure in a sealed space, which contained oxygen and the material to be tested. The plan of such an apparatus is shown in Fig. 5. The apparatus consists of a 10 cc. pipette (A) fused to a capillary tube (B) 900 mm. in length. The capillary tube is connected at the bottom by a Y-tube to another capillary tube (C) which serves as a leveling tube and through which oxygen can be introduced into the apparatus. The Y-tube is connected by a rubber tube (E) to a leveling bulb (F).

In setting up the apparatus the procedure was as follows: One gram of extract (or residue) was introduced into the pipette and the end sealed off. Mercury was then placed in E and F, and a pinch clamp placed on E. The height of the mercury was such that it did not disconnect B and C. The apparatus was then evacuated repeatedly and filled five or six times with commercial oxygen which was dried by passing it through sulphuric acid. This procedure was followed to displace the air in the apparatus with oxygen. Finally a very slightly diminished pressure was maintained in the apparatus, the pinch cock on E opened, and the leveling tube raised. If this procedure were carried out properly, the mercury in B would stand a few mm. above the rubber connection and the mercury in both the capillary tube C and the leveling bulb F would stand at the same level. At certain intervals simultaneous temperature and barometric readings were taken and the true pressure calculated.

Table 12 shows the results of an experiment on the extract.

TABLE 12.

SHOWING THE AVIDITY OF THE EXTRACT FOR OXYGEN MEASURED IN DIMINISHED PRESSURE (mm.).

| Time after Standing. | Diminished Pressure in mm. |
|---|---|
| 0 | 0 |
| 23 hours | 36 |
| 64 “ | 56 |
| 88 “ | 76 |
| 112 “ | 91 |
| 136 “ | 96 |
| 208 “ | 125 |

TABLE 13.

SHOWING THE AVIDITY OF THE RESIDUE FOR OXYGEN MEASURED IN DIMINISHED PRESSURE (mm.).

| Time after Standing. | Diminished Pressure in mm. |
|---|---|
| 0 | 0 |
| 26 hours | 86 |
| 49 “ | 123 |
| 90 “ | 154 |
| 114 “ | 188 |
| 138 “ | 209 |
| 160 “ | 220 |
| 232 “ | 269 |

For purposes of comparison of the absorption rates of extract and residue, the above results were plotted. Plotting time against diminished pressure in millimeters, it is seen that the rate of absorption was faster in the case of the residue.

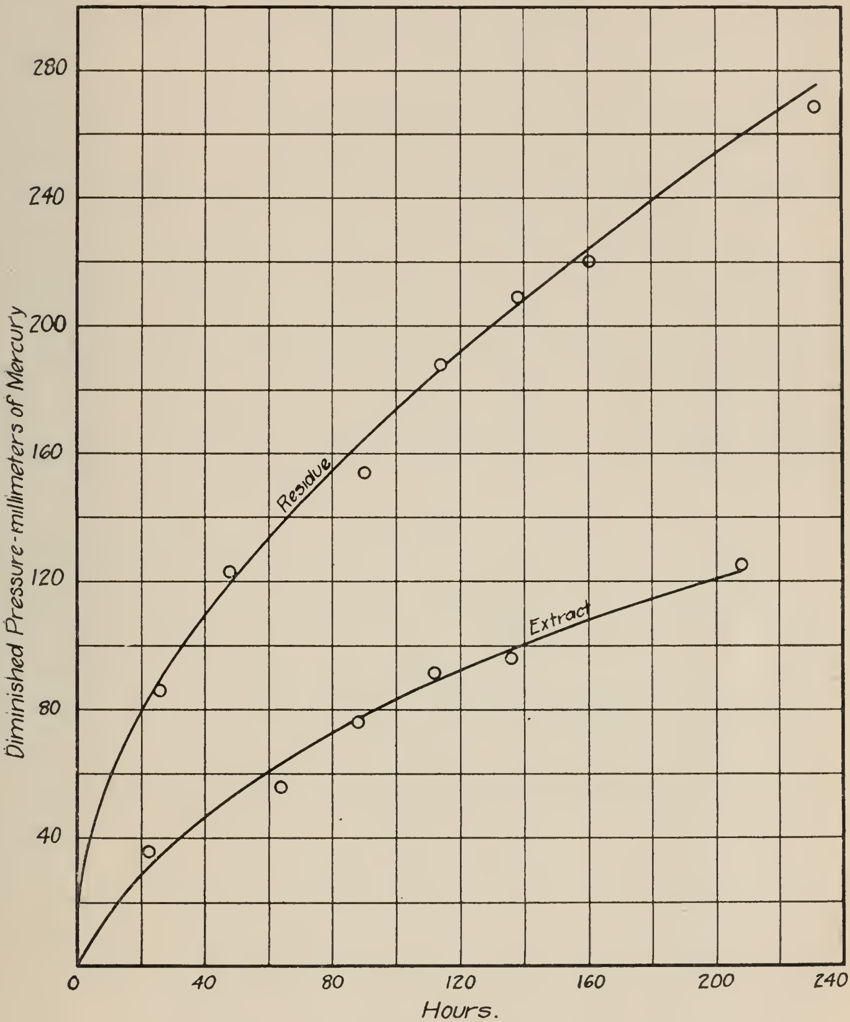

FIG. 6.
RATE OF ABSORPTION OF OXYGEN BY RESIDUE AND EXTRACT.

The results as given in Tables 12 and 13 show that oxygen was taken up by both the residue and extract. The amount of oxygen that had been absorbed when the experiment was stopped was about 4-5 cc. in the case of the residue, and 2-2½ cc. in the case of the extract. The experiment does not show absolutely that oxidation had taken place, but from our knowledge of coal we would be led to suspect that such was the case. At any rate it can be said that the coal was either oxidized or that it was holding the oxygen in an occluded state. Further experiments given below show that the reaction was one of oxidation.

13. *Volatile Matter Determination of Coal, Residue, and Extract.*—Something of the relation of residue and extract to coking properties has already been discussed. Further data as to the relative amounts of volatile matter in the two divisions were desired.

The percentage of volatile matter and fixed carbon in the residue and extract from different coals was therefore determined and for purposes of comparison, volatile matter determinations were also made on the coals under examination. These results are shown in Table 14.

The fuel ratio or the ratio of fixed carbon to volatile matter is also given in Table 14. The use of this ratio is advantageous for the reason that we have in it a method of comparison which does not readily appear from the percentages of fixed carbon and volatile matter alone.

TABLE 14.

SHOWING THE PERCENTAGE OF FIXED CARBON AND VOLATILE MATTER IN COAL, RESIDUE, AND EXTRACT.

| Table No. | Substance | Fixed Carbon per cent | Volatile Matter per cent | Fuel Ratio |
|---|---|---|---|---|
| 6 | Lincoln Coal | 49.27 | 41.99 | 1.173 |
| | Residue | 47.15 | 39.63 | 1.190 |
| | Extract | 47.25 | 52.39 | .902 |
| 7 | New Kentucky Coal | 59.06 | 34.93 | 1.691 |
| | Residue | 57.90 | 34.51 | 1.678 |
| | Extract | 51.52 | 48.42 | 1.064 |
| 8 | Springfield Coal | 50.34 | 44.48 | 1.132 |
| | Residue | 50.26 | 41.90 | 1.200 |
| | Extract | 47.61 | 51.98 | .916 |
| 10 | Carterville Coal | 55.34 | 36.78 | 1.504 |
| | Residue | 54.83 | 34.77 | 1.577 |
| | Extract | 51.79 | 48.10 | 1.077 |
| 9 | Montgomery Co. Coal | 46.02 | 42.12 | 1.093 |
| | Residue | 43.91 | 39.88 | 1.101 |
| | Extract | 48.82 | 50.45 | .968 |
| 11 | Catlin Coal | 49.00 | 47.70 | 1.027 |
| | Residue | 49.02 | 46.01 | 1.066 |
| | Extract | 47.35 | 52.30 | .905 |
| 14 | Pocahontas Coal | 69.82 | 16.80 | 4.156 |
| | Residue | 68.68 | 17.72 | 3.876 |

The fuel ratios of coal, residue, and extract are separately shown in Table 15. Here the results appear in such a form that the fuel ratio of one coal can easily be compared with that of another. Likewise, the fuel ratio of the residue and extract can be compared.

The fuel ratios for the several extracts are uniformly lower than the ratios for the residues. It is to be remembered that a decrease in the ratio number denotes an increase in the relative amount of volatile constituent. The table is made to show in the last column the relation between the various samples, the fuel ratio for the residue being used in each case as the basis of reference. It is quite possible that when reduced to this basis some idea may be had as to the coking property of a coal. While more data would be required on which to base a conclusion, it may be said that the high percentage differences as shown in the table are consistent in representing the more pronounced coking property of the samples listed.

TABLE 15.

SHOWING THE FUEL RATIO OF COAL, RESIDUE, AND EXTRACT.

| Table No. | Description | Coal $\frac{C}{vol}$ | Residue $\frac{C}{vol}$ | Extract $\frac{C}{vol}$ | Percentage of decrease of ratio in extract over ratio for residue referred to residue as basis |
|---|---|---|---|---|---|
| 7 | New Kentucky | 1.705 | 1.678 | 1.062 | 40.3 |
| 10 | Carterville Nut | 1.504 | 1.578 | 1.077 | 38.3 |
| 9 | Montgomery Co. | 1.093 | 1.101 | .968 | 26.5 |
| 6 | Lincoln | 1.173 | 1.190 | .902 | 32.2 |
| 11 | Catlin | 1.027 | 1.066 | .904 | 13.7 |
| 8 | Springfield | 1.133 | 1.200 | .916 | 24.1 |

14. *Ultimate Analysis of Coal, Residue, and Extract.*—A difference in the chemical constitution of the residue and extract would be expected, when we consider the marked differences in their physical properties. In order to find out if there was a difference in the chemical constitution of these two substances, ultimate analyses were made of extract, residue, and coal from a number of mines as follows:

TABLE 16.

VERMILION COUNTY COAL.

SHOWING THE ULTIMATE ANALYSIS OF COAL, RESIDUE, AND EXTRACT.

| Substance | Carbon | Hydrogen | Nitrogen | Sulphur | Ash |
|---|---|---|---|---|---|
| Coal | 72.40 | 5.35 | 1.27 | 2.72 | 8.68 |
|  | 72.36 | 5.34 | 1.16 |  |  |
| Residue | 69.16 | 4.92 | 1.29 | 3.72 | 13.03 |
|  | 69.08 | 4.90 |  |  |  |
| Extract | 81.14 | 5.88 | 1.24 | 1.33 | .53 |
|  | 80.97 | 6.04 |  |  |  |

These results were averaged and calculated to an ash free basis.

### TABLE 17.

### VERMILION COUNTY COAL.

### SHOWING THE ULTIMATE ANALYSIS OF COAL, RESIDUE, AND EXTRACT ON AN ASH FREE BASIS.

| Substance | Carbon | Hydrogen | Nitrogen | Sulphur | Oxygen |
|---|---|---|---|---|---|
| Coal | 79.26 | 5.86 | 1.32 | 2.97 | 10.59 |
| Residue | 79.48 | 5.68 | 1.48 | 4.28 | 9.12 |
| Extract | 81.48 | 5.99 | 1.25 | 1.34 | 9.94 |

The results of the ultimate analyses of Williamson County coal, residue, and extract are given in Table 18.

### TABLE 18.

### SHOWING THE ULTIMATE ANALYSIS OF COAL, RESIDUE, AND EXTRACT.

| Substance | Carbon | Hydrogen | Nitrogen | Sulphur | Ash |
|---|---|---|---|---|---|
| Coal | 74.58 | 4.72 | 1.63 | 2.35 | 7.85 |
| | 74.42 | 4.89 | | | |
| Residue | 72.46 | 4.70 | 1.53 | 2.52 | 11.35 |
| | 72.17 | 4.73 | | | |
| Extract | 82.69 | 5.54 | 1.73 | 1.42 | .56 |
| | 82.60 | 5.54 | | | |

These results were averaged and calculated to an ash free basis.

### TABLE 19.

### WILLIAMSON COUNTY COAL.

### ULTIMATE CONSTITUENTS OF COAL, RESIDUE, AND EXTRACT CALCULATED TO THE ASH FREE BASIS.

| Substance | Carbon | Hydrogen | Nitrogen | Sulphur | Oxygen |
|---|---|---|---|---|---|
| Coal | 80.85 | 5.21 | 1.77 | 2.55 | 9.62 |
| Residue | 81.57 | 5.32 | 1.72 | 2.84 | 8.55 |
| Extract | 83.11 | 5.57 | 1.74 | 1.43 | 8.15 |

On examining the results of the ultimate analysis, it will be seen that there was little difference in the proportion of the elements in the coal, residue, and extract. The percentage of carbon and hydrogen is slightly higher in the extract. However, because of the small variation no special significance can be attached to this fact.

The oxygen content was the chief point of interest to be considered in the analysis. Previous work with solvents such as pyridine[1] and benzene[2] indicated that the material dissolved from coal by these solvents contained a smaller proportion of oxygen. Professor Lewes[3] states that "degradation products of the original vegetation are to be found in the bituminous coals, the residual body and humus forming the basis, which is luted together by the hydrocarbons and resins, and the characteristics of the various kinds of coal are dependent upon the proportion in which the four groups of the conglomerate are present". Assuming that the hydrocarbons and resins were the more soluble in phenol and would be dissolved by the phenol, it would be only natural to conclude that the extract would be low in the amount of oxygen it contained. Furthermore, if the bodies low in oxygen were dissolved, the residue would on that account contain a larger percentage of oxygen. When we consider that all the errors of the analysis are combined in the oxygen value, it would be impossible to state from the results of the above analyses that there was any difference in the percentage of oxygen in the extract from that of the residue.

It is to be noted that the percentage of sulphur in the residue is high. This is to be accounted for by the fact that the residue contained practically all of the ash that was present in the coal. That there was some sulphur in the extract was taken to indicate that a part of the sulphur in the coal was present in the organic form.

15. *Thermal Decomposition of Residue and Extract.*—The residue and extract were subjected to destructive distillation at 800° and the resulting gases analyzed. For purposes of comparison the coal was also treated in the same manner. The method for the destructive distillation of the material may be briefly described in the following manner.

One gram of the substance was placed in a hard glass test tube. Immediately above the substance was placed a short layer of asbestos which served the purpose of a tar scrubber and as a means of holding the extract in the lower part of the tube thus counteracting its tendency to foam. The tube was then connected by a rubber stopper, a glass stop cock, and a short piece of small rubber tubing to a 300 cc. gas holder which contained mercury. Previous to heating, the air was exhausted from the tube and from the connections by means of a filter pump. The test tube was then introduced into a gas furnace which had been previously heated to 800°.

The temperature was maintained between the temperature limits of 780° and 810° throughout all of the experiments. The upper part of the test tube holding the rubber stopper was well insulated against the heat. As soon as the pressure in the tube was equal to the atmospheric pressure, the stop cock was opened and the gas collected. At the end of one hour the distillation was stopped. The gas was analyzed according to the usual procedure.

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[1] Clark and Wheeler, J. of Chem. Soc., 103, 1709 (1913).
[2] Pictet and Ramseyer, Arch. sci. phys. nat., 34, 234 (Chem. Abstracts 7, 1496) (1913).
[3] Progressive Age, 29, 1032 (1911).

The average of duplicate determinations gave the following results for gases which were produced by the destructive distillation of Vermilion County coal, residue, and extract.

### TABLE 20.

### SHOWING THE COMPOSITION OF GASES PRODUCED BY DESTRUCTIVE DISTILLATION OF COAL, RESIDUE, AND EXTRACT.

| Components of the Gas | Vermilion Co. | Residue | Extract |
|---|---|---|---|
| Carbon dioxide | 18.1 | 15.1 | 12.9 |
| *Ethylene | 3.4 | 3.5 | 3.1 |
| Oxygen | .5 | .9 | .1 |
| Carbon monoxide | 20.6 | 17.4 | 16.4 |
| Hydrogen | 35.6 | 39.2 | 47.8 |
| Methane | 17.4 | 18.1 | 13.4 |
| **Ethane | 1.6 | 1.5 | 2.8 |
| Nitrogen | 2.8 | 4.3 | 3.5 |

Coal, from Williamson County, residue and extract were also destructively distilled. The results of the analysis of the gas which was produced are given in Table 21.

### TABLE 21.

### SHOWING THE COMPOSITION OF GASES PRODUCED BY DESTRUCTIVE DISTILLATION OF WILLIAMSON COUNTY COAL.

| Components of the Gas | Coal | Residue | Extract |
|---|---|---|---|
| Carbon dioxide | 7.1 | 6.9 | 7.2 |
| *Ethylene | 3.1 | 2.9 | 2.6 |
| Oxygen | .6 | .8 | .7 |
| Carbon monoxide | 12.8 | 16.5 | 11.8 |
| Hydrogen | 50.7 | 46.0 | 51.8 |
| Methane | 17.3 | 20.5 | 18.2 |
| **Ethane | 1.9 | 2.8 | 2.7 |
| Nitrogen | 6.5 | 3.6 | 5.0 |

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* Including higher homologues and benzene.
** Including higher homologues calculated as ethane.

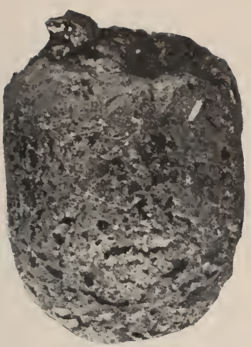

Fig. 7.
Coke from Vermilion County Coal.

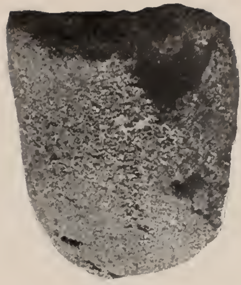

Fig. 8.
Insoluble Residue from Vermilion County Coal—After Destructive Distillation.

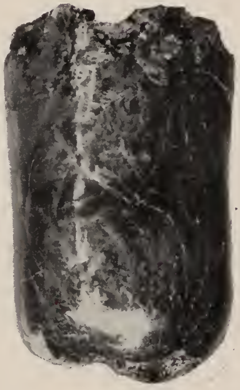

Fig. 9.
Phenol Soluble Extract from Vermilion County Coal—After Destructive Distillation.

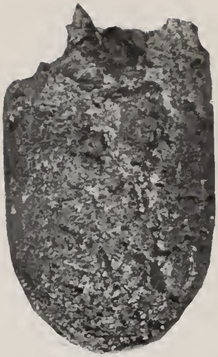

Fig. 10.
Coke from Vermilion County Coal.

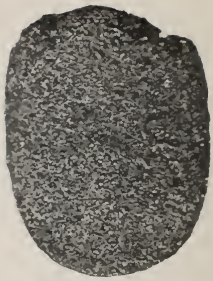

Fig. 11.
Insoluble Residue from Williamson County Coal—After Destructive Distillation.

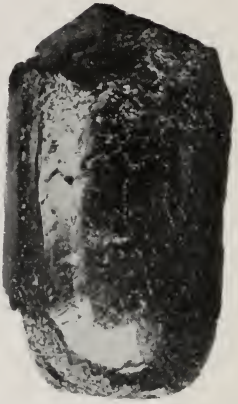

Fig. 12.
Phenol Soluble Extract from Williamson County Coal—After Destructive Distillation.

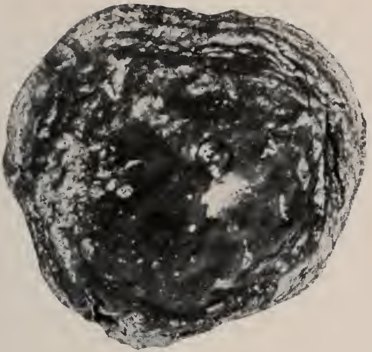

FIG. 13.
COKE FROM VERMILION COUNTY COAL
WITHOUT PREVIOUS HEATING

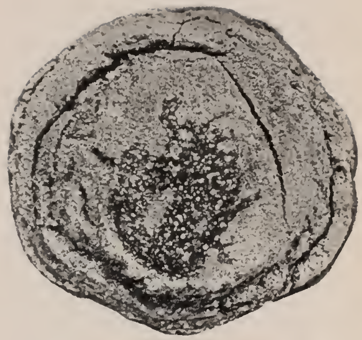

FIG. 14.
SAME AS IN FIG. 13.
PREVIOUSLY HEATED 26 HOURS AT 105°.

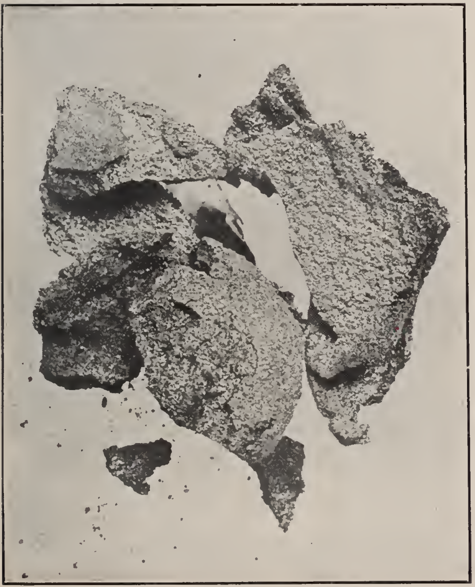

FIG. 15.
SAME AS IN FIG. 13.
PREVIOUSLY HEATED 50 HOURS AT 105°.

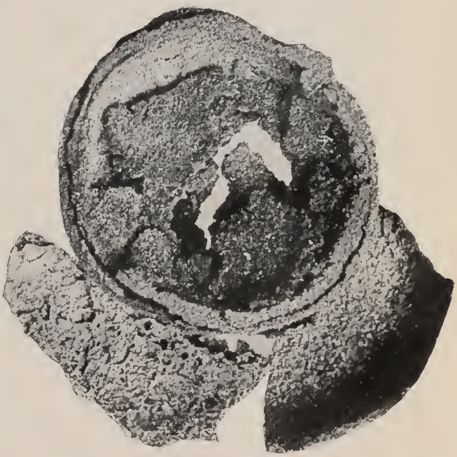

FIG. 16.
SAME AS IN FIG. 13.
PREVIOUSLY HEATED 120 HOURS AT 105°.

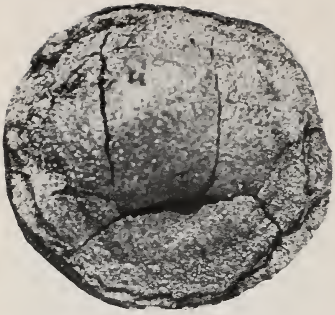

Fig. 17.
Coke from Williamson County Coal
Without Previous Heating.

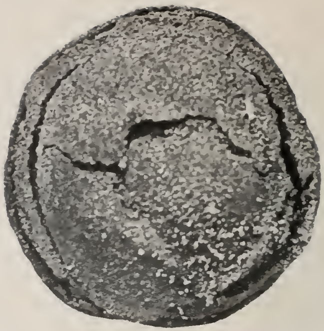

Fig. 18.
Same as in Fig. 17.
Previously Heated 26 Hours at 105°.

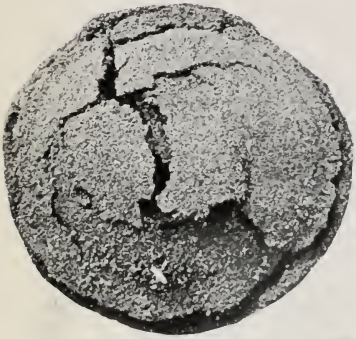

Fig. 19.
Same as in Fig. 17.
Previously Heated 50 Hours at
105°.

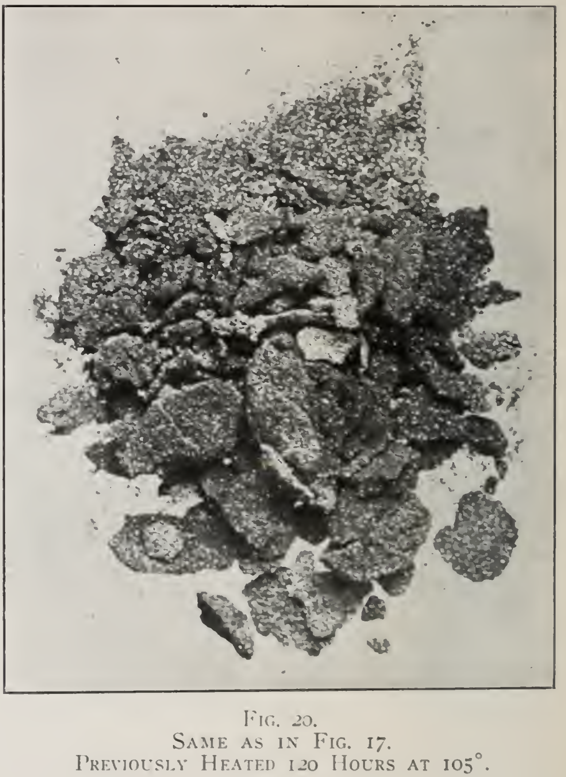

Fig. 20.
Same as in Fig. 17.
Previously Heated 120 Hours at 105°.

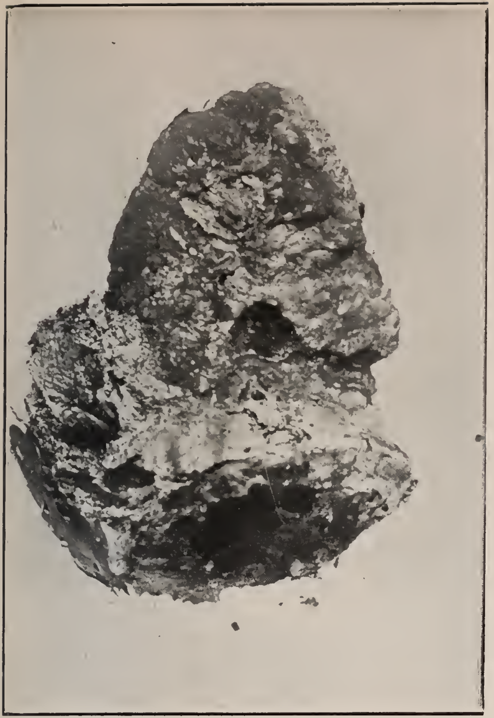

Fig. 21.
Coke from Soluble Extract, Vermilion County Coal. No Previous Heating of Extract.

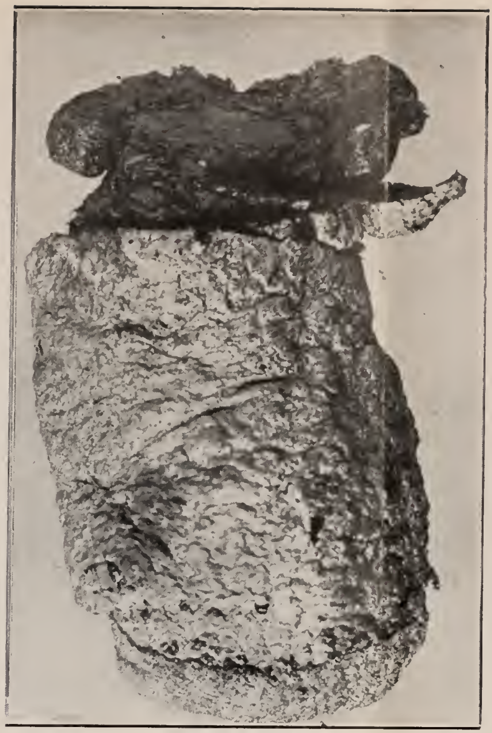

Fig. 22.
Same as in Fig. 21.
Previously Heated 50 Hours at 105°.

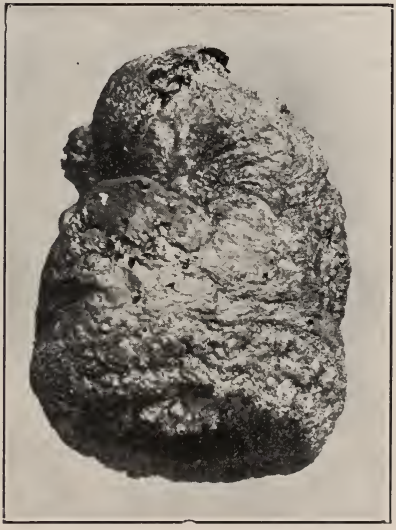

Fig. 23.
Same as in Fig. 21.
Previously Heated 122 Hours at 105°.

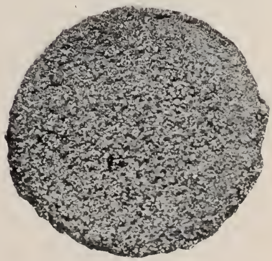

Fig. 24.
Same as in Fig. 21.
Previously Heated 216 Hours at 105°.

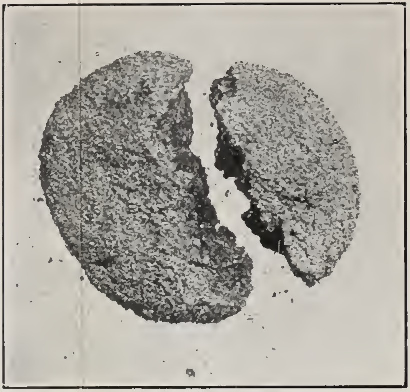

Fig. 25.
Coke from Insoluble Residue, Vermilion County Coal. No Previous Heating of Residue.

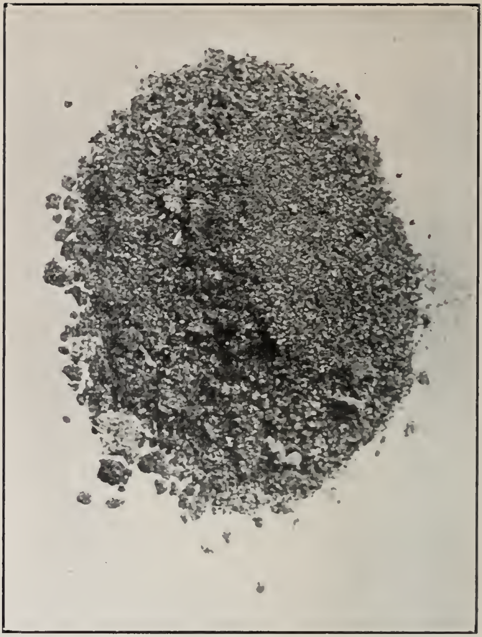

Fig. 26.
Same as in Fig. 25.
Previously Heated 50 Hours at 105°.

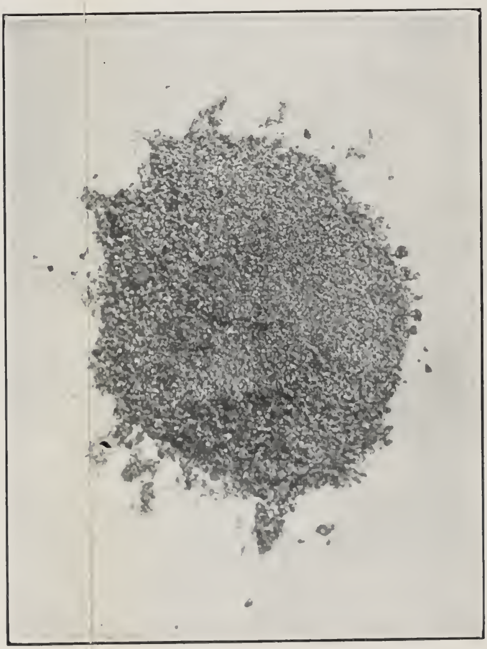

Fig. 27.
Same as in Fig. 25.
Previously Heated 122 Hours at 105°.

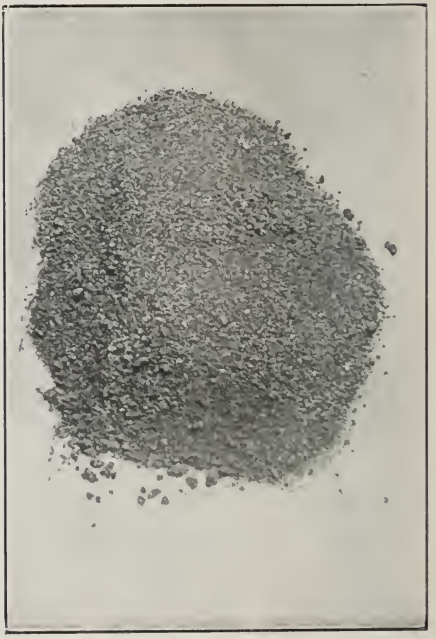

Fig. 28.
Same as in Fig. 25.
Previously Heated 216 Hours at 105°.

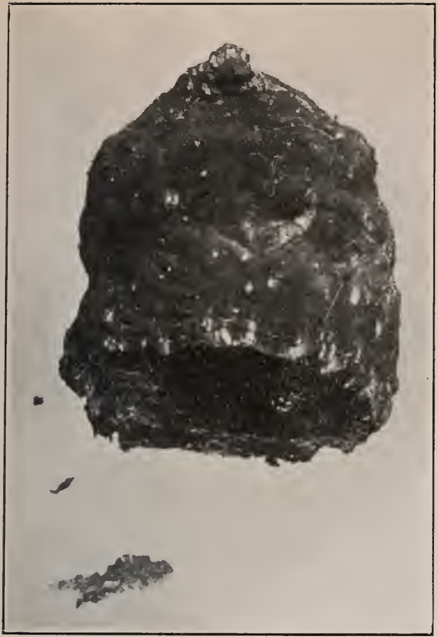

Fig. 29.
Coke from Soluble Extract, Williamson County Coal. No Previous Heating of Extract.

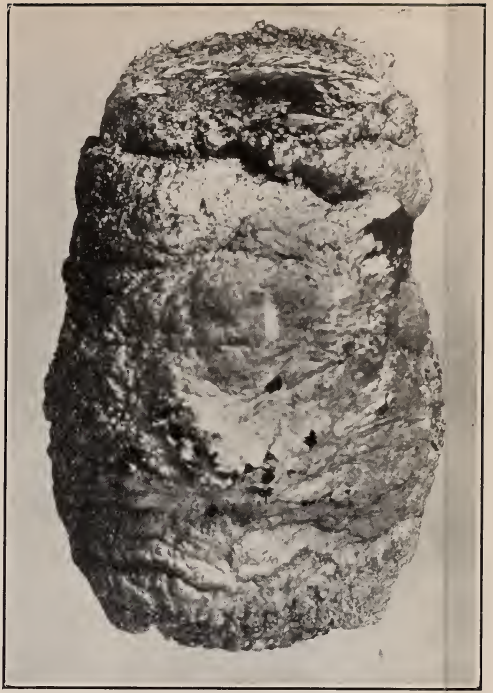

Fig. 30.
Same as in Fig. 29.
Previously Heated 90 Hours at 105°.

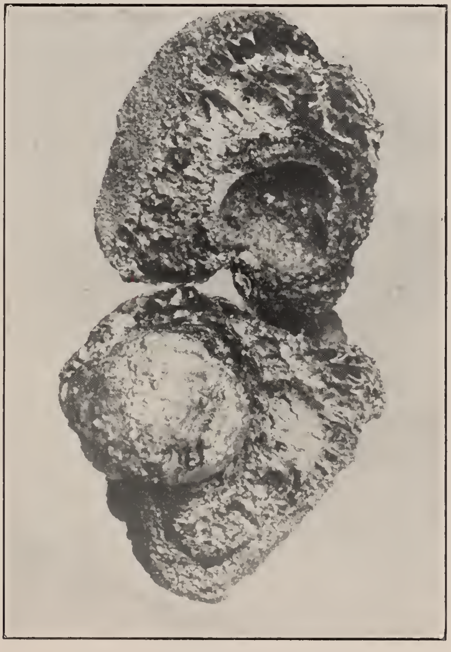

Fig. 31.
Same as in Fig. 29.
Previously Heated 242 Hours at 105°.

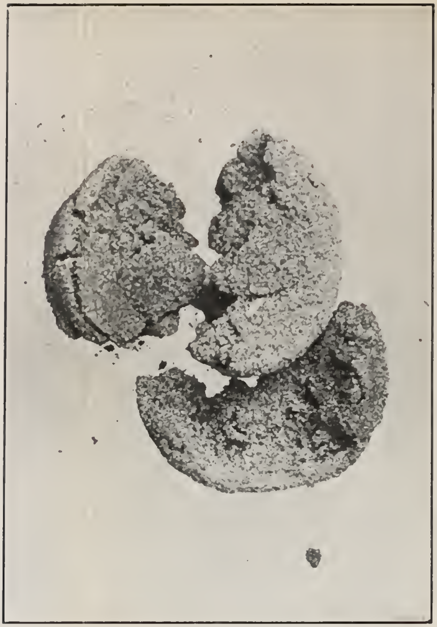

Fig. 32.
Coke from Insoluble Residue, Williamson County Coal. No Previous Heating of Residue.

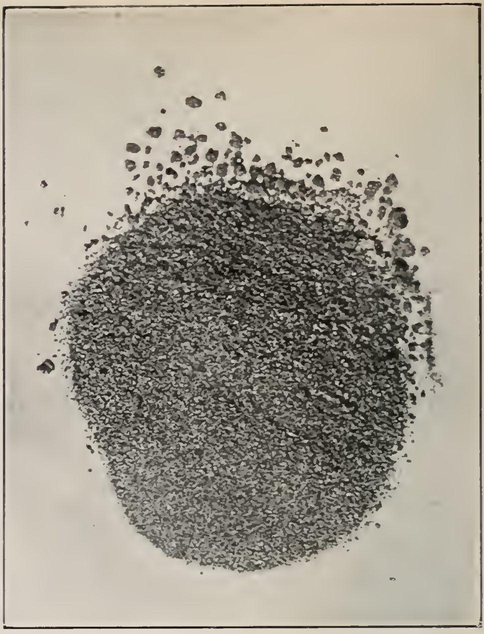

Fig. 33.
Same as in Fig. 32.
Previously Heated 90 Hours at 105°.

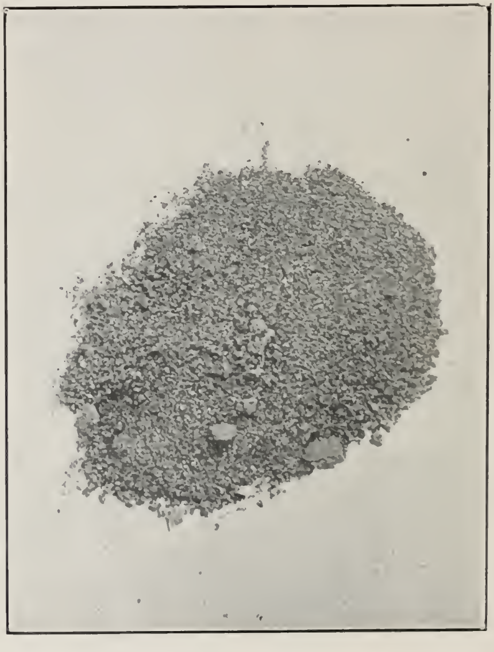

Fig. 34.
Same as in Fig. 32.
Previously Heated 242 Hours at 105°.

The work of Clark and Wheeler[1] on the extract and residue which was obtained by the extraction of coal with pyridine, showed a marked difference in the percentage composition of the gases which were produced by the destructive distillation of these substances. The methane and ethane in the gas from the extract were larger in amount than that obtained from the gas of the residue.

From their results it may be supposed that similar results might be obtained by the destructive distillation of the residue and extract obtained by extraction of coal with phenol. The above results as indicated in Tables 20 and 21 do not show an increase in the amount of methane in the gas from the extract but instead there was a decrease in this constituent and an increase in the hydrogen content.

It will be observed in Table 20 that the amount of carbon dioxide was high in all of the gases. This fact taken in conjunction with the fact that the Vermilion County coal had been in a finely ground condition for some time suggested that oxidation had taken place. This led to a consideration of that problem which will be treated under a subsequent heading.

The properties of the material which remained in the test tube after thermal decomposition, were very characteristic.

The coal gave a good sound coke of shiny appearance.

The residue gave a very poor coke. It could be easily broken and possessed a dull appearance.

The extract gave a very light and friable material. It had a very smooth surface after the glass was broken away. The surface was much brighter than that of the coke produced from the coal.

Photographs of the residues are shown in Figs. 7-12. Fig. 7 is of Vermilion County coal; Fig. 8, residue; and Fig. 9, extract. Fig. 10 is of the Williamson County coal; Fig. 11, residue; and Fig. 12, extract.

16. *The Effect of Oxidation on the Volatile Matter Determination.*—From some of the previous work it became evident that oxidation was playing an important role in producing changes in the residue and extract. It is a well-known fact that when coal is exposed it gradually loses its coking properties, and that this is due to the absorption of oxygen. It was desired to know if this was oxidation or simple absorption of oxygen. The first experiments were made on the coal itself. It was desired to see what effect heating at 105° would have on the percentage of volatile matter.

Coals from Vermilion County, and Williamson County were heated in air at 105° for different lengths of time and volatile matter determinations were made at successive stages of oxidation. The results on these two coals are given in Table 22.

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[1] J. Chem. Soc., 103, 1710 (1913).

TABLE 22.
SHOWING THE PERCENTAGE OF VOLATILE MATTER IN COAL AFTER CONTINUED HEATING AT 105°.

| Time after Heating. Hours | Vermilion County Coal | | Williamson County Coal | |
|---|---|---|---|---|
| | Volatile Matter in coal | Difference | Volatile Matter in coal | Difference |
| 0 | 44.36 | 0.00 | 37.85 | 0.00 |
| 26 | 42.39 | —1.97 | 39.22 | +0.37 |
| 50 | 41.78 | —2.58 | 36.75 | —1.10 |
| 120 | 40.51 | —3.85 | 35.91 | —1.94 |

These results show that the volatile matter decreased with progressive heating. The plus result which was obtained in the case of the Williamson County coal was in all probability a variable in the volatile matter determination. From the results of these two coals the decrease in volatile matter was more rapid in the case of the coal which had the higher percentage in the beginning. The material remaining in the crucible after the determinations indicated very plainly that the length of time of heating affected the coking properties as shown below.

The Vermilion County coal gave a very poor coke after 120 hours' heating. Figs. 13 to 16 show the material after the volatile matter determination. Fig. 13, coal without heating; Fig. 14, after heating 26 hours; Fig. 15, after heating 50 hours; Fig. 16, after heating 120 hours.

The Williamson County coal after heating 120 hours would not coke. Figs. 17 to 20 show the material after the volatile matter determination. Fig. 17, coal without heating; Fig. 18, after heating 26 hours; Fig. 19, after heating 50 hours; Fig. 20, after heating 120 hours.

The study of the effect of oxidation on the amount of volatile matter was extended to the residue and extract. The residue and extract from the Vermilion County coal were heated for various lengths of time and volatile matter determinations made. The results are given in Table 23.

TABLE 23.
SHOWING THE PERCENTAGE OF VOLATILE MATTER IN THE RESIDUE AND EXTRACT AFTER HEATING AT 105°, VERMILION COUNTY.

| Time of Heating. Hours | Volatile Matter in Residue | Difference | Volatile Matter in Extract | Difference |
|---|---|---|---|---|
| 0 | 43.53 | 0.00 | 52.79 | 0.00 |
| 50 | 42.75 | — 0.78 | 52.22 | —0.57 |
| 122 | 61.08 | +17.55 | 47.68 | —5.11 |
| 216 | 50.45 | + 7.92 | 48.23 | —4.56 |

On examining these results it will be noted that the percentage of volatile matter in the extract decreased with the length of time of heating. It thus acted similarly to coal. On the other hand, the residue increased in the amount of volatile matter. This increase may be accounted for by the fact that during the volatile matter determination there was an excess of sparking. The material acted similarly to lignite coal when an attempt is made to coke a finely ground sample. There was evidence of considerable mechanical loss. The greater avidity of the residue for oxygen would also tend to increase the indicated amount of volatile matter by reason of the ultimate formation of $CO_2$ from the oxygen absorbed.

The material which remained after the volatile matter determination was of special interest. Figs. 21 to 24 show the extract after the volatile matter determination; Fig. 21, extract without previous heating; Fig. 22, after heating 50 hours; Fig. 23, after heating 122 hours; Fig. 24, after heating 216 hours. Heating the extract for 50 hours yields a more voluminous material by the volatile matter determination than is produced by the extract without heating. After heating 216 hours the extract had lost all tendency to swell but it still retained a slight coking tendency, i. e., the material did not fall to pieces.

Figs. 25 to 28 relate to the same source of material from Vermilion County, and show the material that remained after the volatile matter determination was made on the residue; Fig. 25 is the coke from residue without previous heating; Fig. 26, after heating 50 hours; Fig. 27, after heating 122 hours; Fig. 28, after heating 216 hours. If a volatile matter determination is made on the residue previous to heating a material is produced that will just adhere. After heating, the residue loses what little coking ability it originally possessed.

The residue and extract from Williamson County coal were also subjected to prolonged heating at 105°. The results are presented in Table 24.

TABLE 24.

SHOWING THE PERCENTAGE OF VOLATILE MATTER IN THE RESIDUE AND EXTRACT AFTER HEATING AT 105°, WILLIAMSON COUNTY COAL.

| Time after Heating. Hours | Volatile Matter in Residue | Difference | Volatile Matter in Extract | Difference |
|---|---|---|---|---|
| 0 | 37.44 | 0.00 | 47.21 | 0.00 |
| 90 | 56.40 | +18.96 | 44.19 | —3.02 |
| 242 | 67.33 | +29.89 | 42.68 | —4.53 |

A discussion of these results on Williamson County coal will not be taken up for they are identical in character with those obtained from the Vermilion County coal.

Figs. 29 to 31 show the material which remained after the volatile matter determination of the extract from Williamson County coal. Fig. 29, extract without heating; Fig. 30, after heating 90 hours; Fig. 31, after heating 242 hours.

Figs. 32 to 34 show the material which remained after the volatile matter determination was made on the residue from Williamson County coal; Fig. 32, residue without heating; Fig. 33, after heating 90 hours; Fig. 34, after heating 242 hours.

Reviewing this phase of the study it is seen that Figs. 7 to 34 inclusive, illustrate the effect of oxidation such as may occur upon simple exposure to the air at room temperature and at 105°, and for varying lengths of time. The temperature employed for the destructive distillation was approximately 800°. Before applying this temperature the samples were subjected to a vacuum for the purpose of removing the surrounding oxygen as well as that which might have been occluded by the sample. The effect upon the insoluble residues of long exposure to a temperature of 105° is hardly noticeable so far as modifying the coking property of that material is concerned, for the reason that this substance, even without exposure to oxidation, is practically devoid of any tendency to coke when highly heated. Quite the contrary is true in the case of the soluble extract. Its tendency to coke and so to ultimately produce coke is marked, but after long exposure to oxidizing conditions this property is practically lost. This feature also characterizes the coal, the unseparated extractive material being sufficiently pronounced in its behavior to govern the effect of oxidation on the unheated coal.

17. *The Effect of Oxidation on the Amount of Material Extracted from Coal by Phenol.*—The further effect of oxidation was noticed in a number of different ways. For example, the amount of material which could be extracted from coal depended to a certain degree on the extent of oxidation.

Williamson County coal was heated for five days at 105° in an atmosphere of oxygen. The amount of material extracted from this coal is shown in Table 25.

TABLE 25.

SHOWING THE EFFECT OF OXIDATION ON THE AMOUNT OF RESIDUE AND EXTRACT IN COAL, WILLIAMSON COUNTY.

| | Coal not previously heated | Coal heated before extracting | Change |
|---|---|---|---|
| Weight of residue | 3.777 | 4.102 | +.325 |
| Weight of extract | 1.484 | 1.094 | —.390 |

A finely ground sample of Vermilion County coal was kept in a closed jar for four months. This coal was extracted with phenol and the amount of residue and extract determined. The coal was also heated at 105° for varying lengths of time in oxygen and subsequently extracted with phenol. The results of these experiments are shown in Table 26.

TABLE 26.

SHOWING THE EFFECT OF OXIDATION ON THE AMOUNT OF RESIDUE AND EXTRACT IN COAL.

| Condition of Coal | Weight of Residue | Increase in Weight of Residue | Weight of Extract | Loss in Weight of Extract |
|---|---|---|---|---|
| Fresh | 3.236 | 0.000 | 1.826 | 0.000 |
| Standing 4 mo. | 3.494 | + .258 | 1.601 | — .225 |
| Heated 3 days | 3.821 | + .585 | 1.325 | — .501 |
| Heated 6 days | 3.860 | + .624 | 1.259 | — .567 |
| Heated 10 days | 3.802 | + .566 | 1.242 | — .584 |
| Heated 15 days | 3.795 | + .559 | 1.353 | — .473 |

From the results in Tables 25 and 26 it is seen that the oxidation of coal increases the amount of residue insoluble in phenol, and that the decreased solubility becomes very marked when the coal is completely oxidized. Also it should be noted that in the sample of coal which was preserved in the sample bottle there was a decrease in the amount of soluble material. Thus the coal was being slowly oxidized although it was not directly exposed to the air. The fact that coal is so readily oxidized should be kept in mind in any work on coal and especially in work of the present nature.

18. *The Effect of Oxidation on the Composition of the Gases Produced by Destructive Distillation.*—During the work on the destructive distillation of coal as noted on page 28 one gas was produced which was high in the amount of carbon dioxide. This result, when considered in connection with the fact that the sample of coal was not

fresh, and in the light of the preceding experiment, would indicate that oxidation had taken place. It was desired to see what effect oxidation of the extract and residue would produce in the composition of the gases when destructively distilled. In order to have a sample of coal which was not already partially oxidized, a fresh sample of Vermilion County coal was secured. The residue and extract from this coal were destructively distilled and the gases analyzed. The residue and extract were also heated at 105° for 24 hours and then destructively distilled. The results from the analyses are as follows:

Table 27.

Showing the Composition of Gases Produced by Destructive Distillation of Residue and Extract, Vermilion County Coal.

| Gas | From Residue not oxidized | From Residue oxidized by heating | From Extract not oxidized | From Extract oxidized by heating |
|---|---|---|---|---|
| Carbon dioxide | 5.2 | 8.7 | 6.8 | 10.0 |
| *Ethylene | 4.8 | 4.6 | 5.1 | 4.2 |
| Oxygen | .3 | .3 | .2 | .4 |
| Carbon monoxide | 15.1 | 15.2 | 11.9 | 19.0 |
| Hydrogen | 44.1 | 43.0 | 40.5 | 44.8 |
| Methane | 26.2 | 24.0 | 24.0 | 17.4 |
| **Ethane | 3.6 | 3.3 | 7.3 | 3.2 |
| Nitrogen | .7 | .9 | 4.2 | 1.0 |

The residue and extract from the old sample of Vermilion County coal were heated for 48 hours at 105° in the air. They were then destructively distilled and the gases analyzed. For purposes of comparison some of the residue and extract which had not been heated were destructively distilled and the gases analyzed. The results are presented in Table 28.

Table 28.

Showing the Composition of Gases Produced by Destructive Distillation of Residue and Extract, Vermilion County Coal.

| Gas | From Residue oxidized by standing only | From Residue oxidized by heating | From Extract oxidized by standing only | From Extract oxidized by heating |
|---|---|---|---|---|
| Carbon dioxide | 8.3 | 14.4 | 10.1 | 12.4 |
| *Ethylene | 5.2 | 3.5 | 4.0 | 3.2 |
| Oxygen | .9 | .1 | .4 | .3 |
| Carbon monoxide | 14.2 | 18.5 | 21.0 | 14.6 |
| Hydrogen | 39.8 | 40.5 | 40.8 | 50.0 |
| Methane | 23.1 | 20.4 | 16.3 | 15.6 |
| **Ethane | 3.4 | 1.4 | 2.8 | 2.5 |
| Nitrogen | 5.1 | 1.2 | 4.6 | 1.4 |

---

* Including higher homologues and benzene.

** Including higher homologues calculated as ethane.

Extracts from the Vermilion County coal and also from Williamson County were heated for a much longer time and then destructively distilled. The extract from the Vermilion County coal was heated for 216 hours at 105° and that from Williamson County for 264 hours. The results are given in Table 29.

### TABLE 29.
### SHOWING THE COMPOSITION OF GASES PRODUCED BY THE DESTRUCTIVE DISTILLATION OF EXTRACT.

| Gas | From Vermilion County Extract after being heated for 216 hours. | From Williamson County. Extract after being heated for 264 hours. |
|---|---|---|
| Carbon dioxide | 9.1 | 7.9 |
| Ethylene | 3.7 | 3.8 |
| Oxygen | .3 | .4 |
| Carbon monoxide | 17.9 | 16.1 |
| Hydrogen | 44.2 | 47.5 |
| Methane | 19.9 | 20.9 |
| Ethane | 3.1 | 2.1 |
| Nitrogen | 1.8 | 1.3 |

On examining the data in Tables 27 and 28 it is to be noted that the amount of carbon dioxide in the gas is higher after the material has been heated. Also, the percentage of carbon monoxide increased in every case except one. The fact that the percentage of methane and ethane in the gas from the material that had been heated was reduced in every case, should also be noted. The above results seem to indicate that the oxygen taken up during the heating at 110° was not held in an absorbed condition but that it had entered into chemical combination with the material. Furthermore the apparatus was exhausted to 15-20 mm. before the destructive distillation was started, and when the temperature was raised, any oxygen which was merely occluded would be given off as oxygen before a temperature was reached at which active oxidation with the formation of $CO_2$ would take place. As a result of these experiments it may be stated that upon exposure to oxidizing conditions oxygen enters into chemical combination with both residue and extract.

The results in Table 29 seem to show that the oxidation may reach a point of saturation so far as the ability of the molecular structure is concerned and that beyond this point, if the oxidizing conditions are continued, a dissipation of the oxygen compounds may occur, presumably being discharged as $H_2O$. However, further proof of this reaction after such long continued periods of heating should be made.

## III. Summary and Conclusions.

(1) A study of solvents for the purpose of subdividing coal constituents without decomposition was carried out. Phenol was adopted as the most active solvent for the purpose of this investigation.

(2) An apparatus was devised for carrying out the extraction so that the temperature of the solvent would be above 110°.

(3) The application of the method to different types of coal showed that coal varied widely in the amount of material dissolved by phenol. The amount of soluble material seemed to differentiate sharply between subdivisions of coal types. On the ash and moisture free basis the high volatile coals of Vermilion County give 35-40 per cent of soluble material; the coals from Madison and Montgomery Counties give 30-35 per cent and the low volatile coals of Williamson County show 20-30 per cent of soluble material.

(4) Extraction of coal leaves a residue which will not coke. The coking constituent of the coal is in the extract.

(5) Residue and extract oxidize at room temperatures and more readily at 100°, the rate of oxidation being more rapid in the residue.

(6) Residue and extract possess an avidity for water as well as oxygen at ordinary temperatures. The residue shows the greater avidity in both cases.

(7) Volatile matter determinations show that the extract contains more volatile matter than the residue.

(8) The ultimate analysis of the coal, residue, and extract shows that the percentage composition of carbon, hydrogen, nitrogen, and oxygen is substantially the same.

(9) Destructive distillation of coal residue and extract gave gases of practically the same composition.

(10) Oxidation of coal, residue, or extract produces a lowering in the percentage of volatile matter.

(11) Oxidation decreases the amount of material which may be extracted from coal by phenol and the coking properties are decreased in proportion to the extent of the oxidation.

(12) Oxidation of coal, residue, and extract is shown by an increase in the percentage of carbon dioxide in the gases produced by thermal decomposition. It is concluded, therefore, that the oxygen absorbed is chemically held.

## IV. Historical.

Professor Bedson[1] refers to work which was done by Dr. Smythe on coal at Göttingen in 1851. This work was published in a report printed for the Commissioners of the 1851 Exhibition. A brown coal from Brühl, near Cologne, was used in the work. The following solvents were used in the extraction in the order named: benzene, chloroform, alcohol, ethyl ether, petroleum ether, and acetone. Benzene extracted 3 per cent of the coal, chloroform 1.8 per cent, and alcohol 2.4 per cent. The other extracts were very small. Attempts were made to obtain pure substances from these extracts by the ordinary methods of purification used in organic chemistry.

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[1] J. Soc. Chem. Ind., 27, 149 (1908).

Reinsch[1] in 1885 supposed that coal was composed of two substances, which could be distinguished by their action towards alkaline solution. With an alkaline solution he was able to isolate substances which were very characteristic in not being attacked by mineral acids.

Baker[2] in 1901 examined the solvent action of pyridine on certain classes of coals. The extraction was carried out for 50 hours in a Soxhlet extractor. Anthracite coal from South Wales gave very little material soluble in pyridine. A bituminous coal from Durham was soluble in pyridine to the extent of 20.4 per cent. The pyridine extracts were dark brown in color and showed fluorescence in some cases. An ultimate analysis did not show a concordant change in the proportions of elements present in the coal, residue, and extract.

Andersen and Henderson[3] in 1902 extracted Bengal and Japan coal with pyridine. They selected samples from Boiaker (Bengal), Paronai and Yubari (Japan) and also, for comparison two Scotch coals: Linrigg Lower Dumgray and Bannockburn Main. They did not give the amounts which were dissolved. The pyridine extract, after the removal of the solvent, possessed a black lustrous appearance similar to bitumen. The extract from all of the coals was similar in character. Ultimate analyses of the extract were given and the results showed that there was little difference in the chemical constitution of the coals of Bengal and Japan, from those of Scotland. They stated that pyridine was the best solvent that had been used. The percentage of carbon, hydrogen, and nitrogen in the extract was about the same as that in the original coal. The coking properties of a poor coal could be entirely removed but that it was only partially removed in a strong coking coal.

Professor Bedson[4] in 1908 worked upon the pyridine extract from certain gas coals. The coals varied from 22 to 33 per cent of pyridine extract. Also, four Cannel coals were extracted giving from 7 to 29 per cent of extract. Bedson called attention to the fact that in the first six coals the percentage of volatile matter is not far removed from the percentage of pyridine extract. After commenting upon this fact he decided that it would be dangerous to base any definite conclusions upon this fact alone. Although those coals were "gas" coals and similar in character, frequently substances identical in chemical constitution, give marked differences in physical properties. So that in coal work any problem may not be considered from the purely statistical standpoint alone.

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[1] Dingl. Poly. J., 256, 224. (Chem. Soc. A, 48, 876; 1885).

[2] Trans. North Engl. Inst. Mining and Mech. Eng., 50, [2], 23 (1901). (J. Soc. Chem. Ind., 20, 789; 1901).

[3] J. Soc. Chem. Ind., 21, 242 (1902).

[4] J. Soc. Chem. Ind., 27, 147 (1908).

Through Bedson, Blair[1] became interested in the proximate constituents of coal. Blair extracted coal from the Busley Seam of the Botley Colliery. After freeing the extract from pyridine, it was extracted with solvents in the following order: petroleum ether, ethyl ether, absolute alcohol and chloroform. Only a scanty description of these compounds was given. A complete account of the results was reserved for another communication.

Professor Lewes[2] in 1912, in a series of lectures on coal carbonization has reviewed some of the work upon the extraction of coal with pyridine. He himself had done some work but the results with the exception of one analysis were not given. He states that in all probability the coking property of a poor coking coal can be entirely removed by pyridine and that such will not be the case with a strong coking coal, for the reason that in the latter case some of the resinic bodies resist the solvent action of pyridine. Lewes in his criticism of pyridine as a solvent, points out the following facts as objections to it. He has worked on some coals that gave a higher percentage of volatile matter after extraction than they did before. He gives some results of Andersen and Henderson and claims that they show the same property. However, the results which he compared were not apparently considered by the authors as comparable. Professor Lewes states that the increase in volatile matter in the residue over the original volatile matter in the coal can only be taken to mean that pyridine has formed an additional product in some way. However, Clark and Wheeler, as well as Wahl, in work to be mentioned later, found that the residue did not increase as to the percentage of volatile matter. It was noted that in some cases when an ultimate analysis was made of the extract, the amount of nitrogen was greater than it was in the coal, in spite of the fact that the extract had been carefully washed free from pyridine. Furthermore, the two most successful solvents, according to Lewes, were aniline and pyridine,—both organic bases. The same effect is produced upon coal by these solvents as with NaOH, i. e., a non-coking residue remains. Sodium hydroxide is known to saponify resinic substances, and it is probable that pyridine and aniline form a compound with some of the resin, which is soluble in an excess of the solvent. Of course, this would be shown in the high nitrogen content of the extract.

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[1] J. Soc. Chem. Ind., 27, 150 (1908).

[2] Progressive Age, 29, 1030 (1911).

Lewes gives the following conclusions: "These considerations seem to make it clear that the resin constituents condition the coking of coal during destructive distillation, and that they are of at least two kinds—the one easily oxidizable, soluble in pyridine and saponifiable by alkalies, and which on weathering is oxidized into a humus body with the evolution of water and carbon dioxide, and is responsible for the heating of coal in storage; the other class non-oxidizable, not saponified by alkalies, and forming with pyridine a compound insoluble in excess of the reagent, and this class may be the hydrocarbons from decomposed resins, as the residue in which they are present yields rich liquid hydrocarbons, as tar and pitch, but not rich in gas".

A. Wahl[1] in 1912, extracted a number of coals with pyridine. Volatile matter determinations were made before and after the extraction. The percentage of volatile matter in the coal was from 13 to 55 per cent and after extraction, the volatile matter in the residue was slightly lower, in some cases as much as 5 per cent. The amount of material extracted was from 6 to 26 per cent. The coke which was produced from the residue was harder and more compact than that of the coal. The material dissolved by the pyridine was an amorphous brown solid, which produced a voluminous coke. The ultimate analyses of coal and pyridine extract were given in some cases, but these showed scarcely any difference in composition.

In 1912, Frazer and Hoffman[2] published a paper on "The Constituents of Coal Soluble in Phenol". They tried the effect of a number of reagents and organic solvents on coal and found that pyridine, aniline and phenol removed the largest amount of soluble material. The published report was concerned with those constituents of coal soluble in phenol. Illinois coal from Franklin County was used in the extraction. The amount extracted was 10.87 per cent, calculated on a moisture and ash free basis. After freeing the extract from phenol by distillation an analysis gave ash 2.16 per cent. In the light of the present investigation this amount of ash in the extract seems to be abnormally high. By the use of sodium hydroxide and organic solvents such as ether, acetic acid, methyl alcohol, acetone, benzene and petroleum ether, they were able to separate the extract into certain

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1. Comtes rend., 154, 1094 (1912).
2. Technical Paper 5, Bureau of Mines.

portions, which when distilled in vacuo, gave varying physical properties. Analyses of these substances were made. The authors concluded that in the absence of evidence to the contrary coal substance soluble in phenol was present as such in the coal itself. They also believed that some of the substances isolated were approximately pure compounds, but this could not be proven absolutely on account of the small amount of material which they had at their disposal. They intend to follow the investigation using pyridine as a solvent.

Pictet and Ramseyer,[1] in 1913, extracted large quantities of washed coal from Montrambert (Loire) with benzene. From 248 kg. coal, almost 244 grams of almost black, quite fluid oil were obtained. They distilled this at various temperatures and pressures and worked on the different fractions. From their studies they think that coal contains among other constituents polymerized hydroaromatic hydrocarbons.

In 1913, Clark and Wheeler[2] in a paper on the "Volatile Constituents of Coal" described some of their work on the extraction of coal with phenol. They contend that coal is composed of two different types of bodies, which possess different degrees of ease of decomposition. They called the two constituents, the "hydrogen-yielding" and the "paraffin-yielding", respectively. However, they did not mean to say that above a certain temperature, one type of body decomposes and that below that temperature it will not decompose. From their experiments on coal they concluded that because of an increase in the amount of hydrogen evolved between 750° and 800°, that there was a hydrogen-yielding constituent in coal. They did not mean to designate by a hydrogen-yielding constituent one which gave all hydrogen, or by a paraffin-yielding constituent one that gave all paraffin, but one which gave a predominance of hydrogen or paraffin. It was also shown by them that the temperature of the destructive distillation influenced to a large extent the kind of gas given off. Clark and Wheeler extracted coal with pyridine and then the pyridine extract was itself extracted with chloroform. It was noted from the ultimate analysis that the increase in the amount of nitrogen in the residue and extract was 2.75 per cent, showing that nitrogen was being added from some source. Data were given for the analysis of gases which were produced by the destructive distillation of the pyridine extract, pyridine residue, chloroform extract and chloroform residue, and for dehydrated cellulose. The gas which was produced from the pyridine extract contained a larger amount of methane than the gas from the residue. The results

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1. Arch. sci. phys. nat., 34, 234 (Chem. Abs. 7, 1496; 1913).
2. J. Chem. Soc., 103, 1704 (1913).

in the case of the present investigation did not show this difference. From the results obtained they did not hesitate to identify the pyridine residue as a degradation product of cellulose but they were doubtful if all of the pyridine extract was resinous in character. However, by extracting the pyridine soluble material with chloroform, they thought they were able to get out material consisting almost entirely of resinous matter. Detailed methods of extraction and of destructive distillation were given.

42

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